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JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto." -BACON.

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On the Value of Selection amongst Assured Lives, and its effect upon the Adjustment of a Scale of Premiums, as between Persons Assuring at Different Ages. By JOHN ADAMS HIGHAM, Esq., *Fellow, and one of the Members of the Council, of the Institute of Actuaries of Great Britain and Ireland.*

This paper was read before the Institute of Actuaries on 25 March 1850, and was printed by order of the Council. As probably very few of our readers have seen the paper, and it has been long out of print, we believe we shall be rendering them a service by reprinting the paper at a time when attention has been again directed to the effect of selection.—ED. *J. I. A.*

I CANNOT better introduce the subject of this paper than by a quotation from Mr. Morgan's Preface to his Tables of the Experience of the Equitable Society. He writes as follows:—

"In a body of lives of the same age, all selected as healthy from the general mass of mankind, it is obvious that the rate of mortality must be considerably less for the first ten or twenty years after selection, than amongst those from whom they are thus chosen. As, however, these selected lives advance in age, their general health and the rate of mortality amongst them will naturally approximate to the common standard. This approximation cannot be accurately estimated, if the observations be taken from a blended mass of

lives, selected at the age of twenty, for instance, which have attained the age of fifty, and of lives selected at the age of fifty. The rate of mortality will for the former class be less, for the latter greater, than that which occurs in either class separately.

"The correct method, therefore, (if sufficient data existed), would unquestionably be to make distinct tables from the mortality of each distinct class. The numbers of each separate class in the tables now presented are insufficient for that purpose, and there is a variance observable in the rates of mortality at different ages when estimated in this manner, which renders it scarcely possible to graduate a table from them."

It would appear that the opinion here expressed, of the insufficiency of the existing materials for the required purpose, has been shared by subsequent writers; for although several (and particularly Mr. Farren) have shown in a very interesting manner the extent to which the rate of mortality among a body of assured lives increases with the period which has elapsed since their selection, independently of the increase resulting from their growing age; I am not aware that any one has computed the values of annuities on lives selected at particular ages and kept apart from others during their subsequent existence; or has added to the six values of the expectation of life, similarly determined, which Mr. Morgan has given in his book.

The Committee of Actuaries of 1838, did indeed append to their Tables, printed in 1843, a table described as "showing the expectations of life for persons admitted at particular ages in the Equitable Society"; but inasmuch as the whole of the lives admitted between the ages of twenty-five and sixty-five, are divided into but four sets, each extending over ten years of age,—during which period, the effect of selection must have been well-nigh exhausted as respects the youngest lives in each set;—the classification is not sufficiently minute for practical purposes.

In computing the tables which I have now the honour to lay before the Institute, and which show the expectation of life, and the value of an annuity at 3 per-cent, for every fifth year of age from twenty to sixty-five; I have endeavoured to obtain more exact results than those published by the Committee of Actuaries, and at the same time to avoid the irregularities to which Mr. Morgan adverts, by determining the probabilities of life from the data supplied by persons who entered the Equitable Society at the age under observation, and at the ages one year younger and one year older. For instance, the value of an annuity on a life selected at

the age of forty, is calculated from the experience of the Society in respect of persons who entered at the ages of thirty-nine, forty, and forty-one.

The results are still somewhat irregular, but I think they will be found sufficient to determine a question of considerable importance, by pointing to the general table of mortality, which is more suitable than any other in common use for the calculation of the premiums to be charged for life assurance.

The form in which the calculations have been made is more convenient than any which I recollect to have met with elsewhere. By the use of the columnar method, of Mr. Peter Gray's new and useful Tables, and of Professor De Morgan's card of four-figure logarithms, a great degree of rapidity and an almost complete system of check have been attained.* The conversion of the natural numbers into logs. and of logs. into natural numbers, is the only part of the process which it is necessary to perform in duplicate. The same method could of course be followed with a more extended table of logarithms, but it was sufficient for my purpose to use one which would give results to two places of decimals. I have not attempted to adjust the irregularities which appear in the columns, except where it was necessary to do so in order to avoid positive absurdities. A comparison of the expectations of life, given on the 7th page of Mr. Morgan's Preface, with those given on the 28th page of his book, will show how materially the character of a table is sometimes altered under the process of adjustment, even when applied by the most skilful hands.†

The following short tables contain a summary of the results which I have obtained, compared with the corresponding values taken from five of the tables most frequently employed by actuaries.

* The Council have been deterred from printing the very judiciously constructed tables appended to this paper, in consideration of the expense it would involve. They trust that the omission will be less felt inasmuch as the *results* are all given in the following pages.

† For example:—

Age.	Unadjusted Expectation.	Adjusted Expectation.	Difference.
20	41·77	41·67	·10
40	27·55	27·40	·15
60	14·11	13·91	·20
80	5·51	4·75	·76

Expectation of Life. Deduced from the Mortality of Particular Classes.

Age.	886 Persons assured at ages 19, 20, 21.	1779 Persons assured at ages 24, 25, 26.	2330 Persons assured at ages 29, 30, 31.	2221 Persons assured at ages 34, 35, 36.	1844 Persons assured at ages 39, 40, 41.	1402 Persons assured at ages 44, 45, 46.	1008 Persons assured at ages 49, 50, 51.	638 Persons assured at ages 54, 55, 56.	594 Persons assured at ages 58, 59, 60, 61, 62.	250 Persons assured at ages 63, 64, 65, 66, 67.
20	38·31									
25	34·72	35·69								
30	31·12	31·95	32·68							
35	27·64	28·61	28·78	30·06						
40	25·20	24·96	25·22	26·57	27·27					
45	22·17	21·51	21·66	23·01	23·64	23·93				
50	18·35	18·20	18·25	19·49	20·00	20·17	20·28			
55	14·96	15·55	15·21	16·28	16·71	16·83	17·01	17·40		
60	11·73	13·18	12·55	13·55	13·51	13·77	14·00	14·29	14·85	
65		11·13	10·76	11·17	10·80	10·90	11·19	11·36	11·59	11·74
70			(9·50)	8·93	8·55	8·42	9·03	9·32	9·16	8·92
75			7·02	7·02	7·02	7·02	7·02	7·02	7·02	7·02

Expectation of Life.

Age.	Selected Lives (Equitable Society).	Δ	General Table (Equitable Society).	Mr. Davies's Equitable Table.	Carlisle (Mr. Milne).	Experience of Life Offices (Com. of Actuaries).	English Life Table, Males (Mr. Farr).
20	38·31	−3·46	41·77	41·06	41·46	41·49	39·88
25	35·69	−2·57	38·26	37·44	37·86	37·98	36·48
30	32·68	−1·92	34·60	33·98	34·34	34·43	33·13
35	30·06	− ·77	30·83	30·66	31·00	30·87	29·84
40	27·27	− ·10	27·37	27·40	27·61	27·28	26·57
45	23·93	+ ·11	23·82	24·10	24·46	23·69	23·31
50	20·28	− ·02	20·30	20·83	21·11	20·18	20·03
55	17·40	+ ·30	17·10	17·85	17·58	16·86	16·68
60	14·85	+ ·78	14·07	15·06	14·34	13·77	13·59
65	11·74	+ ·50	11·24	12·35	11·79	10·97	10·87
	(1)		(2)	(3)	(4)	(5)	(6)

Value of an Annuity at Three per-cent.

Age.	Selected Lives (Equitable Society).		General Table (Equitable Society).		Mr. Davies's Equitable Table.	
	Value.	Proportion.	Value.	Proportion.	Value.	Proportion.
20	20·88		21·87		21·53	
25	20·09	1·000	20·93	1·000	20·47	1·000
30	19·18	·953	19·80	·945	19·37	·945
35	18·17	·904	18·44	·881	18·23	·890
40	17·13	·852	17·11	·817	16·99	·829
45	15·69	·780	15·56	·743	15·58	·760
50	13·82	·687	13·82	·660	14·02	·685
55	12·34	·614	12·12	·579	12·49	·610
60	10·89	·541	10·35	·494	10·94	·534
65	8·89	·442	8·51	·406	9·29	·453

Value of an Annuity at Three per-cent.—(continued).

Age.	Carlisle (Mr. Milne).		Experience of Life Offices (Mr. Jenkin Jones).		English Life Table (Mr. Farr).	
	Value.	Proportion.	Value.	Proportion.	Value.	Proportion.
20	21·69		21·80		21·18	
25	20·67	1·000	20·84	1·000	20·19	1·000
30	19·57	·945	19·75	·946	19·13	·947
35	18·43	·890	18·52	·888	17·98	·889
40	17·14	·829	17·12	·821	16·72	·827
45	15·86	·766	15·54	·746	15·32	·759
50	14·30	·691	13·82	·663	13·74	·680
55	12·41	·600	12·02	·577	11·91	·589
60	10·49	·507	10·19	·489	10·06	·497
65	8·92	·430	8·38	·400	8·29	·410

The latter table shows a close agreement, as regards the proportions which the terms bear to each other, between the values of annuities taken from Mr. Davies's tables and those computed from lives selected at particular ages and kept apart from others during their subsequent existence. If we only ascribe to the principle of selection a somewhat increased force, we shall have a table almost identical with that of Mr. Davies; and that the lives proposed to assurance companies are now selected with much better effect than formerly I have no doubt, considering the increased pains which are taken for that purpose.

A comparison of columns two and five will show a still more marked resemblance between the two great tables of mortality founded on the experience of assurance companies. These neither of them take into account the effect of selection, and consequently the premiums calculated from them favour young lives at the expense of old ones. For a young man's chance of life is exaggerated in the tables, by the adventitious circumstance that an infusion of new blood takes place at every stage, for the greater part of the future lifetime through which he is supposed to pass. On the other hand, a man assuring at a mature period of life finds his expectation underrated by his being classed with the mixed body who have lived over from the earlier ages.

From what has been stated I conclude that Mr. Davies's table is more suitable than any other in common use, for the calculation of life premiums; and with this assumption I proceed to consider the method of fairly adjusting a scale of charges, as between persons assuring at different ages.

Here are three things first to be settled.

1. The rate of interest.
2. The addition to cover expenses.
3. The addition to afford profit.

1. Under the first head I follow the dictum of Mr. De Morgan, namely: that "no office would be justified in assuming more than 3 per-cent with tables that come any ways near to the actual experience of mortality". (Probabilities, p. 261.)

2. With regard to the office expenses I follow the same authority, but with a slight modification. His opinion is, that "the yearly contribution of every member to this fund ought to be the same". (Probabilities, p. 271.) But this cannot be carried out exactly —to tax all policies alike would be to prohibit small assurances. I think the yearly contribution of every member to the fund for expenses ought to be the same percentage on the amount of his assurance. That is to say, the addition for expenses ought to be a constant addition to the *Annual* Premium. I think that five shillings per-cent per annum upon the sum assured would be about the amount required by an office of average magnitude.

3. The addition to afford profit.

It appears to me that the present value of all future profit to be derived on a policy should be the same, whatever the period of life at which it be effected. If I hold property on two lives, one old and the other young, and propose to assure only one, it ought to be a matter of indifference to the company which I select. It follows that the addition for profit should be a constant addition to the *Single* Premium*.

I subjoin two tables of premiums constructed on these principles, the one calculated from the annuities on selected lives, the other from Mr. Davies's values of annuities. I have taken as a standard the Northampton 3 per-cent rate for age sixty. It will be borne in mind that my object is to determine the proportions which should exist between the rates at different ages, and not to consider what should be their absolute amount.

* I am aware that both these modes of addition have been employed by insurance companies, although I do not know that any have combined them.

Annual Premium per-cent.

Age.	Difference.	Annuities on Selected Lives.	Northampton 3 per-cent Premiums.	Davies's Annuities.	Difference.
	s. *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	*s.* *d.*
20	+1 4	2 4 11	2 3 7	2 2 5	−1 2
25	+ 6	2 8 7	2 8 1	2 7 2	− 11
30	− 3	2 13 1	2 13 4	2 12 8	− 8
35	−1 1	2 18 9	2 19 10	2 19 0	− 10
40	−2 9	3 5 2	3 7 11	3 6 8	−1 3
45	−2 6	3 15 5	3 17 11	3 16 11	−1 0
50	+1 0	4 11 7	4 10 7	4 10 4	− 3
55	+1 4	5 7 8	5 6 4	5 6 7	+ 3
60	0	6 7 4	6 7 4	6 7 4	0
65		(8 3 10)	7 16 8	(7 16 4)	

It will be observed that both tables agree very closely with the common scale of premiums calculated by the Northampton Table of Mortality at 3 per-cent, and I will confess that it was with a desire to defend that scale of premiums from the reproaches with which it has of late been the fashion to assail it; and in the hope of suggesting some considerations which might check the too common disposition to lower to a dangerous extent the premiums on young lives, that I have been induced to submit to the Institute the present paper. I say for the Northampton table of premiums all that the warmest admirer of the Carlisle table of mortality can urge in its favour, namely, that while I own the mode of its construction was faulty, and the data on which it was founded are insufficient, it *happens* to agree very nearly indeed with the best and fairest table I can frame.

Having shown that the Northampton premiums are fairly adjusted, I proceed to show that they are moderate in amount. This question of course turns upon the number who insure at the younger and older ages respectively. I have therefore extracted and classed under their ages at entry, in quinary periods, the whole of the persons recorded in the reports of the Equitable and Amicable Societies, and in the report of the Committee of Actuaries for Town, Country, and Irish Male Lives. The following table shows the annual premium which would be payable on a policy for £100 effected on each life, the rate per-cent being determined for each quinary class by the mean age.

Age.	Number of Persons Assured at each Age.	Annual Premium.						
		Northampton 3 per-cent	Carlisle 3 per-cent	Carlisle 4 per-cent	Experience 3 per-cent	Experience 4 per-cent	English Table 3 per-cent	English Table 4 per-cent
		£	£	£	£	£	£	£
Under 14	658	1,235	851	746	850	744	910	805
15 to 19	1,050	2,135	1,462	1,285	1,445	1,267	1,560	1,385
20 to 24	4,034	9,140	6,336	5,611	6,232	5,490	6,761	6,041
25 to 29	8,428	21,089	15,170	13,586	14,791	13,113	16,008	14,397
30 to 34	10,729	29,919	22,039	19,859	21,651	19,355	23,250	21,052
35 to 39	10,451	32,840	24,777	22,512	24,612	22,218	26,073	23,778
40 to 44	8,861	31,763	24,440	22,383	24,784	22,649	25,770	23,688
45 to 49	6,721	27,746	21,675	19,988	22,791	21,097	23,203	21,523
50 to 54	4,754	22,920	18,798	17,575	19,852	18,613	20,016	18,772
55 to 59	3,086	17,588	15,481	14,676	16,121	15,296	16,387	15,570
60 to 64	1,761	12,126	11,064	10,599	11,708	11,232	11,902	11,431
65 to 69	656	5,645	5,198	5,019	5,640	5,460	5,702	5,525
70 to 74	126	1,406	1,342	1,309	1,419	1,384	1,422	1,387
75 to 79	23	340	313	306	345	338	339	333
80 to 84	2	41	35	34	40	40	39	38
85 to 89	2	54	46	45	57	56	51	50
Total	61,342 Persons	215,987	169,027	155,533	172,338	158,352	179,393	165,775
		Annual Premium on £6,134,200 assured.						

The Northampton 3 per-cent premium is equal to the
Carlisle 3 per-cent increased . . . . 27·8 per-cent.
„ 4 per-cent „ . . . . 38·9 „
Experience 3 per-cent increased . . . . . . 25·3 „
„ 4 per-cent „ . . . . . . 36·4 „
English Table, 3 per-cent increased . . . . . 20·4 „
„ 4 per-cent „ . . . . . 30·3 „

Now I apprehend that if I had asked the opinion of any member of this Institute as to the propriety of adopting Mr. Farr's 3 per-cent table, with an addition of 20 per-cent, he would have replied that the premiums might be safe for a non-participating scale, but would not be high enough to allow bonus. Such would appear to be Mr. Farr's own opinion, for he has published his rates with various percentages of addition, ranging as high as 40 per-cent.

How is it, then, that the Northampton Scale has acquired its reputation of being excessive in amount?—that, in point of fact the companies using it have been so remarkable for large accumulations? The answer is supplied by the foregoing investigation. The Northampton rates are profitable because they approximate so closely to a fairly adjusted scale.

It is not enough for the interests of an assurance company that a deficiency of its charges at one period of life should be compensated by excess at another; for the great bulk of its

business will be transacted where its rates are lowest. Now it is very nearly a matter of indifference to the Northampton Offices, at what period of life the assurances effected with them are taken out; and to that circumstance, mainly, I attribute their prosperity.

I think it is clear that one objection which Mr. Farr makes to the Northampton table as the groundwork of a scale of premiums, applies also, in a measure, to his own. That is to say, the profit to the company employing them will arise, to an uncertain extent, from their overrating the mortality it is likely to sustain. I respect greatly the ability which characterises all Mr. Farr's papers; especially that appended to the 8th Report of the Registrar-General; but it does appear to me unfair that his strictures on the premiums charged by most of the leading assurance companies, which he points out by name, should have been published with the authority of a parliamentary paper and at the national expense. He states that "in his money calculations and examples no allowance is made for expenses, fluctuations, and profits", but that is just begging the whole question.

It will be observed that I have confined my remarks to the premiums for assurance for the whole term of life. I believe that most of the offices named by Mr. Farr have discontinued the use of the Northampton tables for short period and contingent assurances; yet one would suppose from his paper that they were still generally employed for those purposes, and even for granting endowments and annuities.

The words just quoted from Mr. Farr suggest three reasons for the margin of protection which assurance companies are in the habit of taking in determining their rates of premiums, namely, they have to allow for "expenses, fluctuations, and profits". I have not yet referred to the second of these—the contingency of fluctuations—because it is not possible to measure its value. But, if properly taken into account, it will require a larger amount of addition, proportionately, to the premium for assurance on a young life than to that on an old one, because the former contract may extend far into the future, while the latter must be determined comparatively soon. It is more easy to form an opinion whether the value of money is likely to be materially altered, or the system of direct taxation materially extended, or whether any breach of the public faith is likely to be committed, during the next seven years, than during the next seventy years. For this reason I think that if a company were formed to assure only very old

lives, it might be justified in assuming a higher rate of interest in its calculations, than one which should assure only very young lives.

If the contingency of fluctuations be introduced into the calculation of life premiums, it should be in the form of a further constant addition to the annual premium, so that the charge on each policy should depend on the period of its duration. The effect will of course be to raise the proportion which the premiums on young lives bear to those required at the older ages.

In concluding these observations, I beg to suggest that a distinction between the addition for expenses and the addition for profit, in the calculation of life premiums, would point the way to a more exact mode of dividing profits, than any which, I believe, has yet been adopted. If two persons of different ages are now about to effect assurances, entitling them to a share of profit at the end of a certain number of years, the present values of the *endowments* which will represent their cash bonuses ought to be proportionate to the present values of the *temporary annuities* of profit which they are to contribute to the general fund until the period of distribution. Thus making $p =$ the annual profit charged to a person entering at any age, the formula for the calculation of what is technically termed the "Scale of Appropriation" will be

$$p\,\frac{\text{Value of temporary annuity}}{\text{Value of endowment}} = p\,\frac{N_{m-1} - N_{m+n-1}}{D_{m+n}}$$

This appears to me correct in principle, but I do not assert that it would be possible to carry it out in practice. Our whole course of business is a system of compromise between that which is theoretically correct, and that which is practically convenient; and a mode of dividing profits which should fully satisfy both requirements would exhibit a degree of perfection to which we have not hitherto attained.

Royal Exchange Life Office,
18 *February* 1850.

Postscript.

The foregoing remarks upon the proper adjustment of a scale of premiums apply only to assurance companies transacting business without the intervention of agents. If an allowance of commission, in the usual form of a percentage upon the annual premiums, has to be taken into account, the formula should be as follows:—

$$\left(\frac{A+p}{1+a}+\epsilon\right)\frac{1}{1-c}.$$

$$\text{The above}=\left\{\frac{1-\delta\,(1+a)+p}{1+a}+\epsilon\right\}\frac{1}{1-c}$$

$$=\left\{\frac{1}{1+a}(1+p)-\delta+\epsilon\right\}\frac{1}{1-c}$$

$$=\left\{\left(\frac{1}{1+a}-\delta\right)(1+p)+\delta p+\epsilon\right\}\frac{1}{1-c}$$

$$=\mathrm{P}\frac{1+p}{1-c}+\frac{\delta p+\epsilon}{1-c}^{*}.$$

As an assurance company cannot, of course, have two scales of premium, one to pay commission and the other not, I think the charge should be regarded as affecting the business at large, and that the rate of allowance assumed in the calculation should be that proportion which the whole amount of commission paid in a year, bears to the whole amount of premiums received. For instance, if the company pay a commission of 5 per-cent upon four-fifths of its premiums, c will be $=\cdot04$.

Let δ be as before $=\cdot029126$ and $\epsilon=\cdot0025$; also let p be arbitrarily fixed $=\cdot04778$, so that the rate for age 60 may agree with the previous table, the formula for the office premium will be—

$$1\cdot09143\text{ (pure premium)}+\cdot004053.$$

The first column of the following table has been calculated by this formula from Mr. Davies's table at 3 per-cent.

Age.	Adjusted Annual Premium.	Northampton Premium.	Difference.
	£ s. d.	£ s. d.	s. d.
20	2 1 5	2 3 7	2 2
25	2 6 2	2 8 1	1 11
30	2 11 8	2 13 4	1 8
35	2 18 1	2 19 10	1 9
40	3 5 11	3 7 11	2 0
45	3 16 2	3 17 11	1 9
50	4 9 10	4 10 7	0 9
55	5 6 4	5 6 4	0 0
60	6 7 4	6 7 4	0 0

* P represents the "pure" annual premium, c the rate of commission, p the constant addition to the single premium, for profit, and ϵ the constant addition to the annual premium, for expenses.

On the Principles to be observed in Life Office Valuations made with a view to Distribution of Profits: being a paper read to the Actuarial Society of Edinburgh. By John M. McCandlish, F.R.S.E., *Honorary President of the Society.*

Some apology is needed, perhaps, for proposing to rediscuss a subject which has been handled so often, and by so many distinguished actuaries. But I am probably addressing gentlemen who have not had occasion to become familiar with all that has been written or said on the subject, and others to whom it may usefully be presented from a fresh point of view; and there are some important practical questions too on which the profession seem scarcely yet agreed, which will admit of further consideration. I shall not attempt to notice the various papers which from time to time have been contributed by actuaries towards the solution of the questions I am to consider. For many of the views I am to express I am necessarily indebted to those who have preceded me; and the aim of this paper is less to introduce what is new, than to enforce, and illustrate if possible, sound and correct opinions. I am anxious to set the subject in a light which will make it intelligible to others than actuaries, and to discuss principles rather than formulas.

With respect to most of the calculations which an actuary may have to make, he will find it important to start with the enquiry, What is the object for which the calculation is desired? Even in so simple a problem as finding the value of an annuity on a single life, it may materially affect his data and his method of proceeding, if he knows that the object is to ascertain what such an annuity can be bought for, or what it can be sold for, or what loan may be given or obtained on the security of the annuity, or whether it is to be the subject of negotiation and compromise. In estimating the liabilities and assets of a life office, several distinct objects may be had in view. The question may be, whether the office is not insolvent, or unlikely to be able to meet its liabilities as they emerge; or it may be, what sum ought to be paid to another office to relieve it of its liabilities; or the object may be to ascertain what surplus or profit it has realized after making provision for all its liabilities, and how that ought to be apportioned. Confusion has sometimes arisen from a well meant endeavour to find some general method of proceeding which will apply to all these cases. There will therefore be some advantage in confining the present enquiry to one of these objects in the first instance; and as the purpose for which such

valuations are most frequently undertaken, especially in Scotland, and with which therefore we here are most likely to have to deal, is the ascertaining and dividing of surplus or profit, I propose to ask your attention exclusively to this aspect of the question.

I have another reason for doing so. The plan which has been followed by some distinguished actuaries, of trying to establish a canon of valuation which shall not in any case be too severe or exacting, presents some grave difficulties. Archbishop Whateley has somewhere remarked, that the evil done by a bad example is not so much that others follow it, as that others think they have done well if they just come short of it. An easy system of valuation, which may be justified in extreme cases, has a great attractiveness, even in cases that are not extreme; and though it may not be followed absolutely, it may be approached unnecessarily. There is a proverbial facility in descent, and there are many temptations in a life office to be content with the least possible Reserve, in order that there may be a larger fund available for present expenditure or for present dividends and bonuses. What, therefore, I should like to see established by the common voice of our profession, is a high standard of Valuation, which shall be regarded as the thing to be desired and aimed at, which may be relaxed, indeed, in one direction or another when there is need, but any departure from which ought to be recognized as a departure, and carefully measured, and have specific reasons assigned for it. That actuaries will agree absolutely on any one combination of data is not to be expected, nor shall I venture to insist on it. I desire rather to offer a contribution towards the recognition of sound principles.

One or two preliminary remarks of some importance may be made. It does not seem always to be remembered (although suggested by so well known a writer as David Jones), that a valuation may affect very materially the rates of premiums received by an office. I think I have heard it said, in extenuation of a too easy system of valuation, that at all events the premiums charged by the office, being equal to those of most other offices, must be amply sufficient. But let us look at the matter in this way. A yearly payment of £25 will, at a given age, assure £1000 payable at death, according to one table or one mode of calculation, and will assure £1200 according to another. An office has issued a policy for £1000 in consideration of a yearly payment of £25, and may be regarded as amply on the side of safety. But a few months afterwards it estimates its liability under this policy by a

method, or according to a table, which brings out £1200 as the reversionary liability equivalent to a premium of £25 a year. It has thus a surplus reversion of £200 to dispose of. The £200 is not added, as it might reasonably be, to the particular policy to which it owes its being, but has gone into the general profit and loss pot, along with other surplus reversions similarly obtained, and is served out to various policyholders, if indeed it has not already been eaten up in the form of costs or extraordinary losses. From that moment, however, the £25 payable yearly under this policy is, throughout its whole existence, weighted with a reversionary liability of £1200, and the effect on the office is precisely the same as if for a policy of £1000 the yearly premium had been no more than £20. 16*s*. 8*d*.

Another question to raise at the outset is, For whose benefit are we about to make the supposed valuation? Whose interests are we to have in view? It seems still the fashion, in works which profess to instruct the public about life assurance, to divide life offices into three classes, Mutual, Proprietary, and Mixed; but I am not aware that there is in this country at the present time any proprietary office in the sense here intended. Practically, in all British offices, the assured are either the only partners in the concern, or have far the largest share in it. One can imagine that in a purely proprietary company, where no portion of the surplus or profits went to the assured, the proprietors might not be unwilling to issue policies at rates to yield them a profit, and then to estimate, as nearly as they could, the present value of that profit in all time coming, and proceed at once to divide the value among themselves. But in a mixed company, no less than in a purely mutual office, there is little temptation to this course. Suppose the case of a mixed office, which appropriates to its shareholders one tenth, or possibly so much as a fifth of its profits. A too sanguine valuation, and consequently an over-estimate of accrued profits, might, no doubt, lead immediately to increased dividends, but at the cost of an undue increase of the liabilities of the shareholders to their partners, the assured. For every pound of excessive profits unwisely credited to themselves, they would unduly increase, by £9 or by £4, as the case might be, the claims upon them of the assured, who are at once their partners and their creditors. The interests of the assured, on the other hand, may be best considered by assuming them to belong to a purely mutual office. What motive can they have to over-estimate future profits, or even to sail as close to the wind as they can in estimating them? An agreement among

persons wishing to assure, that they shall all pay premiums on the lowest possible scale, and take their chance, might be a not unreasonable proceeding; but one can scarcely imagine their agreeing to pay rates known to be more than sufficient, and then proceeding to appropriate to themselves at once the whole expected profits from these over-payments, and possibly something more. It seems clear, therefore, that alike in the interests of shareholders and assured, a system of valuation is needed which will make ample provision for all liabilities to come.

Having thus glanced at the views which may be supposed to influence shareholders and policyholders, it may be allowable to look for a moment at those which may affect the mind of the actuary. It is undoubtedly a pleasant thing to find ourselves able to report, respecting any society which has done us the honour of consulting us, that its funds are sufficient to meet its obligations, and perhaps to yield a good bonus in addition; and there may be many subtle influences at work to induce us to take too sanguine a view of the situation. Some persons might imagine that the best safeguard against this was to employ an outside actuary, who, not being responsible for any mismanagement or misfortune in the past career of the office, might feel himself free to give an unbiassed opinion, and thereby, perhaps, to contribute towards putting the office in a sounder position for the future. But the history of life assurance does not, I think, confirm this view; and it seems to me that those offices are fortunate whose valuations are made on the responsibility of officers permanently attached to them, who must be conscious that they may have the future to answer for as well as the present, that they may have to face another and yet another valuation, and that any error committed now will tell against them with accumulating force at these successive epochs of test and trial.

Proceeding now to consider the method of valuation, I propose to confine my attention to that one which undoubtedly is, beyond any other, the prevailing method—that, namely, by which we estimate separately the present equivalent of the liabilities of an office and the present value of its premiums receivable, and hence ascertain the sum which it ought to have in hand to give it the prospect of meeting all its engagements. This method is suggested, at all events, by the Life Assurance Companies Act of 1870, and is enforced in certain cases by the Act of 1872; but—what is of more importance—it enables us to resolve into their elements the data on which we are proceeding, to test and probe each of them sepa-

rately, to see with precision the effects which any change in the data or in our mode of handling them are fitted to produce, and to examine the component parts of our valuation, one by one, alongside the corresponding elements out of which our premiums have been constructed.

Alike, in calculating rates of premium and in estimating the value of policies, there are three well known centres round which all the questions we can consider group themselves,—the table of mortality to be used, the rate of interest to be assumed, and the method of dealing with the "loadings".

I—*Tables of Mortality.*

The actuarial sentiment of the day seems to favour as near an approach as possible to what is sometimes called a "true" table of mortality. The form of expression occasions, I think, a little confusion sometimes, for it suggests the idea that a table may be found which will indicate beforehand with certainty the rate at which any given number of persons will die. A moment's consideration will recal the fact, that no "true table" in this sense is available to us. All that any table can do for us is to formulate the experience of the past, and to excite in us a more or less reasonable expectation that the future will be not dissimilar. Whatever abstract "law" of mortality may exist in the nature of things, it is obviously liable to so many exceptional influences, so many caprices even, if we may so speak, that although the law were known to us, we could not rely on it as an invariable factor. Those of us who have had a long connection with life assurance, or who have made ourselves familiar with its history, will be least inclined to dogmatize about particular tables of mortality. Long after my first acquaintance with the subject, the Northampton table was in high repute as thoroughly safe, although it might not be used with prudence for all purposes at all ages. The Carlisle table, which succeeded it in popularity, is now pooh-poohed in many quarters, with scant justice, I think. The Experience (17 Offices) table was long the pet of a few actuaries, but distrusted, unduly, perhaps, by the many. The English table promised us the results of a far wider basis of facts than any that had preceded it, but does not seem to be much used by the profession. And now we have, as the favourite, the Institute (20 Offices) table, for which we are sometimes invited to discard all others. It is a highly satisfactory consideration, that all of these tables run so nearly parallel, and that hence we may feel pretty sure that the

law of mortality lies within comparatively narrow limits. Still, if we had the most perfect table that could be formed from the experience of the past, the question would remain whether the future could be trusted to reproduce an exactly corresponding experience. Apart from the circumstance that, for anything we can tell, the general death-rate throughout the country may increase or diminish during the coming years, it is in a high degree improbable that the future experience of any one office will correspond precisely with the general average. Even as a matter of pure chance (if there be such a thing), the average results will be consistent with large deviations, on the one side or the other, in the case of individual offices; but it is plain, besides, that something may depend on the localities or the social classes from which an office draws its business, and on the skill and care of those who conduct it.

A "true table of mortality", then, in the sense that by using it we may reckon confidently on neither profit nor loss arising under this head, is not to be looked for. It is obvious that, of the two possible deviations, the one to be provided for is an excess of mortality, or rather of premature mortality, and that consequently the sort of table to be used is one that can only be described as a "safe" table. I think I have seen in some paper on this subject (by whom written I cannot recal) a strong preference expressed for a true table rather than a safe one; and if the meaning be that a table is not a sound one which is *needlessly* "safe", and does not even approach to what will probably be "true", I shall not dissent. But what I desire to suggest is, that this is not a case where it is possible to say, "You have no need to err on the safe side, for you have no need to err at all on either side." Certainty is unattainable, and error is so highly probable that it ought to be provided for, and provided for on the side of safety.

It seems to me very desirable that the table to be used should be a *known* table, one whose merits or demerits may be canvassed and recognized by actuaries, and not a fancy table, constructed, as it is sometimes said, from the office rates of premium, or compounded and doctored to suit individual tastes. However admirable such a private table may be, it has two great drawbacks: it cannot inspire confidence outside its own birthplace, and it does not impose a sufficient restraint on the fancies or the necessities of ingenious actuaries. We have in these days a choice of so many carefully constructed and well developed tables, that there is little if any excuse for indulging a peculiar or exceptional taste in this matter.

And this leads me to observe that, in my humble opinion, the table of mortality and the rate of interest to be used in a valuation ought to be selected without much reference to the data on which the office rates of premium are calculated. There are said to be some patent logs which will tell you pretty nearly how far a ship has sailed, but if you want to know her true position you must ascertain her present latitude and longitude. A merchant who is taking stock must look, not to what his goods have cost, but to what, according to his present knowledge and judgment, they are now likely to fetch in the market. A contractor who, in the midst of some great operation, the construction of a railway or a harbour, desires to know the state of his affairs, will turn his back on the past, and estimate from a fresh point of view the outlay he has yet to incur, and the proportion of the contract price he has yet to receive. No actuary, of course, will approach a valuation without having regard to the data on which the premiums have been calculated, but these data will be in his opinion either sound or unsound, and if he thinks them unsound, he will scarcely feel at liberty to use them in his valuation, merely because the mistake has already been made of using them to calculate the premiums. He may see good reason for suggesting an alteration in the rates of premium, but he will not aggravate any evil that may already have been done by continuing the use of false data.

At the same time, it seems unwise to alter too readily the basis on which the calculations of an office have hitherto been made. It may be said, Why not use the very best data you can get at the time? But seeing that you cannot attain certainty with any of them, if those you have hitherto used are reasonably satisfactory, there are some advantages in avoiding a change which may appear to the public more important and more unsettling than it really is. I observed, the other day, that a London actuary mentioned it as the practice of some of the Scotch offices to use "check valuations": it seems to me a good practice, and I have adopted it with satisfaction oftener than once. They serve to check, in a broad way, the general accuracy of your calculations, as well as to show the precise effect of variations in your tables of mortality and rate of interest.

Before proceeding to contrast the effects of adopting different data, it seems important to observe that, unless very great mistakes are made in this respect, and funds are parted with, or fresh liabilities are undertaken on the strength of anticipations never realized, the prosperity of an office will depend, in the long run, not on

the rates of mortality or of interest which are used in the calculations, but on the rates actually experienced. Take, for example, the rate of interest. If 4 per-cent is actually earned, the amount finds its way, year by year, into the treasury of the office; and according as 3 or 4 per-cent have been previously reckoned on, the actual benefits are either enjoyed after they are earned, or by anticipation beforehand; they are certain to present themselves once, but can never be enjoyed twice. The longer the interval between two valuations, the less important do variations of data become, for they are more equalized by the facts of actual experience.

I have studied with much interest certain tables presented by Mr. Valentine to the Institute in 1874, founded on a previous paper by Mr. Manly. A supposititious office is taken, and certain assumptions are made as to the amount of its business from year to year, the amount of claims, surrenders, &c. Then the liabilities of this office are valued at successive quinquennial periods, extending from five years after its birth to forty-five years. The valuations are made according to no fewer than twenty-seven different tables or combinations of tables; and the reserves brought out as necessary to meet its obligations are set down for comparison. I do not inquire how far this conjectural office has been rightly constructed. No doubt its figures will differ from those of any real company, as those of real companies will differ from each other; and there is one important deviation from the circumstances which would be present in the case of most real companies—that there are no vested bonuses to provide for. Still, a comparison of the results of these several valuations is of much interest and value.

Of the various tables of mortality employed by Mr. Manly or Mr. Valentine, there are only two with which we need trouble ourselves—the Carlisle and the Institute's new table. Of the two tables prepared by the Institute, the H^{M} and the H^{M5}, I shall deal with the former. If an office were to discontinue taking new business, or if its new business were very limited in amount compared with its old, there would be strong reasons for using the H^{M5} table; but in this paper I am dealing with the case of offices having a certain amount of life in them, and for them the H^{M} table seems to furnish, on the whole and overhead, a nearer approximation to a true table than the other would do.

In comparing, then, the reserves needful according to the Carlisle and the H^{M} tables, I observe, in the first place, that the differences become gradually smaller in proportion as the office

advances in age: and this is the case whenever two sets of tables we contrast with each other. The cause of this is not far to seek; but the fact is worth noting, although it will no doubt be influenced a good deal by the rate at which the business has increased. As in a human life the further it advances the more important comparatively do the facts of the past become, and the smaller become the limits of possible deviation when we forecast its future; so, as an office grows older, and its policies on an average come nearer their end, and the provision laid up for them grows larger and larger, and the provision yet to be made consequently less and less, the variations you can indulge in, in estimating the future, come to have a decreasing influence on the general results. But, as happens also in a human life, the age of an office is not always to be reckoned in years. As it grows older in years it may become more youthful in constitution. If you take, for example, the average age of the lives assured, or the average duration of its existing policies, it is possible to imagine an office enlarging its business at a rate which will give a retrograde movement to these two points, so that at a given date its lives and its policies shall, one with another, be younger than they were several years ago. This is one of the special circumstances about an office which the intelligent actuary will not lose sight of when he determines on the data which he is to use for that office.

Let us now inquire what difference arises in the estimated reserve out of a difference in your assumption as to interest. The following table shows the amount of reserve required (for Mr. Manly's fancy office) at the end of 10, 20, 30, and 40 years, according to the Carlisle tables, and taking interest at 3 per-cent; the difference in amount and the percentage of difference when 3½ and 4 per-cent are taken:—

Years.	Reserve, at 3 per-cent.	Difference in amount, at 3½ per-cent.	Per-cent of No. 2.	Difference in amount, at 4 per-cent.	Per-cent of No. 2.
(1)	(2)	(3)	(4)	(5)	(6)
10	200,261	11,313	5·648	21,874	10·921
20	496,512	24,368	4·908	47,313	9·529
30	744,746	32,782	4·401	63,834	8·572
40	862,122	35,819	4·155	69,838	8·101

In this supposed office, therefore, to adopt at the end of 20 years a 3½ per-cent valuation instead of one at 3 per-cent, would require an addition of just 5 per-cent (4·908) to the amount of the

reserve, or to change from a 3 per-cent to a 3½ per-cent valuation would lessen the sum required by 5 per-cent. To make a difference of 1 per-cent in the rate of interest would involve a difference of reserve varying from 10·92 per-cent at 10 years to 8·10 at 40 years.

The corresponding differences brought out in connection with the H^M tables are not dissimilar. Between 3 and 3½ per-cent, they are, at 10 years, 5·35 per-cent; 20 years, 4·64; 30 years, 4·15; and 40 years, 3·91.

So much for interest alone. Taking now the same rate of interest, but a different rate of mortality, we have the following contrast between the H^M 3 per-cent and the Carlisle 3 per-cent tables:—

Years.	Reserve, H^M 3 per-cent.	Reserve, Carlisle 3 per-cent.	Difference.	Percentage of Difference.
10	216,505	200,261	16,244	7·5
20	530,818	496,512	34,306	6·46
30	791,662	744,746	46,916	5·92
40	914,448	862,122	52,326	5·72

It will appear from this that, so far as this imaginary office is concerned, a change in the assumed rate of mortality at the end of 10 years would make a difference of 7½ per-cent in the results, and the ratio of difference would diminish, till at 40 years it was no more than 5·72. Thus the difference occasioned by this change in the rate of mortality is greater than what a change of one half per-cent in the rate of interest would occasion, and less than a change of 1 per-cent would occasion.

The combinations which approximate most closely to each other are the H^M 3½ per-cent, and the Carlisle 3 per-cent table. The following are the reserves brought out on these data respectively:—

Years.	Reserve, H^M 3½ per-cent.	Reserve, Carlisle 3 per-cent.	Difference.	Percentage of Difference.
10	204,922	200,261	4,661	2·27
20	506,152	496,512	9,640	1·9
30	758,731	744,746	13,985	1·84
40	878,609	862,122	16,478	1·87

So far as these calculations go, the two tables may be regarded as almost identical for practical purposes, the differences amounting only to about 2 per-cent.

Before leaving these interesting figures, for which we are greatly indebted to Messrs. Manly and Valentine, it will be useful to observe that a change from one set of data to another exhausts its importance, for the most part, the first time it is made. Let us suppose that at the end of 30 years the question arises, whether, valuing by the Carlisle tables, the rate of interest ought to be 3 or 3½ per-cent. The difference in the results is found to amount to £32,782; that is, if we change from 3½ to 3, we must set aside a larger sum by £32,782 than we should otherwise have done, or if we change from 3 to 3½ we shall have this amount at our disposal. But if the change had been made five years earlier the difference would have been £29,454. It is obvious that the difference at 30 years practically includes the difference at 25 years, and is not additional to it. At whatever period therefore such a change is made, the benefit that is sought for, whether of increased security or immediate profit, is obtained once for all, and a continuance in the new line adopted yields very little further advantage at successive valuations.

Having availed myself so largely of the figures relating to this supposititious office, I am happily able in some measure to illustrate and confirm them by calculations made for a real office. Four years ago I had occasion to make a periodic investigation into the affairs of a company with which I am connected, and I had the bulk of the policies, those for the whole of life at uniform rates, valued under four different tables, Carlisle 3 per-cent, Carlisle 3½, H^M 3½, and English life table No. 3 with 3 per-cent. Each valuation was complete in itself, the net premiums for every individual policy being filled in according to the tables used in each valuation. The amount of the policies so valued, after deducting reassurances, was close on two millions sterling. The following were the reserves brought out:—

	£
Carlisle 3 per-cent	357,032
" 3½ "	338,635
H^M 3½ "	359,807
English III, 3 per-cent	375,239

The difference under the Carlisle tables between 3 and 3½ per-cent amounted to £18,397, or 5·15 per-cent. The difference between Carlisle 3 per-cent and an H^M 3½ per-cent valuation was no more than £2,775, or less than 1 per-cent (·77). I felt myself justified under these circumstances in adhering to the data hitherto used by the office, namely, the Carlisle tables and 3 per-cent.

These valuations, unlike those we have been previously dealing

with, included vested bonuses. This is an element which might disturb such comparisons as we have been instituting. In valuing a policy without bonuses, the greater or less value of sums payable at death according to different data are subject to modification by a corresponding greater or less value of annuities, or by the change in the net premiums, whereas in valuing a bonus it is only the one calculation which falls to be made. The vested bonuses in the case now under review amounted, after deducting reassurances, to £100,000, and the values were as follows:—

	Value of Bonuses.	Value of Policies, not including Bonuses.
	£	£
Carlisle 3 per-cent	58,876	298,156
„ 3½ „	54,729	283,906
H^M 3½ „	55,642	304,165
English III, 3 per-cent	60,797	314,442

It will be seen that as between the Carlisle 3 per-cent and the H^M 3½ per-cent tables, the former required a higher reserve for bonuses, and the latter for policies free from bonuses. When the bonus element is eliminated from the general results, the difference between these two tables is just about 2 per-cent, or as nearly as possible the difference which Mr. Valentine's figures indicate.

Thinking it possible that the effect of varying the data might be influenced by the age of the assured, I have valued separately the policies of the profit class in which the lives had attained the age of 61 and upwards. The amount of these with bonus additions was £224,600, and the values were as follows:—

	£
Carlisle 3 per-cent	99,545
„ 3½ „	95,818
H^M 3½ „	100,063

The proportions of difference were still less than those formerly brought out. The values of the bonuses taken separately were £22,684, £21,600, and £22,033.*

* Since the above was written, another periodical investigation has been made into the affairs of the office in question, and its whole-life policies at uniform rates of premium have again been subjected to three distinct and complete valuations, according to the Carlisle 3 per-cent, Carlisle 3½ per-cent, and H^M 3½ per-cent tables. The policies and vested bonuses thus valued, amounted to £3,160,233, and the values were as follows:—

	£
Carlisle 3 per-cent	589,105
„ 3½ „	557,915
H^M 3½ „	591,806

The difference therefore between a Carlisle 3 per-cent and a Carlisle 3½ per-cent valuation was only 5·62 per-cent of the former, and between a Carlisle 3 per-cent and an H^M 3½ per-cent valuation only ·45 per-cent of the former. When sums reassured were taken into account, the two last named valuations brought out final results almost identical, the difference being only ·10 per-cent.

I may add here one or two other sets of figures which have come under my notice, confirmatory of those already given. Some years ago a large office, whose obligations amounted to ten millions, reported that between a valuation at 3, and one at 3½ per-cent (both Carlisle), there was a difference in result of £132,164, equal to 4·94 per-cent of the larger sum. In the case of another office, much younger and smaller, the difference was so much as 6·39 per-cent.

II—*The Rate of Interest.*

I shall have occasion to say little as to the rate of interest, beyond what has necessarily been said in connection with the tables of mortality. Some years ago I had the honour of reading a paper to this Society, specially directed to the considerations which ought to guide an actuary in fixing on a rate of interest to be used in his calculations. It is only needful at present to recal such considerations as these:—that the question is not what interest has been earned in the past, but what may be looked for in the future; that past earnings are only an element in the estimate of the future, and that consequently it is not of itself any conclusive argument for a given rate, that it has been earned or has even been exceeded in the past, unless it can also be shown that it may be expected to continue; that the estimate has to be formed for very long periods of time to come, stretching not merely to the average duration of lives, still less of policies, but to the utmost possible limit of their duration; that the probable value of money in the future is a problem of the greatest uncertainty among financiers and political economists, but that so far as appearances go, the present tendency is towards a reduction in the earning power of money invested with the degree of safety and with the sense of safety which ought to attach to the investments of an assurance office; and finally, that the adoption of any given rate in our calculations, assumes that every premium will be invested at that rate on the day it becomes due, and that each year's interest will be similarly invested, and that there will be no expense or loss through the investments, all of which assumptions are too favourable when compared with the facts. The result of such considerations manifestly is, that the rate of interest to be adopted in estimating the liabilities of a life office ought to be amply within the limits of the probable, or that if it approaches these limits nearly, there ought to be sufficient margin somewhere to guard against disappointment in this particular.

III—*The Loading.*

I come now to what is by far the most important question connected with a valuation, namely, how to deal with the loadings. It has long been my conviction that, if there is any one cause to be blamed rather than another for the mischief that has arisen from such failures as those of the Albert and European companies and their allies, it is an erroneous method of handling loadings. No doubt there have often been rash and wasteful expenditure, a facile acceptance of offered transactions, insecure investments, and other features of bad management, but in almost every case of failure these have been encouraged, and their ruinous effects concealed and ultimately aggravated, by a system of valuing gross premiums, which has deluded all parties concerned with the appearance of resources that had no real existence.

I remember very well, when I was introduced to these questions by my most valued predecessor, Mr. Ivory, that my first impressions were in favour of estimating the value of the premiums we actually expected to receive. It struck me as unnatural, that where we were to receive £25 a year from a policy so long as it endured, we should reckon only on receiving £20. But I soon came to see the true principle, and I have never since wavered in opinion as to the great danger which attends a departure from a valuation of net or pure premiums.

I have already referred to the intimate connection between the two processes of calculating the rate of premium and valuing the policy, and here it is necessary to revert to the former process. How is the premium constructed? Of what is it composed? There are, of course, the same three elements in it as in the valuation process,—mortality, interest, and loading. Assuming, as we are bound to do, that the calculation is made with some approach to correctness of principle, obviously the pure premium provides for the risk run on a reasonable assumption as to mortality and interest. Why, then, is the loading added? For expenses, contingencies, and profit. The need of an addition for expenses is obvious; the addition for contingencies and profit may not be so much a necessity. Let us look at them separately.

First, as to the part of the loading intended to provide for expenses, the modes of dealing with it, when we come to value, have been the subject of two important papers contributed recently by Mr. Sprague and Mr. Deuchar, and of a discussion in the Institute of Actuaries. Attention has been drawn to the unquestionable fact, that the expenses chargeable against a policy come

upon it in an unequal manner, with an excessive force in the first year and a mitigated force ever after. But the provision for these expenses is made in the form of a uniform addition to the pure premium. The two portions of the premium—that which provides for the risk and that which provides for the costs—are both, for convenience sake, made uniform throughout the whole endurance of the policy; but in the one case too much is charged at first, and in the other case too little. As the valuation corrects the one error, so it may the other. It ascertains how much of the over-payment for risk ought to be reserved against the time when the current risk, having increased with the age of the assured, will be insufficiently met by the still uniform premiums; in like manner it may determine how much of the over-provision for future expenses may be at once appropriated to meet the excess of the first year's costs.

It has even been suggested that a table of premiums might be constructed on this principle, which would yet differ in no outward respect from any ordinary table. Suppose that an assurance effected at age x were treated as one to commence a year hence, at age $x+1$. The premium is calculated for age $x+1$, and suitably loaded for costs. But what of the intervening year? You have a one year's assurance to provide for, the net premium for which is of course comparatively small, and will admit of a very heavy loading before it comes up to the amount of the whole-life rate for age $x+1$ with its loading. It is plain that calculations made on this principle might be so adjusted that the rate for the first year, with its heavy loading, would be exactly the same as the rate for future years, with its lighter loading; and that this uniform rate, though calculated for age $x+1$, need not be different from what on the more usual system would be charged at age x.* If this could be done in constructing the premium, and done with a reasonable regard to principle, it would be equally easy and legitimate, when we come to a valuation, to apply the corresponding analytic process, and produce identically the same result. That result would simply be, that a smaller proportion of each actual premium would be reserved for future expenses than under the ordinary net premium valuation plan, and yet the system might be described, with some degree of truth, as a net premium valuation.

* This plan might be carried the length of dividing the transaction into a term assurance for t years and a whole life assurance deferred for t years. The limit of t would be found in this, that the premium calculated for age $x+t$, with the loading required for expenses and contingencies still future, would require to be not greater than the public would pay at age x.

I cannot say that I see much to object to in these theories; and if the question of the solvency or insolvency of an office turned on the application of them, I should not hesitate to apply them. They have, at all events, done good service in exposing a fallacy, which has received encouragement from the Board of Trade, and from other quarters, from which more correct views might have been expected. In those comparisons between office and office, which, unfortunately, it has been the fashion to make of late years, the question has been raised as to the relative extravagance or economy with which different offices conduct their business. An uninformed public may be excused for adopting the readiest method of comparison which comes to hand. Very modest arithmetical acquirements will enable anyone, by a sum in simple proportion, to reckon what percentage each society's expenditure bears to its premium income. But those who desire, by a scientific treatment of such questions, to arrive at the simple truth, will perceive that gross injustice may be done, and false conclusions supported, by such rough comparisons. The share of a company's whole expenses which belongs to each policy, may, for convenience sake, be provided for by a uniform loading, but its real incidence is not uniform. The uniformity of that portion of the premium which provides for the sum assured does not, to the actuary, indicate any uniformity in the risk run, or in the portion of each year's premium to be reserved; nor ought the uniformity of the loading to conceal from an actuary the inequality with which the expenses are justly chargeable against each year's premiums. As matter of fact, the costs of an office consist of a heavy ratio of expense applicable to new policies, and a comparatively small ratio applicable to older policies. No comparison of office with office can be scientific or fair that does not recognize this distinction, and break down premiums and expenses alike into their component parts, so as to answer the questions, at what ratio of cost to premiums is new business obtained, and at what other ratio is old business conducted.

There is a good deal to be said, therefore, against reserving the whole of the loading, which has been added for expenses, when a disproportionate part of these expenses has been already incurred and paid. For all that, it seems extremely undesirable for an office, valuing with a view to a division of profits, to avail itself of this process. But before going further into the question, let us look for a moment at those portions of the loading, if any, which are added for contingencies and with a view to profits.

An addition made to a net premium to provide against contingencies, infers an acknowledgment that there are contingencies to be provided against; and what has been said as to the impossibility of measuring exactly beforehand either rates of mortality or rates of interest, will have indicated some of the sources of these contingencies. But in any view it seems a manifest absurdity to provide for possible losses or disappointments, and then at once to reckon the provision so made as an asset capable of immediate appropriation and distribution.

Whether it is a good and wise thing to provide in part for future bonuses by an addition to the net premiums, made deliberately with this object, is a question a little beyond the scope of this paper. But certainly no good reason could be assigned for this procedure, if the addition were at once to be converted into a present asset.

It is difficult therefore to see a plausible reason for valuing any part of the loadings with a view to creating or increasing a surplus for immediate distribution, except it be a part of the addition made for expenses. There is much to be said, however, against valuing a jot more than the net premium. Any ease or benefit to be got is confined to the first valuation occurring after a policy is issued, and there is introduced into our valuation an element that is undefined and arbitrary. The immense importance of the question will be seen from the following illustration.

I gave you, some time ago, the results of an actual valuation made under different tables of mortality and at different rates of interest. We found that to value by the Carlisle tables, a difference of one half per-cent on the rate of interest made a difference in the results of £18,400, equal to 5·15 per-cent of the net reserve; and of two valuations, both made at $3\frac{1}{2}$ per-cent, but one according to the H^M and the other according to the Carlisle tables, the difference was £21,170, or 5·88 per-cent. But the value of the net premiums in that case (Carlisle 3 per-cent) was £680,000. To have valued no more than 5 per-cent of the loading would have diminished the reserve by £34,000, or 9·52 per-cent. In that case, therefore, an encroachment to the extent of 5 per-cent on the loadings would have made nearly twice as much difference in the result as a change from 3 to $3\frac{1}{2}$ per-cent, or from the H^M to the Carlisle tables. The comparative effects would vary with the age of an office, but valuing the loading tells most, and is therefore most dangerous, when an office is young, and when there is most temptation to keep down the amount of the reserve.

If we entertain the notion of valuing and appropriating any part of the loading, at what point are we to stop? If we find that taking five per-cent of the loading into account does not produce the amount of bonus we have set our hearts on, there is a manifest temptation to take more, and there is no well-defined landmark to stop us till we arrive at the gross premium. One effect of such a system is to produce negative values. The unsoundness and danger of these have been ably pointed out in a recent paper by a member of this Society, and I need not dwell on them further than to say that the condemnation which negative values justly receive is really deserved by the process which evolves them. When that process brings out a negative value, its erroneous character can be understood by any ordinary man of business; but the process is scarcely less at fault when it ends in an insufficient positive value. The error may not be so glaring, but it is little less mischievous.

I may notice here a method of valuing which, in effect, includes in the assets a portion of the loadings, although it is sometimes very erroneously described as a net valuation. The office premiums, we shall suppose, are on a 3 per-cent basis, the valuation is made at 3½ per-cent, but the pure premium at 3 per-cent is multiplied by the value of an annuity at 3½ per-cent. A little consideration will show that a valuation is not correctly made under any given table, unless all the elements of it—premium, assurance values, and annuity values—are equally derived from that table. A pure premium at 3 per-cent is equal to a pure premium at 3½ per-cent loaded, and to include the former in a valuation made otherwise at 3½ per-cent is to value a portion of the loading. In the actual valuation to which I have more than once referred, the pure premiums at 3 per-cent amounted to £47,000, and at 3½ per-cent to £45,000, so that the former was equal to the latter with a loading of 4·44 per-cent.

Not many actuaries will be found who would approve of the whole loading being valued, or even any very large portion of it, in order to distribute as realized profits the so-called surplus which might be brought out by the process. But actuaries of some reputation are of opinion that, even when no part of the loading is to be reckoned on in calculating the divisible profits, the whole of it ought to be valued and brought into account, not to be divided, but to be set aside as a reserve fund. The form of the schedules of the Life Assurance Companies Act of 1870 rather favours this view, as they appear to call for the value of the gross premiums imperatively, but to ask for the value of the net premiums

only "if computed." While the Act was being passed, I tried to induce some members of the profession to get this altered, but unsuccessfully. I take this opportunity of recording my protest against the system.

Two reasons have sometimes been given for the practice now under consideration. It is said, that to exhibit the value of the actual premiums receivable is to be consistent with fact, and that the surplus thus established is a real surplus, though not immediately available. This argument seems to me to involve an actuarial mistake. In a valuation, we are engaged in estimating the future liabilities and the future assets of an office, and setting the one against the other in order to a balance with the realized assets. Now there are four elements in this computation:—(1) the sum assured; (2) the portion of the actual premiums intended to provide for the sum assured; (3) the remainder of the premium, *i.e.*, the loading; and (4) the expenses, contingencies, and future profits, which the loading is intended to provide for. The actuary reduces to definite figures his estimate of the present equivalent of three of these elements, but he leaves the fourth out of view. If he makes one side of the equation matter of definite account, he is surely bound to deal in the same way with the other. He might almost as reasonably value the pure premiums and leave out the values of the sums assured, as value the loadings and leave out all estimate of the liabilities and contingencies which they may be required to meet.

If, for any purpose, we value the total future premiums, including alike what is intended to provide for sums assured and for expenses, contingencies, and profits, we are bound in valuing the liabilities to take the same elements into account. If we load the premiums and value the loading, we ought to load the sums assured and value that loading also. The desired result might thus be arrived at by adding 10 or 20 per-cent, or any other proportion that might be thought needful, to the value of the company's liabilities; but it is a sounder plan, in my opinion, as matter of principle, and more convenient in practice, to omit from both sides the values of these contingent additions to assets and liabilities, and exhibit simply the value of the sums assured (with the already vested bonuses), and the value of the pure premiums intended to provide for the sums assured.

There is a subtle element of uncertainty in the value of loadings, which does not apply to the value of the pure premium, owing to the contingency of a policy not being kept up; but where the

gross premium is valued, both portions of it are reckoned on with the same confidence.

It is sometimes argued that if pure premiums alone are valued, an office with high rates and one with low rates will show the same results. Thus, if the Carlisle tables and 3 per-cent are the data, and the rates of both offices are founded on these data, but one has a loading of 5 and the other of 25 per-cent, the estimate of their future receipts will be the same. No doubt it will be, but there will be two very important differences, not likely to escape observation. One office will, *cæteris paribus,* have a larger realized fund than the other; and its larger provision for the future may be made abundantly manifest, though it is not reduced to precise figures.

I need scarcely notice another argument, for it would have no bearing on a valuation made for division of profits. It is said that the actual premiums of an office might be less than the net rates of the table by which you were valuing. If you value, as you will certainly do in these circumstances, on data which come as close to the verge of safety as you can venture to go, and the actual premiums charged by the office are lower than the net premium which even these data would bring out, the object of the valuation can have no reference to profits, or even to solvency. So exceptional a case, if there ever should be one, may be left for exceptional treatment. That it sometimes occurs in America, as it is said to do, under State regulations, is a condemnation either of the rates of premium or of the enforced data of valuation, or of both.

When a gross valuation is set forth in the accounts of an office, the actuary's report comes to be in some such terms as these,—that he estimates the liabilities of the office at so much, its assets, realized and contingent, at so much, that consequently the surplus is so much, but that as that surplus is subject to reduction for future expenses and contingencies (perhaps a convenient "&c." is introduced to indicate future bonuses) the whole of the surplus is not available for immediate distribution, but that he estimates that such and such a sum (perhaps about a fifth of the gross surplus) is available, and it is recommended for division, and that the balance ought to be held over as a reserve.

The objections to this system are—

1. It presents the actuary as a sort of conjuror, or "medicine man", who distinguishes between what part of a surplus may and what may not be divided, either arbitrarily, or according to some occult rule which he is unable or unwilling to explain

to ordinary men of business. The days are gone by when men can command respect by being mysterious, and, in my humble opinion, actuaries will find it to their advantage to carry the public with them, as far as possible, in their processes. Few persons may be able to understand how a surplus has been ascertained which you recommend for division, they will take it on your word; but to accept on your authority the existence of a surplus, only a small portion of which you recommend as safe for division, seems to demand a larger faith.

2. The indefiniteness of the line which is to divide the so-called surplus into two portions, divisible profits and reserve, and the circumstance of the actuary not being committed to a distinct principle in drawing it, offer a great temptation to himself to stretch it, where his own interest or ambition, or the pressure of other influences, suggests such a course.

3. Supposing the actuary to be conscientious and firm, there is yet great reason to avoid dangling before directors and shareholders and the assured a larger apparent surplus than it would be safe practically to deal with. You may tell them that it is imprudent to touch more than a portion of it, but the surplus is there on your high authority, set down in precise figures, even to the pence. It is difficult for them to reconcile this precision of statement with the unsubstantial character which practically you desire to assign to it. The reserve you wish to establish for future expenses and contingencies will seem far too large, especially as new business may be trusted to bear its own charges. In short, your skill as an actuary, which has elicited the existence of this large surplus, is more to be trusted than that over-caution, which would reserve it for the enjoyment of others than those present shareholders or policyholders to whom alone it seems justly to belong. Arguments like these are employed to support an immediate appropriation of this supposed surplus, and all you have to set against them is what seems an arbitrary, empyrical, and fanciful distinction. All who are acquainted with the history of life offices know how often such arguments have been used, how much trouble they have given when they have been successfully resisted, and how often and how mischievously they have prevailed, sometimes by leading to actual encroachment on the surplus, in other cases by encouraging sanguine and extravagant management.

4. The use of this method of valuation by sound offices, which might themselves resist all temptation to the abuse of it, would sanction and encourage its use by offices of a different description.

You challenge the position of an office, and its conductors answer you somewhat in this way, "We have made our valuation by the Carlisle tables, the majority of offices do the same; we have assumed 4 per-cent interest, we are not without good authorities for that; we have valued our gross premiums; some first-class offices do so. It is true they are so fortunate as to show a large surplus; our surplus is very small, but if we make both ends meet, if our liabilities and assets balance each other on principles of valuation which have so high a sanction, we are content to wait for surpluses and profits, and who, in the meantime, dare question our present solvency?" And yet, as most actuaries will admit, an office in such circumstances may be almost hopelessly insolvent.

As I have already said, I believe that very great mischief has been done by this practice of gross valuations. Suggestions have been made that the legislature should restrain it in some way, as, for example, wherever it produces negative values. An easier and more effective method of preventing such mischief would be found if leading actuaries were to agree that the basis, at all events, of all investigations into the condition of life offices ought to be a net valuation. If the valuation is required for other purposes than that which in the present paper I have had in view, the distribution of profits, it will be an easy thing to modify the process so as to adapt it to the object then in view.

I have not hitherto referred to the case of policies for which the consideration has been received in the form of a single payment at the outset, or of a yearly payment to endure for five, ten, or other limited number of years, nor to the case of endowment assurances, or others of special character. These form, in most offices, a comparatively small proportion of the whole business, but, small or great, their case demands more consideration than I am able, at so late a stage of this paper, to give it. Each class will be dealt with by the discriminating actuary according to its own circumstances. I will only give one case, as an illustration of the sort of questions which may arise. When a policy issued in respect of a single payment comes to be valued, there are, of course, no future premiums to be taken into view, as to which any question of gross or net can arise. If the sum assured be valued at the net rate of the table on which the premium was based, no reserve will be made for any future expenses or contingencies, and the value of any loading, which may have been put on expressly for future profits, will be at once appropriated. So far as expenses are concerned, the future demands of this particular policy need scarcely

be regarded. Still, it seems inconsistent with sound principle to make no reserve whatever in such a case, for future expenses contingencies or bonuses. The plan which I have generally adopted has been to convert the original loading into an equivalent annuity, and to value that annuity from time to time according to the increasing age of the assured, and to add the value of this annuity to the value of the original sum assured in estimating the reserve required for this policy. I have dealt in a corresponding way with policies carrying five or ten yearly payments, but, except for the pleasure of adhering to sound principle, such refinements might be disregarded, on the principle *de minimis non curat lex.*

I ventured to suggest, at the outset, that this subject might be made intelligible to ordinary men of business, and I see no reason why it should not be. Yet there can be no doubt that, even among persons greatly interested in the safety and prosperity of life offices, not one man in a thousand, outside of the actuarial profession, will ever take the trouble of mastering these subjects, so as to command the bearings of the various points which are habitually present to the mind of an actuary. It is on the actuary, therefore, that the responsibility will always devolve of guiding the public in these matters. It is a very grave responsibility; for upon the success or failure of one insurance office may depend the poverty or comfort of thousands of families. Of the duties which fall upon an actuary, none, perhaps, approaches in importance the valuation of the liabilities of a life office. Besides the largeness in general of the sums he has to deal with, and the number of persons interested in the result, there is no part of his work which makes greater demands on his professional skill and knowledge; nor is there any that needs a more rigorous conscientiousness, or a greater watchfulness against subtle forms of self-seeking.

But great as is the importance of right principles being adopted in such valuations, it ought never to be lost sight of that, for the most part, the prosperity of an office depends ultimately on the successful accumulation of its funds. No doubt an erroneous estimate of liabilities may lead to an undue enlargement of them, or to an undue expenditure; but where policies are only issued at adequate rates, and where the funds are laid out only on safe investments, and where the expenses are moderate in proportion to the results obtained, there is comparatively little risk of an office being ruined through false principles of valuation.

On the Mortality among Innkeepers, Publicans, and other Persons engaged in the Sale of Intoxicating Liquors,—being the Experience of the Scottish Amicable Life Assurance Society during Fifty Years, 1826–1876. *By* John Stott, F.S.S., *&c., Manager of the Society.*

THE influence of Occupation upon health and longevity, is, I think, a subject deserving of even more attention than has hitherto been given to it by the Actuarial and Medical Professions. As the benefits conferred by Life Assurance are now almost universally recognized, they are being taken advantage of by classes of the community which, until the last few years, did not seem fully to appreciate them. It is therefore of no little consequence to have, as far as possible, a clear understanding of the relative risks attaching to different occupations and to the various branches of industry.

The Registrar-General, in his Fourteenth Annual Report, gave a Table showing the mortality per-cent in twelve distinctly defined Occupations, occurring in the year 1851. For the sake of reference it is here inserted:—

Table showing the percentage of Mortality, during the year 1851, *in the following Occupations.*

Occupations.	Ages.					
	25-35	35-45	45-55	55-65	65-75	75 and above.
1. Farmers . .	1·015	·864	1·199	2·490	5·530	14·802
2. Shoemakers. .	·912	1·059	1·503	2·869	6·505	16·446
3. Weavers . .	·797	1·056	1·537	3·299	7·459	17·308
4. Grocers . .	·763	1·046	1·579	2·265	4·972	12·457
5. Blacksmiths .	·812	1·240	1·651	3·724	7·443	16·710
6. Carpenters . .	·945	1·032	1·667	2·966	6·586	14·286
7. Tailors . .	1·163	1·415	1·674	2·818	7·647	15·528
8. Labourers . .	·979	1·252	1·730	2·920	6·790	17·394
9. Miners . .	·849	1·135	2·015	3·450	8·051	17·867
10. Bakers . .	·759	1·475	2·121	3·301	6·678	15·066
11. Butchers . .	1·130	1·653	2·310	4·149	6·647	15·449
12. Innkeepers . .	1·383	2·045	2·834	3·897	8·151	18·084
All England . .	·948	1·236	1·787	3·031	6·396	14·055

From this we learn that, taking the mortality at middle life (ages 45 to 55), out of every 1,000 farmers, 12 die annually; out of every 1,000 shoemakers and weavers, 15; out of every 1,000 grocers, 16; out of every 1,000 blacksmiths, carpenters, tailors, and labourers, 17; out of every 1,000 miners, 20; out of every 1,000 bakers, 21; and out of every 1,000 butchers, 23; whilst

out of every 1,000 innkeepers and publicans, no fewer than 28 die annually:—the yearly mortality at the ages stated being rather under 18 per 1,000 in the general population of the country.

Ten years afterwards similar observations were made by the Registrar-General as applicable to the deaths of males upwards of 15 years of age in the years 1860–61, and published by him in the supplement to his Twenty-fifth Annual Report. Some slight transpositions are here apparent, but, as in the Table of 1851, the Farmer class remained at the head of the list, and the Innkeeper class at the bottom. Again, in the supplement to his last Report—the Thirty-fifth—the deaths occurring in the year 1871 are tabulated for various occupations, and added to those previously given for 1860–61; but with the unvarying result that the highest rate of mortality is to be found among Publicans,—this mortality exceeding that of the general population (males above 15) by about 55 per-cent.

The following remarks apply to the experience of the Scottish Amicable Society during its first fifty years, in connection with this class of lives; and it is hoped that this small contribution to the statistics of the subject, may, notwithstanding the comparatively limited number of lives under observation, aid in determining, with some approach to accuracy, the addition which should be made to such risks, so as to place them on a more equitable footing with the general body of assurers.

As the occupation under review is not well defined, and the unsettled nomenclature of its various branches makes classification somewhat difficult, it is right to state that in the following returns are included all innkeepers, licensed victuallers, publicans, spirit-merchants, beer-sellers, travellers in connection with any of these trades, steam-boat stewards, and generally all persons connected with the sale of intoxicating liquors. Wine merchants, not carrying on any retail business, are not included.

The Society, I find, has assured 361 persons directly engaged in the liquor traffic. Of these 102 have died, 115 have surrendered or relinquished their policies; leaving 144 members of this class still on the Registers.

It was at first thought scarcely worth the labour of taking out the exact experience at the different ages, and consequently an approximate calculation was made by extracting from the "Journal of Benefits", an equal number of lives entering next before and next after each publican. This basis of comparison, experience had shown, would probably be sufficiently accurate to enable an estimate to be made of the aggregate experience among Publican

as compared with Non-Publican Lives. The following Table gives the result:—

Scottish Amicable Society's experience of "Publican" as compared with Non-Publican Lives.

Lives Entering.	Number.	Whereof		
		Died.	Cancelled.	In Force.
Next before each Publican . .	361	76	104	181
Next after " " . .	361	60	144	157
	722	136	248	338
Mean of these	361	68	124	169
Publicans	361	102	115	144

Showing no fewer than 34 deaths, (102—68), or, singularly enough, exactly fifty per-cent excess of mortality in the Publican class.

But, on second thoughts, it was deemed better to adopt the more exact method of ascertaining the actual years of risk, and it is somewhat striking to observe how closely the results correspond with the approximate method referred to. The following Table shows the numbers at risk, and the actual deaths as compared with the deaths to be expected by the Carlisle, the Actuaries, and the English Tables of Mortality respectively:—

Table showing the Actual Mortality Experience of the Scottish Amicable Society among Publicans and Innkeepers.

Ages.	Number at Risk.	Deaths.	Deaths expected by		
			Carlisle Table.	H^M Table.	English No. 2 (Males).
20 to 25	25	...	·176	·168	·211
25 " 30	107	3	·918	·758	·992
30 " 35	314	5	3·179	2·561	3·216
35 " 40	547	13	6·012	5·192	6·350
40 " 45	675	16	9·527	7·321	9·097
45 " 50	630	13	9·060	8·603	10·095
50 " 55	488	19	7·335	8·559	9·426
55 " 60	331	12	7·136	7·880	8·317
60 " 65	213	9	7·796	7·296	7·821
65 " 70	114	5	5·047	5·625	6·008
70 " 75	68	1	4·550	5·031	5·177
75 and upwards	17	6	1·744	1·853	1·814
	3529	102	62·480	60·847	68·524

The mortality being thus 63 per-cent in excess of the Carlisle Table; 68 per-cent in excess of the Actuaries Table; and 49 per-cent in excess of the English Table No. 2 (Males).

Having arrived at this conclusion, it will be interesting to enquire into the causes of death of these Publicans, and so endeavour to account for the excessive mortality among their class. The following table has accordingly been prepared, showing the causes of death and the percentage thereof under each class of disease to the whole number of deaths. This percentage again is compared with two Tables, namely—the Society's own General Mortality Experience, which was published shortly after its Fifth Septennial Investigation, and the Ordinary English Death Rate between ages 15 and 75, as given in the Eighteenth Report of the Registrar-General. The classification of the Registrar-General in his earlier reports has been adopted as more convenient for analytical comparison than the form since introduced by him. It will be observed that there were seven cases of "Dropsy", and in accordance with Dr. Farr's classification these fall to be entered under the Diseases of Uncertain Seat. As, however, it is now generally conceded that Dropsy is not an independent or primary disease, but rather the *sequela* of Cardiac, Hepatic, or Renal complaints,—and would scarcely now be accepted by the Medical Adviser of a Life Office as a sufficiently explicit term,—the method adopted by Dr. J. G. Fleming in his "Medical Statistics of Life Assurance" has been followed—the seven cases of Dropsy being distributed over the three diseases mentioned, giving to "Liver" (as perhaps best entitled to it) the odd seventh.

NOSOLOGICAL TABLE, *showing the Deaths among Publicans assured by the Scottish Amicable Society,—and the percentages thereof, as compared with the General Mortality Experience of the Society, and with the Ordinary English Death Rate.*

Causes of Death (Registrar-General's Classification).		Deaths.	Percentage under each Disease.	Percentage of Deaths as compared with The Society's own Experience.	Percentage of Deaths as compared with The English Life Table.
1. ZYMOTIC DISEASES—					
Small-pox	1	9	8·824	19·405	16·030
Diarrhœa	3				
Fever	3				
Erysipelas	2				
2. DISEASES OF UNCERTAIN SEAT—					
Hæmorrhage	1	5	4·902	3·881	3·820
Cancer	4				
Dropsy 7 (see above)	–				
3. TUBERCULAR DISEASE—					
Phthisis		16	15·686	13·842	25·170
4. DISEASES OF NERVOUS SYSTEM—					
Apoplexy	3	22	21·569	16·817	9·800
Paralysis	3				
Disease of Brain	11				
Delirium Tremens	2				
Epilepsy	3				
5. HEART DISEASE		13	12·746	8·667	7·050
6. BRONCHITIS		10	9·804	10·092	11·310
7. DISEASES OF DIGESTIVE ORGANS—					
Disease of Stomach, &c.	9	21	20·587	12·549	8·770
Obstruction of Bowels	1				
Disease of Liver	11				
8. DISEASE OF KIDNEY		3	2·941	2·975	3·000
9. GENERATIVE ORGANS		...	...	·388	1·800
10. RHEUMATISM		1	·980	·905	·980
11. INTEGUMENTARY TISSUES		...	...	·517	·210
12. MALFORMATIONS		...	...	...	·010
13. PREMATURE BIRTH, &c.		...	...	...	·460
14. ATROPHY		...	...	...	2·060
15. OLD AGE		2	1·961	2·070	2·740
16. UNASCERTAINED CAUSES		...	...	2·329	2·450
17. VIOLENT DEATHS		...	...	5·563	4·340
Total		102	100·	100·	100·

After giving effect to the correction referred to, it will be seen from the Nosological Table given above, that the greatest number of deaths occurred from diseases of the Nervous System, the Organs of circulation, and the Digestive Organs. The mortality in these three classes is greatly in excess of that of the general population, and about 30 per-cent, 50 per-cent, and 60 per-cent

respectively in excess of the general mortality experience of the Society itself. The deaths from Diseases of Uncertain Seat are also in excess of the percentage under both Tables, notwithstanding the elimination of the seven cases of Dropsy. On the other hand Tubercular Disease and Bronchitis are as much as 40 per-cent, and 13 per-cent respectively under the expectation. The deaths by Contagious and Infectious Diseases are greatly under the expectation, being no less than 45 per-cent below the ordinary death rate, and 55 below the Society's general experience. This surely is an unexpected result in a class a large proportion of which must of necessity be specially exposed to such diseases by coming in contact with numbers of people of a degraded type.

As it is the Digestive Organs and the Nervous System which suffer most from the abuse of stimulants, so we are prepared to find that the greatest mortality occurred in connection with them—no less than 43 out of the 102 deaths being traceable to these diseases. Thus—

Deaths among Publicans from Diseases of the Nervous System and of the Digestive Organs.

	Deaths.	Percentage of Total.	Percentage by	
			The Society's own Experience.	The English Table.
Nervous System . .	22	21·569	16·817	9·800
Digestive Organs . .	21	20·587	12·549	8·770
Together . . .	43	42·156	29·366	18·570

The excessive development of these diseases making the deaths 45 per-cent above the general experience of the Society, and about 130 per-cent above the ordinary English death rate.

Another unlooked for result in investigating this subject is, that it is not in the lowest branches of the trade that the greatest mortality is to be found. The returns from time to time bring out the embarrassing fact that the humble beer-seller belongs to the healthiest section of the trade, whilst the innkeeper belongs to the most unhealthy. It would also appear that the mortality is greater in the large towns than in the country. The following figures are obtained from the last supplement issued by the Registrar-General.

Annual Mortality per-cent among Publicans in London,—and in England exclusive of London,—as compared with the Mortality of the Male Population of all Classes.

Ages.	London.		England ex London.	
	Publicans.	General Population.	Publicans.	General Population.
15 to 25	·686	·703	1·003	·727
25 „ 35	1·642	1·086	1·407	·972
35 „ 45	2·324	1·714	1·981	1·281
45 „ 55	3·766	2·568	2·797	1·812
55 „ 65	5·487	4·385	4·228	3·154
65 „ 75	10·383	8·283	7·088	6·489
75 and above	32·692	18·451	21·034	16·288

Although somewhat apart from the subject, reference may be here made to Mr. F. G. P. Neison's "Contributions to Vital Statistics", in which he gives some curious data collected as to confirmed Drinkers. If the returns,—obtained as they were,—may be relied on, they go to show that the mortality among confirmed drinkers is, between the ages of 20 and 30, fully five times, and between the ages of 30 and 50, fully four times that of the community at large; and that the average duration of life after falling into intemperate habits is, among working men, about 18 years; among merchants and tradesmen, 17 years; among professional men, 15 years; and among females, 14 years. The observations extended over 6,111 years of life, in which 357 deaths had occurred, as against 110, the rate for the general population. They also showed that the intemperate indulgence in distilled liquors is, as might have been expected, more hurtful than the abuse of fermented liquors;—the rate of mortality among the Beer-drinkers being about 4½ per-cent yearly, and among the Spirit-drinkers 6 per-cent.

The Publican class is of course a very important section of the community, numbering, as it does, nearly ONE HUNDRED THOUSAND MEN, representing in the aggregate, as was somtime ago stated in Parliament, the enormous capital of ONE HUNDRED AND SEVENTEEN MILLIONS STERLING. We know that in this large and useful body there are many respectable and strictly temperate men, and no one would wish to detract from their merits, or try to make out a case against the class. Still, as Dr. Farr observes, if we admit that many of them are temperate, the mortality of the intemperate among them must be excessive, seeing that the mortality of the trade continues to be so great; and without doubt, as has been

shown, the excess of mortality arises chiefly from diseases induced or aggravated by excessive drinking.

Dr. B. W. Richardson, who has made the National Health his special study, in a recent article in one of the magazines, makes the following striking remarks:—

"The most startling fact of all in reference to occupation and health is that which is told of the innkeepers and publicans. This class of the community is really at the lowest of the vital scale. The cause, unhappily, is not difficult to discover. There is nothing in the occupation of an innkeeper, as an occupation, which can account for its unhealthiness on ordinary grounds of labour. It is not an occupation which exposes those who fill it to physical danger, as the work of the miner or the engine-driver does. It is not an occupation which makes great demands on the physical organism, like that of the blacksmith or rope-maker. It is not an occupation which leads men into solemn charges and responsibilities, like those of the physician, solicitor, or clergyman. It is not an occupation which brings those who follow it to the miseries of want and starvation. Why, then, is it the occupation most nearly allied to death? The answer is simply told. The occupation is the one the most nearly allied to alcohol. This agent of death, which diffuses danger more or less amongst all classes of workers in our part of the garden of life, tempts most rapidly into destruction those who are the dispensers of it."

Notwithstanding all that has been said, we find that Dr. J. A. Allen, Medical Adviser to several of the American Companies, in his "Examinations in Life Insurance"—a treatise which has gone through half-a-dozen editions—in dividing the different occupations into four classes, places in the highest or "least dangerous" class, innkeepers, saloon-keepers, brewers, rectifiers, and steam-boat stewards. As we are not aware that there is any great difference in the habits of the vendors of intoxicating drinks on this and on the other side of the Atlantic, it is not easy to see how such a conclusion is arrived at. It will be satisfactory, however, to the profession, to know that he also places "Actuaries" (as well as Physicians) in this select class!

Let us now endeavour to elicit from the interesting facts obtained some deduction of a practical character. We know that each particular pursuit has its own attendant risks, and that the lives daily accepted by Assurance Offices at the same premiums, are by no means of the same vitality. But it would be impracticable to frame Tables of Rates for each department of skill and industry.

The deviation in other occupations is perhaps not so great as to justify the suggestion of such a course. Still, in the class of lives now under review, it is obvious that the marked excess in the mortality proves the necessity for a higher rate of premium than is adopted for the general body of the community. The Publican class is undoubtedly the lowest in the scale of vitality, and consequently the most disadvantageously placed in relation to the value of life. The figures of the Scottish Amicable Society, it is thought, afford the means of determining, at least approximately, the proper rate of extra to be imposed. The result of the first rough estimate, as explained, agrees almost exactly with the more scientific investigation thereafter made, and proves that the mortality in the Publican class is just about 50 per-cent above that of the English Life Table, or the general male population of England above the age of 15.

Consequently, applying the figures, we have

$$\frac{102-68{\cdot}524}{3529}\times 100 = {\cdot}948$$

showing that an extra rate of premium, amounting to 19*s*. per-cent per annum, is necessary to cover the extra risk attaching to the class. It may be added, that the Scottish Amicable for a number of years past has been in the habit of imposing an extra of £1 per-cent per annum in all such cases, and it is therefore satisfactory to know that this loading so closely meets the extra risk now indicated by its actual experience of half a century.

HOME AND FOREIGN INTELLIGENCE.

THE NATIONAL MUTUAL LIFE ASSOCIATION OF AUSTRALASIA, LIMITED.

REPORT BY THE ACTUARY, MR. J. M. TEMPLETON,

On the Investigation of the affairs of the Association, made with a view to the ascertainment of the Surplus Fund at 30th September 1874, and the Division thereof among the Members.

I.—Introductory Narrative.

Since the Association was established on the 20th August 1869, it has experienced not only all the difficulties which usually surround a young institution, but at the very outset of its career, it had to contend with circumstances peculiarly trying to all life assurance companies, even those of the oldest standing. Before the first year had elapsed, news was received of the stoppage of the Albert Assurance Company—a company whose policyholders were to be

found in nearly every part of the world—and a wide-spread distrust of the safety of life assurance was thereby engendered in the public mind. Scarcely had the first impression of this died away, before it was followed by the disastrous failure of the European Assurance Society—a society whose business was immense and distributed through every part of the globe, and which in the two years preceding its failure had done a greater amount of new business than almost any other life assurance company in Great Britain. These two failures, which are without parallel in the history of life assurance, and which resulted in serious loss to many persons resident in this and the neighbouring colonies, brought life business almost to a stand still for some time. Many persons who had policies on their lives either surrendered them or allowed them to lapse, and the uninsured were heard to congratulate themselves that they had never insured their lives.

Summary of New Business transacted.

Year ending.	No. of Policies issued.	Amount Assured.	Annual Premiums.	Single Premiums.
		£	£ s. d.	£ s. d.
30th September 1870	202	85,400	2,978 16 4	Nil.
" " 1871	94	32,450	1,205 12 8	Nil.
" " 1872	161	60,670	1,995 14 3	Nil.
" " 1873	289	92,780	3,028 2 6	Nil.
" " 1874	395	108,200	3,736 16 3	57 9 0
Total . . .	1,141	£379,500	£12,945 2 0	£57 9 0

Extension of Agencies.

Since the Office was established, the Directors have taken every opportunity of extending the Agencies of the Association throughout Victoria. Considerable difficulty has been experienced in obtaining suitable agents; and in consequence of this the Association is still unrepresented in many country towns, where there are fair prospects of business. Two or three travelling agents are, however, kept continually employed, and with their assistance the Directors hope before the lapse of many months to cover the colony of Victoria with a network of Agencies.

Interest Realized.

A large part of the funds has been lent to the members of the Association on their Life Policies with personal security,—a kind of investment, not only safe, but highly profitable, and at the same time peculiarly adapted to the requirements of a young Office. These loans being made repayable by quarterly instalments limited to three years, preclude the necessity of keeping large sums of money uninvested for the purpose of meeting claims as they arise; and, while the members are only charged a moderate rate of interest, the funds are made to yield a great profit by the continual re-lending of the quarterly instalments as they come in.

During the five years, the average rate of interest realized on the

funds has grown from slightly under 4 per-cent to 9¼ per-cent. This high rate of interest is the result of developing the loan business concurrently with the issue of Life Policies.

Expenses of Management.

The Expenses of Management of Life Offices are necessarily much heavier in proportion to the income during the first years of their business than during any subsequent period. This arises partly from the preliminary and other expenses incidental to the establishment of the business, and partly from the fact that the first year's premiums of all policies entail much heavier expenses upon the office than renewal premiums; for instance, Medical Fees, Expenses of Travelling, Agents, and Advertising, are all incidental to getting new business—these expenses fall upon the first year's premiums, and are not afterwards incurred in the collection of renewal premiums. To these items may be added Commission to Brokers; for since the office was established, the Directors have paid a commuted commission out of the first year's premiums for the introduction of new business, and have thus avoided the liability to pay commission on renewal premiums. This method involves additional expenditure in the first year, but saves an expenditure of a very much greater amount spread over future years.

Eliminating the items above mentioned, the other expenses during the five years have been at the rate of 15 per-cent on the annual premium income. This is, to a great extent, the result of the agreement made with The National Insurance Company of Australasia, Limited, for the joint use of offices and staff—an arrangement which has considerably reduced the expenses of both institutions. Every item of expenditure has been included in the accounts and written off; therefore, no part of the expenses of the first five years is left as a charge upon the income of future years. No members have been admitted at lower rates of premium than those published in the ordinary prospectus; hence, the liabilities under policies are those only for which adequate premiums have been, and are charged.

Increase of Resources.

The Net Funds and Revenue have increased during the five years as shown below:—

	Funds.	Annual Revenue.
	£ s. d.	£ s. d.
1869—August 20th (commencement)	Nil.	436 8 9
1870—September 30th	781 3 5	2,739 8 10
1871 " "	2,283 18 2	3,650 18 8
1872 " "	3,873 4 11	5,546 11 5
1873 " "	8,585 17 11	8,635 17 4
1874 " "	15,464 16 9	12,068 4 2

Mortality Experience.

During the five years only eight deaths have occurred among the members of the Association, involving claims to the amount of £8,200,

which have been duly paid. According to the "Institute of Actuaries H^M Table" the expected deaths were 20 in number, and the expected claims were £6,650 in amount.

The difference between the expected and actual claims amounts to £3,450, but it would be improper to consider this amount to be profit, as the payment of the expected claims is not saved but only deferred.

II.—INVESTIGATION OF THE AFFAIRS OF THE ASSOCIATION AT 30TH SEPTEMBER 1874.

In fixing the principles upon which the valuation of Policies should be made, I had to keep in view the fact that the Association, by the issue of policies, had entered into contracts, many of which would endure thirty or forty years, or even longer; I had therefore to assume a rate of interest which we might expect our funds to realize during these long terms, and fixed 4 per-cent per annum as the basis of calculation. I do not anticipate that this rate will be objected to as too high; to those who consider it too low, I have to say it is on the safe side, and any error thereby caused will be rectified at the periodical investigations, when the surplus interest realized is divided among the members. As regards the *proportion of annual premiums to be reserved for future expenses and profits*, the only sound method appeared to me to be to reserve the whole of the loading contained in the office yearly premiums, as this was the only course by which future profits would be left for future division.

In the valuation of policies, all the calculations were made by two independent methods, and thus all errors were discovered and rectified. My labour in making these calculations was very much lessened by the use of the Arithmomêtre imported by me a few years ago; and it is due to the inventor and patentee, M. Thomas de Colmar, of Paris, that I should make this acknowledgment.

Referring to the summary and valuation of policies, it will be observed that I have inserted two items as liabilities which are not usually found in the accounts of life assurance companies. The first is, "*Reserve to provide for increased mortality in the future.*" My reason for making this reserve is, that I firmly believe that the "H^M Table" used in this valuation represents very closely the mortality which will be experienced by this Association, and that if during one short period the mortality experienced falls short of the expectation, it will be counterbalanced in another period by a corresponding excess. The amount reserved is the difference between the amount which the Association would have had to pay, if the number of deaths since the office was established had coincided with the number expected according to the "H^M Table", and the amount of the claims actually incurred and paid in the same period. The second item is "*Reserve for lives below the average standard.*" For an explanation of this, see pages 51, 52. The reserve thus made is the total amount of additional loading paid by those lives who did not exercise the option of paying at rate for age, with interest added at the rate of 4 per-cent per annum. The contingent debts endorsed upon the policies of those who did exercise the option of paying at rate for age, amount to £2,838, which forms a contingent reserve for

the supposed additional risk undertaken; but the item, for obvious reasons, does not appear in the accounts.

It will also be observed that the amount of office yearly premiums as stated on page 52 differs from the annual premium income as stated in the Report submitted to the members on the 19th November 1874. This is explained by the fact that in the former, the annual premiums according to the tables of the association are only taken into account; while in the latter, the half-yearly, quarterly, and monthly instalments of the annual premiums actually payable, are taken into account. The difference, amounting to about £240 per annum, is practically an additional provision for future expenses and profits, over and above that mentioned on page 50.

The Valuation Balance Sheet shows that the business of the five years has produced a clear surplus of £6,098. 13*s*. 4*d*.

III.—Division of Surplus among the Members.

Of this surplus of £6,098. 13*s*. 4*d*., the Directors have decided to set aside £3,000 as a Reserve Fund to meet unforeseen contingencies. The amount thus reserved from division upon this occasion with the interest accruing thereupon will, however, if no unforeseen contingency arise, be divided amongst the present members at the next investigation; and the *pro rata* share of those who may die in the interval, will then be paid to their representatives, unless they may have previously received it. By treating the Reserve Fund in this way every member has the assurance that he or his heirs will receive the full value of his contribution thereto. This plan is almost identical with that adopted by the Scottish Widows' Fund in dealing with its Guarantee Fund.

The balance of the surplus £3,098. 13*s*. 4*d*. will, in accordance with Clause 88 of the Articles of Association, be divided among the members whose policies are dated previous to the 1st day of October 1873, "in proportion to the amounts contributed towards such profit on account of such policies." On this occasion the amount of loading paid on account of each policy is taken as the measure of its contribution to the profits, and is made the basis of the division.

At future investigations this mode of apportioning profits will be modified so as to provide equitably for the division of the profit arising from the interest realized upon the Assurance Fund exceeding the assumed rate (4 per-cent). The profit from this source will be considerable, and should be divided among the members in proportion to their several interests in the Assurance Fund which produces it.

IV.—Conclusion.

In submitting this Report it is a source of gratification to me to refer to the results now declared as evidence of the soundness of the basis upon which the Association is established. In 1869 when it was announced, on my recommendation, that the Association about to be established would apply the surrender values of policies in the payment of premiums which might be overdue from any cause whatever, and thus keep the policies in force for the benefit of the policyholders, many persons took alarm at this evidence of liberality on the part of a new Office, and prophesied utter failure. The promoters of the

Association were prepared for opposition to this feature, as it had never previously been adopted by any company or society doing life assurance business in these colonies, nor, so far as could be ascertained, had it ever been adopted by any British life assurance company. In 1875, such has been the effect of the introduction by the Association of this just and equitable feature, that four other offices now advertize that they have adopted it, a striking illustration of the echo which the principle has produced in the public mind.

REPORT OF THE CONSULTING ACTUARY (Mr. G. E. COWLEY) TO THE MEMBERS.

I have the honour to report that I have examined the formulas used by Mr. Templeton, your Actuary, for the valuation of the policies of various classes of Assurance granted by the Association, and certify that the same are correct. I have also inspected the Schedules containing the detailed valuations and certify them to be in proper form.

In my opinion the special reserve to be made on account of the extra and special premiums received is equitable, and the further reserve proposed to be made to provide for increased mortality is both correct and equitable, and more than usually prudent in an office so comparatively short a time in existence.

The writing off of the whole of the commuted commission as well as the whole of the Medical Fees, though both these items might be spread over a much longer period, is equitable to all parties, and, I think, should be a strong inducement to future entrants, as also should be the extinguishment of the amount paid to the guarantors of the sums assured while the Association was in its infancy.

The Mortality table upon which the valuations have been made is, I think, by far the most applicable to the first valuation of your policies. In future valuations I think the $H^{M(5)}$ Table should be used for policies which have been in existence four years and had five premiums paid.

With the method of dividing the profits, proposed by Mr. Templeton, I perfectly concur, and consider it the most equitable at the first division, but it will not be applicable to subsequent divisions.

Balance Sheet *on the* 30*th September* 1874.

Liabilities.				£	s.	d.
Assurance Fund, as per Third Schedule				15,464	16	9
Claims admitted but not paid				500	0	0
Other Sums Owing by the Company—						
Sums held on Deposit	£1,300	0	0			
Accrued Interest thereupon	28	3	3			
				1,328	3	3
Directors' Fees	£95	11	0			
Office Expenses	262	10	0			
Expenses of Agencies	77	5	9			
				435	6	9
				£17,728	6	9

ASSETS.		£ s. d.
IN VICTORIA—		
New Zealand Government Debentures		910 0 0
Mortgages		2,459 15 3
Loans on Policies within their Surrender Value		117 10 0
Loans on Policies with Personal Security		11,818 2 6
Agents' Balances		184 17 4
Outstanding Premiums		598 12 10
Outstanding Interest		172 17 2
CASH—		
On Deposit	£1,000 0 0	
In hand and on Current Account	466 11 8	
		1,466 11 8
Total Assets in Victoria		£17,728 6 9
ELSEWHERE THAN IN VICTORIA		Nil.
Total Assets		£17,728 6 9

This Association has not yet granted any Annuities or Endowments.

The following additional particulars are taken from the answers to the Eighth and Ninth Schedules of the (Victoria) Life Assurance Companies Act, 1873.

The principles upon which the valuation and distribution of profits are to be made are not determined by the Articles of Association or by any regulations or bye-laws; but are left to the discretion of the Actuary of the Association. The Articles of Association, however, require that the profits (after reserving such an amount as the Directors may consider necessary to meet unforeseen contingencies) shall be rateably apportioned as a bonus amongst the policies entitled to participate in profits, in proportion to the amounts contributed towards such profit on account of such policies since the next preceding investigation.

The policies for the whole term of life were valued in classes according to the year of birth of the persons assured; thus, for example, all persons born between the first April 1830, and the 31st March 1831, have been treated as of one age, and as if all born on the 30th September 1830.

The future premiums were valued by annuities deferred six months, that being the average time at which the next renewal premiums would fall due.

Policies on Joint Lives and on Last Survivor were valued each separately, the ages being calculated to the nearest birthday as above described, and the future premiums were valued by annuities deferred the exact proportions of a year to the dates of next renewal respectively.

All other policies were valued separately, the values both of sums assured and of future premiums being calculated to the nearest month; and in these cases it has been assumed that the persons assured attained their office ages exactly on the days on which their policies were issued.

In cases where the premiums were payable by half-yearly or quarterly instalments, the corresponding annual premiums according to the

Tables of the Association were valued, and the instalments necessary to complete the current year of policies were valued separately, as their payment is secured by the conditions of the policy.

The liability of the Association is determined by taking the difference between the present value of the sums assured and the present value of the future net premiums. The net premiums valued were those upon which the Tables of Rates of the Association were originally constructed, and were deduced from the Tables of Mortality known as "The English Life Table", with 4 per-cent interest.

The profits will be divided among the members entitled to participate, in proportion to the loading contributed by members in respect of premiums paid during the quinquennium, and interest thereupon to 30th September 1874. A proportionate part of the loading paid in respect of policies under Table II (single and annual premiums limited in number) will, however, be reserved for future division.

The "Institute of Actuaries H^M Table" is the only Table used in the valuation.

In all the calculations the rate of interest assumed has been 4 per-cent per annum.

The whole of the "Loading" has been reserved for future expenses and profits. This amounts on the average to 23·58 per-cent of the premium income, and its present value is £35,699.

A policy must be twelve months in force, in order to entitle it to share in the profits.

Specimen bonuses to policies of £100 each which have been in force for five years (six premiums paid)—

Age at Entry.	Reversionary Addition.	Cash Value.	Reduction of Premium for whole term of Life.
	£ s. d.	£ s. d.	£ s. d.
20	6 8 5	1 7 1	0 1 10
30	5 18 6	1 12 3	0 2 4
40	5 12 7	2 0 8	0 3 2
50	5 16 0	2 14 6	0 5 2

N.B.—The amount of Profit reserved from division on this occasion (£3,000) is sufficient to provide further bonuses nearly equal to the above, and these will be allotted to the present members at the next investigation in 1877, if no unforeseen contingency arise. In comparing the bonuses with those allotted by other Societies, therefore, the amounts in the above Tables should be doubled.

CONSOLIDATED REVENUE ACCOUNT *for the period commencing 20th August* 1869 *and ending* 30*th September* 1874.

	£ s. d.
Amount of Funds on 20th August 1869	Nil.
Premiums (no Reassurance Premiums)	28,671 0 10
Consideration for Annuities granted	Nil.
Interest (no Dividends)	2,294 2 6
Fines and Fees	159 7 11
	£31,124 11 3

		£ s. d.
Claims under Policies (no Reassurances)		3,200 0 0
Surrenders		82 7 11
Annuities		Nil.
Commission	£2,337 11 7	
Agency Expenses	933 8 9	
		3,271 0 4
Expenses of Management		7,029 7 5
Preliminary Expenses		517 5 11
Commission on Original Guarantee		1,537 2 11
Bad and Doubtful Debts (written off)		22 10 0
Amount of Funds on 30th September 1874		15,464 16 9
		£31,124 11 3

VALUATION BALANCE SHEET, *as at 30th September* 1874.

	£ s. d.
Net Liability under Assurance and Annuity transactions	9,366 3 5
Surplus	6,098 13 4
	£15,464 16 9

	£ s. d.
Life Assurance Fund	15,464 16 9
	£15,464 16 9

SUMMARY *and* VALUATION (see p. 52).

Since the Association was established, the average rate of interest at which the life assurance fund was invested has been as follows:—

On 30th September	1870—£3	19	0	per-cent per annum.			
"	"	1871	5	3	0	"	"
"	"	1872	7	8	0	"	"
"	"	1873	8	3	5	"	"
"	"	1874	9	6	8	"	"

Up to the present time there has been no table of minimum surrender values adopted by the Directors, but the minimum surrender values of policies for the whole term of life, with annual premiums payable during life, have been fixed at one third of the premiums actually paid by the members. No applications have been received for the surrender values of endowment assurance policies, or for policies under other classes of assurance.

This Association has not transacted business at other than European rates. But if members go to reside between the 25th parallels of north and south latitude, an extra premium varying from 10*s*. to £2 per-cent per annum is charged, according to the locality of residence. This extra premium is remitted when the members come to reside south of the 25th parallel of south latitude, but the amount of extra premium thus paid is not taken into consideration in fixing the surrender values.

This Association does not grant policies on unhealthy lives; but when the personal or family history, the occupation, or any other

Summary *and* Valuation.

Description of Transactions.	Particulars of the Policies for Valuation. Number of Policies.	Sums Assured. (No Bonuses.)	Office Yearly Premiums.	Loading Contained in Office Yearly Premiums.	Net Yearly Premiums.	Valuation. Value by the "Institute of Actuaries H^M Table". Interest 4 per-cent. Sums Assured. (No Bonuses.)	Office Yearly Premiums.	Loading Contained in Office Yearly Premiums.	Net Yearly Premiums.	Net Liability.
ASSURANCES.										
1.—With Participation in Profits.		£	£	£	£	£	£	£	£	£
For Whole Term of Life	784	269,163	8,541	1,969	6,572	102,926	128,267	29,576	98,691	4,235
Do. Limited Number of Premiums	15	5,550	254	37	217	2,022	1,991	291	1,700	322
Endowment Assurances—"One Life"	127	31,400	1,414	295	1,119	15,274	17,015	3,551	13,464	1,810
Do do. "Joint Lives"	1	100	(a) Nil.	(a) Nil.	(a) Nil.	52	(a) Nil.	(a) Nil.	(a) Nil.	52
Joint Lives	41	10,200	480	113	367	4,963	5,932	1,404	4,528	435
Last Survivor	1	600	19	6	13	242	331	104	227	15
Extra Premiums Payable	...	...	32	32	...	(b) 74	...	...	...	(b) 74
Instalments of Annual Premiums to complete current year of Policies	...	...	...	...	...	...	(c) 3,136	721	2,415	1,035
Reserve to provide for increased mortality in the future	...	...	...	...	...	(d) 3,450	...	...	...	
Reserve for Lives below the Average Standard	...	...	...	...	...	(e) 1,349	...	...	...	(e) 1,349
Total Assurances With Profits	969	317,013	10,740	2,452	8,288	130,352	156,672	35,647	121,025	9,327
2.—Without Participation in Profits.										
Short Term Policies	4	2,700	41	16	24	97	127,817	52	75	22
Extra Premiums Payable	...	...	17	17	...	(b) 17	...	...	...	(b) 17
Total Assurances Without Profits	4	2,700	58	33	24	114	127	52	75	39
Total Assurances	973	319,713	10,798	2,485	8,311	130,467	156,800	35,699	121,101	9,366
No Annuities or Endowments	...	...	...	...	...	...	...	...	...	...

No Reassurances. All Policies have been issued in Victoria.

(a) This policy was paid for by a single premium of £57. 9s. 0d. (b) These are the whole amounts received for extra premiums since the office was established. (c) The payment of these instalments is secured by the conditions of the policy. (d) The amount thus reserved is the difference between the expected claims, according to the H^M Table of Mortality, and the actual claims since the office was established. (e) This is the actual amount paid by those lives in excess of the minimum premium since the office was established, with interest added at the rate of four per-cent per annum.

circumstances connected with the persons proposed for assurance, lead the Directors to believe that their prospects of longevity are not quite so good as those of persons in every respect unexceptionable, their proposals are accepted at rates for ages somewhat in advance of their actual ages. These persons, when accepted, however, have the right to complete their policies at the minimum rates for their actual ages, subject to a contingent debt, equal to the value of the additional premium sought to be charged, to be deducted from the amount of the policy if the person assured die under the average age attained by persons of his age; but when he attains the said average age, the contingent debt is expunged and no additional premium is charged. If such persons elect to pay the rates for such advanced ages, the surrender values of their policies are fixed as if such advanced ages were their actual ages.

The Directors have now adopted a table of minimum surrender values for future use, and specimens of its application are given in the following table:—

Minimum Surrender Values of Policies of £100 under Table I payable during the whole term of life, exclusive of Bonus additions.

Age at Entry.	Minimum Surrender Value at the end of				
	3 Years.	5 Years.	10 Years.	15 Years.	20 Years.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
20	1 12 0	3 2 7	6 18 7	11 4 2	16 1 6
30	2 8 0	4 12 9	9 18 2	16 0 8	22 16 2
40	3 11 2	6 17 8	14 9 11	22 16 6	31 13 2
50	5 4 1	9 18 7	20 9 2	31 0 5	41 10 2
55	6 5 3	11 16 8	23 14 2	35 9 11	46 8 2

The publication of this report gave rise to considerable correspondence in the insurance and other papers, in the course of which the following facts were stated by the actuary:—

1. If the expenditure of the office had amounted to 80 per-cent of the first year's premiums and 7½ per-cent of the renewal premiums, the total expenses would have been 40 per-cent of the premiums; but the actual expenditure was 43 per-cent of the premiums, of which 7 per-cent is represented by the preliminary expenses in establishing the Association and the expense of a guarantee for the payment of expenses and claims during the first three years of the business.

2. The accumulated fund, at the date of valuation, was nearly 54 per-cent of the total premiums received, or over 58 per-cent of the total premiums received on account of existing policies.

3. The premiums payable for the whole term of life were originally calculated on the basis of the English Table No. 1, the interest assumed being 4 per-cent; and the addition for expenses, contingencies, and profits, was made in the form of a constant 4*s.* and a percentage of 15 per-cent of the net premiums; and the loading reserved has been

in all cases (both in cases accepted at the ordinary rate of premium and in cases where an extra premium was charged) the difference between the English 4 per-cent net premium for the real age at entry and the premium actually charged.

Mr. Templeton also states that he was conscious that his valuation produced negative values for a considerable number of the policies, but adds, that his special reserve of £3,450, being the difference between the actual claims during the first five years and the amount of claims which was to be expected according to the Institute H^M table of mortality, was more than sufficient to eliminate the whole of the negative values. Mr. Templeton further states that the Victoria Life Assurance Companies Act gives to any five policyholders power to apply to the Supreme Court to wind up a company; and if such application be made, the Act provides that the policies must be valued by the 17 Offices' Experience Table, 4 per-cent, and the pure premium by the same table only is to be valued. "In the face of this Act, no prudent actuary would endanger his company by dividing more profit than the above method of valuation will produce." This is a point of so much interest that we have reprinted at length the Act in question, in order that our readers may have the opportunity of referring to it for themselves.

In consequence of the comments made on Mr. Templeton's report, the Directors deemed it desirable to obtain and circulate widely the opinion of Mr. A. H. Bailey, Actuary of the London Assurance Corporation, as to the principles on which the valuation was made and the results it exhibited, and the following extracts are taken from his opinion:—

The principles on which the valuation has been made are as follow:—

The rate of interest assumed has been for all purposes 4 per-cent per annum.

The present value, according to the Institute of Actuaries' H^M Table of Mortality, has been calculated of the sums assured, and of the future annual premiums; and there has been reserved for future expenses and profits the present value by this table of the excess of the premiums actually charged over the corresponding premiums of the English Table No. 1. The "Net liability", therefore, of the Report is the difference between the present values by the H^M Table of Mortality of the sums assured and of the annual premiums which by the English Table corresponded to these sums at the ages of the lives when the policies were effected. Further reserves have been made—(1) of the amount which has been received for extra premiums on policies where the lives assured were considered to be below the average, together with interest thereon; (2) of the difference between the calculated amount of claims by the H^M Table and the amount of claims that actually arose; (3) of a sum of £3,000 for contingencies. On the other hand, credit has been taken for what is stated to be the amount of net premiums corresponding to the unpaid half-yearly, quarterly, and monthly instalments required to complete the current year's premiums stipulated for by the terms of the policies.

The following is the result of a valuation made in this manner:—

Assurance Fund			£15,464
Estimated Liability . . .		£6,981	
Further Reserves for—			
Increased Mortality . .	£3,450		
Under-Average Lives. .	1,350		
Contingencies . . .	3,000		
	£7,800		
Less for Instalments of current year's Premiums .	2,415		
		£5,385	
			£12,366
Surplus divided			£3,098

These principles of valuation have now to be considered.

The rate of interest has, I think, been judiciously chosen.

I agree also that the Institute of Actuaries H^M Table is the most suitable one; and that, for the purpose of ascertaining the amount of surplus divisible, what is called a "net premium", or better, a "pure premium" valuation should be made. For this purpose the premiums payable have to be divided into two parts, the "net" or "pure" premium, and the "loading"; the former to provide the sum assured, the latter for expenses, profits, and contingencies. In making the valuation, either the present value of the net premiums only is taken into account; or, what will produce the same result, the gross premiums are valued, and the present value of the loading appears on the other side of the account.

In the present case the office premiums are those of the English Table No. 1, with certain additions; and it is therefore contended that the "net premiums" of the valuation must be the premiums of the English Table, and none other. This, I think, is a mistake. In whatever way the premiums of any life office are formed, they are controlled, and that very effectually, by competition. The additions made to the pure premiums, and termed the "loading", are by no means arbitrary. Such rates of premium must be charged as are obtainable in practice, and these are by no means always what are most desirable.

But, in a valuation, the question is not how the premiums have been constructed, but, taking them as they are, what portion will, according to the best judgment that can be formed, suffice for the risk, and what is the margin beyond this? In other words—what is the prime cost, and what the gross profit? Apply these principles to the valuation now under discussion. It is allowed that the H^M Table best represents the mortality among assured lives. But the English Table shows a heavier mortality, so that the premiums deduced from it are higher. And, therefore, when these premiums are taken to be the prime cost of the assurance, and at the same time their present value calculated by the H^M Table, the effect is to overstate the credit side of the account, and to anticipate the surplus of the future, to the extent of the present value of the difference between the English and H^M premiums.

[On this Mr. Templeton comments as follows:—I clearly see the

applicability of this method of reasoning to the valuation of a Proprietory Life Office which sells life insurance for the purpose of making a profit; but I cannot see that it applies to a Mutual Life Office, in which the aim should be to give every member the full value of what he pays, and make no profit. So long as the whole of the "loading" originally intended for future expenses and profits is reserved—unless it be insufficient for these purposes, I cannot see how the surplus of the future can be anticipated in any way.]

If it had been desired to adhere to the data on which the office premiums were based, the valuation should have been made throughout by the English Table, and to this, I think, there would have been no particular objection. It would have been still better, I think, if the valuation had been made throughout by the H^M Table. But the combination of the two tables is inadmissable, and has the effect of treating some policies as assets, while, under a proper pure premium valuation, provision is made as a liability for every policy in force.

Again, under a correct pure premium valuation, I think that the extra reserve for under-average lives would have been unnecessary. The extra premiums on these policies constitute a further loading, which, not being taken into account in the valuation, is a source of income available for the extra claims that may annually arise from the increased mortality of this class of lives.

On the other hand, it seems to me that the sum taken credit for as "instalments of annual premiums" is a somewhat doubtful asset. No doubt these instalments will be deducted in cases of claims, but the payment of them cannot be enforced when policies are dropped. The item is therefore, in effect, a loan on policies without interest; and is a good asset, if at least, the amount in arrear has been reserved in the valuation as a liability under the policies to which it applies. I should hardly think that this can be the case in the present instance, although, of course, I have not sufficient information to speak positively.

[Mr. Templeton says:—This item did not include any "amount in arrear", but consisted entirely of half-yearly, quarterly, and monthly instalments of premiums, amounting to £3,200. 1*s*. 1*d*., not then due, but to become due after the date of my investigation, of which the present value at that date was £3,136, or deducting the loading, £2,415. These instalments have since been collected, and yielded over £3,130 in hard cash.]

From some approximate calculations which Mr. Templeton has furnished, the probable results can be given of a valuation on what, in my judgment, would have been the correct principles—

(*a*) By the English Table—	
Assurance Fund	£15,464
Estimated Liability	13,382
Surplus	£2,082
(*b*) By the H^M Table—	
Assurance Fund	£15,464
Estimated Liability	13,617
Surplus	£1,847

I have subjected Mr. Templeton's calculations to some tests, and am satisfied of their substantial accuracy.

In these valuations no account has been taken of £3,136, the unpaid instalments of the current year's premiums. If this item be estimated at £1,251, then the surplus which has been divided would have resulted from a proper H^M pure premium valuation. Again, if this item be estimated at £1,016, the surplus which has been divided would have been brought out by a like valuation by the English Table. Now, I think it will not be contended that either of these is an unreasonable estimate; and if that be admitted, it follows that the sum of £3,098 was fairly divisible as surplus.

[As to this, Mr. Templeton says:—The Insurance Commissioner of Massachusetts, U.S., an officer appointed by the State Legislature, whose duty it is to strike out doubtful assets from the accounts of Life Assurance Companies, considers this a good asset. While he strikes out such assets as "Cash in the hands of Agents", "Furniture", &c., he allows the whole amount of this asset, with a deduction of 10 per-cent, thus: £3,200. 1*s*. 1*d*., less £320. 0*s*. 1*d*., equal to £2,880. 1*s*.; and therefore, on this basis, it follows that the surplus available for division was, by the H^M Table pure premium method, £4,727, and by the English Table, £4,962.]

The general result, therefore, is that the principles on which the valuation was made are incorrect. Apart from the reasons for this conclusion which I have given, it will be observed that the results of the valuation were not acted upon, but that further reserves were made amounting altogether to nearly 78 per-cent of the estimated liability. This would have been an unreasonable proceeding if the valuation had been properly made; and is, I think, a practical condemnation of the principles which were adopted.

Mr. Bailey concludes his report by saying that, these large extra reserves having been made, there was no justification for the assertion that had been made, that the Association was not in a position to declare a bonus and that its assets were not sufficient to meet its liabilities.

Finally, Mr. Templeton remarks:—It will be observed that Mr. Bailey's method requires larger reserves, and therefore the error, if any, is on the side of safety; if this were its only characteristic, I should at once recommend its adoption. But the following objections present themselves to my mind:—1st. The method is based upon an abstract theory, and takes no cognizance of the future premiums actually payable; *e.g.*, the net liability of £100. 0*s*. 2*d*., in the example calculated above, would have been exactly the same if the annual premiums had been £35 or £40 instead of £30. 15*s*.; and therefore I consider the method inapplicable to an office which seeks to give to its members the full value in assurance of the premiums they pay. 2nd. When applied to the National Mutual, it has the effect of making the members who enter at ages under 40, contribute more than their fair share to future expenses, contingencies, and

profits, and this injustice is greatest at the youngest ages. The following table will illustrate my meaning:—

Age at Entry.	Proportion of Annual Premium originally intended to Provide for Future Expenses and Profits, termed "Loading".	Proportion of Annual Premium Reserved for Future Expenses and Profits by the method proposed by Mr. Bailey.	Percentage of addition to the Original Loading necessitated by the method proposed by Mr. Bailey.
20	22·7 per-cent.	32·2 per-cent.	42 per-cent.
30	20·5 "	28·4 "	38 "
40	18·7 "	23·5 "	26 "
50	17·0 "	18·7 "	10 "
60	15·4 "	17·5 "	13½ "

Thus, by Mr. Bailey's method, the loading would be increased by 42 per-cent to members who enter at age 20, while it would be increased by only 10 per-cent to those who enter at age 50. 3rd. The method is one which, for the majority of policies recently issued, requires reserves of amounts greater than could possibly be saved out of the premiums received on account of such policies, and thus has the effect of treating them as if their issue caused an immediate loss to the Association. For these reasons I am unable to recommend the adoption of the method proposed by Mr. Bailey *so long as we retain our present rates of premium.*

Life Insurance Act of the Colony of Victoria.

We are indebted to Mr. J. M. Templeton, the Actuary and Secretary of the National Mutual Life Association of Australasia, for a copy of the Act that regulates Life Insurance Companies in Victoria. It appears to be based to a great extent on the Life Insurance Acts of the Imperial legislature, but deals with several matters not within the scope of those Acts, and we therefore think a reprint of the Act will be acceptable to our readers. We would specially direct attention to the following enactments:—§§ 8, 9, 10, as to companies "having secured assets in Victoria"; such assets, which may be increased from time to time, are primarily charged with the satisfaction of the liabilities of the company in Victoria, and must not be removed from the Colony until the whole of such liabilities shall be paid in full: § 11, as to companies whose principal place of business is elsewhere than in Victoria; these are required to keep a separate account of the business and of the assets in Victoria; and in the event of the company being adjudged bankrupt or insolvent, or being ordered to be wound up, no part of the assets in Victoria is to be applied in payment of liabilities elsewhere, until the liabilities in Victoria are paid in full: § 26, as to novations by policyholders: § 35, as to the valuation of policies when application is made to wind up a company: § 37, protecting life policies to the extent of £1000 against creditors: § 38, providing

that the amount of a policy not exceeding £100 may be paid without taking out probate or letters of administration: § 39, as to assignments of policies: § 40, as to mortgages and trusts.—Ed. *J. I. A.*

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37 Vict. No. 474.

An Act to amend the Law relating to Life Assurance Companies.

[25th *November* 1873.]

Be it enacted by the Queen's Most Excellent Majesty by and with the advice and consent of the Legislative Council and the Legislative Assembly of Victoria in this present Parliament assembled and by the authority of the same as follows:—

1. This Act may be cited as "*The Life Assurance Companies Act* 1873". Short title.

2. In this Act—

The term "company" means any person or persons corporate or unincorporate who issue or are liable under policies of assurance or endowment upon human life within Victoria or who grant annuities upon human life within Victoria, and whether the head office or principal place of business of such person or persons is in Victoria or elsewhere. Interpretation of terms. 33 and 34 Vict., c. 61, s. 2.

The term "chairman" means the person for the time being presiding over the board of directors committee of management or other managing body in Victoria of the company.

The term "life assurance business" means the granting of policies of assurance or endowment or annuities on human life.

The term "policyholder" means the person who for the time being is the legal holder of the policy for securing the life assurance endowment annuity or other contract with the company.

The term "financial year" means each period of twelve months at the end of which the balance of the accounts of the company is struck, or if no such balance is struck then each period of twelve months ending with the thirty-first day of December.

The term "Court" means the Supreme Court of the Colony of Victoria.

The term "Judge" means a Judge of the Court.

The term "Registrar-General" means the Registrar-General of Victoria.

3. Every company transacting other business besides life assurance business shall keep a separate account of all receipts after the passing of this Act in respect of the life assurance business of the company, and the said receipts shall be carried to and form a separate fund to be called the life assurance fund of the company, and such fund however invested shall be as absolutely the security of the life and endowment policy and annuity holders as though it belonged to a company carrying Life funds separate. 33 and 34 Vict., c. 61, s. 4. 34 and 35 Vict., c. 41, s. 2.

on no other than life assurance business, and shall not be liable for any contracts of the company for which it would not have been liable had the business of the company been only life assurance business. In respect to all existing companies the exemption of the life assurance fund from liability for other obligations than to its life and endowment policy and annuity holders shall have reference only to the contracts entered into after the passing of this Act unless by the constitution of the company such exemption already exists; but this section shall not apply to any contracts made by any existing company by the terms of whose deed of settlement or articles of association the whole of the profits of all the business are paid exclusively to the life and endowment policy or annuity holders, and on the face of which contracts the liability of the assured distinctly appears: Provided always that this Act shall not diminish the liability of the life assurance fund for any contracts of the company entered into before the passing of this Act.

Registration. First Schedule. 4. Any company desirous of being registered under this Act shall lodge in the office of the Registrar-General a memorandum in the form contained in the First Schedule to this Act signed by the chairman and the principal officer or agent managing the life assurance business of the company in Victoria and verified by a statutory declaration of the persons so signing. A copy of the memorandum and declaration shall within seven days after the day of lodgment be published in the *Government Gazette*, and a copy of the *Gazette* containing such publication shall be forwarded by such company to the office of the Registrar-General, to be there retained and filed with the memorandum.

Registrar-General to keep a register-book. 5. The Registrar-General shall keep a register-book to be entitled "The Assurance Companies' Register-book," and on receipt of the said *Gazette* containing such publication as aforesaid shall enter the date of such receipt, and shall write and sign at the foot of the memorandum originally lodged the words "The above company was on the day of 18 registered by me, as a company having [*or* not having] secured assets in Victoria," and thereupon such company shall be deemed to be registered under this Act.

Company to be registered. 6. No company shall after the expiration of six months from the passing of this Act commence any new life assurance business within Victoria unless it be registered as a company under this Act.

Company to deposit sum with Treasurer. 7. Every company which, after the passing of this Act, shall commence to carry on the business of life assurance within the colony of Victoria shall before accepting any premiums in connection with such business deposit the sum of Five thousand pounds with the Treasurer to be invested by him in the particular fund under his control indicated by the company which shall receive the income therefrom, and the registrar shall not issue a certificate of registration until such deposit shall have been made, and the Treasurer shall retain such deposit until the life assurance fund accumulated out of the premiums shall have amounted to Fifteen thousand pounds, when the amount so deposited shall be refunded to the company.

Either as having secured assets in Victoria or not.

8. Every company registered under this Act shall at the option of such company be registered either as a company having secured assets in Victoria or as a company not having secured assets in Victoria.

Registration of increase of secured assets.

9. Every company registered under this Act as a company having secured assets in Victoria may from time to time increase the amount of such secured assets by lodging with the Registrar-General a memorandum in the form contained in the Second Schedule to this Act signed by the chairman and the principal officer or agent managing the life assurance business of the company in Victoria. The Registrar-General shall enter such memorandum at the foot of the registry of such company in the Assurance Companies' Register-book, and a copy of every such memorandum shall be forthwith published in the *Government Gazette*.

Second Schedule.

Secured assets in Victoria if registered company primarily charged with liabilities in Victoria.

10. In the case of every company registered under this Act as a company having secured assets in Victoria such secured assets to the amount at which the same shall from time to time be registered shall be primarily charged with the payment or satisfaction of all the liabilities of the company in Victoria; and no part of such secured assets shall after such registration be removed from Victoria or be applied in payment of any liabilities of the company other than those so charged as aforesaid until the whole of such last mentioned liabilities shall be paid in full, but nothing herein shall limit or affect the application of any such assets in making or varying any investment thereof in Victoria. Any director agent officer or servant of any company wilfully committing or aiding or assisting in the commission of any breach of this section shall be deemed guilty of a breach of trust and be held liable to replace the amount of all such secured assets as shall be by him or with his aid and assistance removed from Victoria or applied contrary to the provisions of this section, and shall also be deemed guilty of a misdemeanour, and being convicted thereof shall be liable at the discretion of the Court to be imprisoned for any term not exceeding three years or to a penalty not exceeding Five hundred pounds.

Assets of foreign companies in Victoria.

11. Every company whose head office or principal place of business is not in Victoria shall keep a separate account of all the business transacted in Victoria and of the entire assets of the company in Victoria, whether registered as secured assets or not, and in the event of the company becoming bankrupt or insolvent or being ordered to be wound up the entire assets of the company in Victoria shall be applied so far as the same will extend in or towards satisfaction of the liabilities of the company in Victoria, and no part of such assets shall be applied in payment of any liabilities of the company incurred elsewhere than in Victoria until the whole of the liabilities incurred in Victoria shall be paid in full. If any such company be adjudged bankrupt or insolvent or be ordered to be wound up elsewhere than in Victoria then the same company so far only as regards its assets and liabilities in Victoria may upon the application of one or more policyholders or shareholders be ordered to be wound up in Victoria in like manner as if such company were registered under "*The Companies Statute* 1864", and proof of such company having been so

adjudged bankrupt or insolvent or ordered to be wound up shall be conclusive evidence that it is unable to pay its debts.

Name of company to be written on policy, &c.

12. Every company registered under this Act shall cause to be written or printed after the name of such company, upon the face of every policy and receipt for premium issued by it after the registration of such company the words "Registered under '*The Life Assurance Companies Act* 1873', as a company having secured assets in Victoria [*or* "not having secured assets in Victoria", as the case may be].

Change of chairman, &c., to be registered.

13. Notice signed by the chairman and the principal officer or agent managing the life assurance business of the company in Victoria of any change of chairman director principal officer or agent in Victoria of such company, and containing the name address and occupation of every newly appointed chairman director principal officer or agent in Victoria, shall within seven days after any such change be lodged with the Registrar-General, who shall enter the same at the foot of the registry of such company in the Assurance Companies' Register-book, and a copy of every such notice shall be forthwith published in the *Government Gazette*.

Statements to be made by companies.
33 and 34 Vict., c. 61, s. 5.
Third and Fourth Schedules.

14. Every company transacting life assurance business only shall at the expiration of each financial year of such company prepare a statement of its revenue account for such year and of its balance-sheet at the close of such year in the forms respectively contained in the Third and Fourth Schedules to this Act.

Statements by companies doing other than life business.
Ib., s. 6.
Fifth and Sixth Schedules.

15. Every company which concurrently with the transaction of life assurance business transacts any other kind of assurance or other business shall at the expiration of each financial year of such company prepare a statement of its revenue account for such year and of its balance-sheet at the close of such year in the forms respectively contained in the Fifth and Sixth Schedules to this Act.

Statement by foreign companies.
Seventh Schedule.

16. Every company whose head office or principal place of business is not in Victoria shall at the expiration of each financial year of such company prepare in addition to all other statements required by this Act a statement of all its policies in force at the close of such year in the form contained in the Seventh Schedule to this Act.

Actuarial report and abstract.
33 and 34 Vict., c. 61, s. 7.
Eighth Schedule.

17. Every company shall once in every five years, or at such shorter intervals as may be prescribed by the instrument constituting the company or by its articles of association regulations or bye-laws, cause an investigation to be made into its financial condition by an actuary and shall cause an abstract of the report of such actuary to be made in the form prescribed in the Eighth Schedule to this Act.

Statement of life and annuity business.
Ib., s. 8.
Ninth Schedule.

18. Every company shall on or before the thirty-first day of December One thousand eight hundred and seventy-five, and thereafter within nine months after the date of each such investigation as aforesaid into its financial condition, prepare a statement of its life assurance and annuity business in the form contained in the Ninth Schedule to this Act, each of such statements to be made up as at

the date of the last investigation, whether such investigation be made previously or subsequently to the passing of this Act: Provided as follows:—

(I.) If the next financial investigation after the passing of this Act of any company fall during the year One thousand eight hundred and seventy-six the said statement of such company shall be prepared within nine months after the date of such investigation instead of on or before the thirty-first day of December One thousand eight hundred and seventy-five.

(II.) If such investigation be made annually by any company such company may prepare such statement at any time so that it be made at least once in every three years.

The expression "date of each such investigation" in this section shall mean the date to which the accounts of each company are made up for the purposes of each such investigation.

19. The Governor in Council may alter the forms contained in the Schedules to this Act for the purpose of adapting them to the circumstances of any company or of better carrying into effect the objects of this Act. Forms may be altered. Ib., s. 9.

20. Every statement or abstract hereinbefore required to be made shall be signed by the chairman and two of the directors or committee of management or by the agent of the company in Victoria and by the principal officer or agent managing the life assurance business of the company in Victoria, and if the company has a managing director in Victoria by such managing director, and shall be printed, and the original so signed as aforesaid together with three printed copies thereof shall be deposited at the office of the Registrar-General within nine months of the dates respectively hereinbefore prescribed as the dates at which the same are to be prepared; and every annual statement so deposited after the first investigation after the passing of this Act shall be accompanied by a printed copy of the abstract required to be made as aforesaid. Statements, &c., to be signed and printed and deposited with Registrar-General. Ib., s. 10.

21. A printed copy of the last deposited statement abstract or other document by this Act required to be printed shall be forwarded by the company by post or otherwise to every shareholder member and policyholder of the company in Victoria. Copies of statements to be given to shareholders, &c. 33 and 34 Vict., c. 61, s. 11.

22. Every proprietary company shall provide a book to be called the "Shareholders' Address Book," in which the company shall cause to be entered from time to time in alphabetical order the corporate names and places of business of the several shareholders of the company being corporations and the surnames of the several other shareholders with their respective Christian names places of abode and descriptions so far as the same shall be known to the company; and every policyholder or shareholder or if such shareholder or policyholder be a corporation the clerk or agent of such corporation may at all convenient times peruse such book gratis, and the company shall furnish on application to every shareholder and policyholder of the company a copy of such book or of any part thereof on payment of a sum not exceeding sixpence for every hundred words to be copied for such purpose. List of shareholders. Ib., s. 12.

Deed of settlement to be printed. Ib., s. 13.

23. Every company which is not registered under "*The Companies Statute* 1864" shall cause a sufficient number of copies of its deed of settlement act or charter of incorporation or other instrument regulating the constitution of the company to be printed, and shall furnish on application to every shareholder and policyholder of the company a copy thereof on payment of a sum not exceeding Two shillings and sixpence.

Amalgamation or transfer. Ib., s. 14.

24. Where it is intended to amalgamate two or more companies or to transfer the life assurance business of one company to another the directors of any one or more of such companies may apply to the Court by petition to sanction the proposed arrangement, fourteen days previous notice of such application being published in the *Government Gazette*, and the Court after hearing the directors and other persons whom it considers entitled to be heard upon the petition may confirm the same if it is satisfied that no sufficient objection to the arrangement has been established.

Before any such application is made to the Court notice of such application together with a statement of the nature of the amalgamation or transfer as the case may be and an abstract containing the material facts embodied in the agreement or deed under which such amalgamation or transfer is proposed to be effected and copies of the actuarial or other reports upon which such agreement or deed is founded shall be forwarded to each policyholder of both companies in case of amalgamation or to each policyholder of the transferred company in case of transfer by the same being transmitted through the post directed according to the registered address or other known address of such policyholder within such period as to admit of being delivered in the due course of delivery fourteen days at least before the day named for the hearing of such application, and in proving such service it shall be sufficient to prove that such notice was properly addressed and put into the post office; and the agreement or deed under which such amalgamation or transfer is effected shall be open for the inspection of the policyholders and shareholders at the office or offices of the company or companies for a period of fifteen days after the issuing of the abstract herein provided.

The court shall not sanction any amalgamation or transfer in any case in which it appears to the Court that policyholders representing one-fifth or more of the total amount assured in any company which it is proposed to amalgamate, or in any company the business of which it is proposed to transfer dissent from such amalgamation or transfer.

No company shall amalgamate with another or transfer its business to another unless such amalgamation or transfer is confirmed by the Court in accordance with this section: Provided always that this section shall not apply in any case in which the business of any company which is sought to be amalgamated or transferred does not comprise life assurance business.

Statement in case of amalgamation or transfer. 33 and 34 Vict. c. 61, s. 15.

25. When an amalgamation takes place between any companies, or when the business of one company is transferred to another company, the combined company or the purchasing company as the case may be shall within ten days from the date of the completion of the amalga-

mation or transfer deposit at the office of the Registrar-General certified copies of statements of the assets and liabilities of the companies concerned in such amalgamation or transfer together with a statement of the nature and terms of the amalgamation or transfer, and a certified copy of the agreement or deed under which such amalgamation or transfer is effected, and certified copies of the actuarial or other reports upon which such agreement or deed is founded; and the statement and agreement or deed of amalgamation or transfer shall be accompanied by a declaration under the hand of the chairman of each company and the principal managing officer of each company that to the best of their belief every payment made or to be made to any person whatsoever on account of the said amalgamation or transfer is therein fully set forth, and that no other payments beyond those set forth have been made or are to be made either in money policies bonds valuable securities or other property by or with the knowledge of any parties to the said amalgamation or transfer.

Regulation as to novations by policyholders. 35 and 36 Vict., c. 41, s. 7.

26. Where a company, either before or after the passing of this Act has transferred its business to or been amalgamated with another company no policyholder in the first-mentioned company who shall pay to the other company the premiums accruing due in respect of his policy shall by reason of any such payment made after the passing of this Act or by reason of any other act done after the passing of this Act be deemed to have abandoned any claim which he would have had against the first-mentioned company on due payment of premiums to such company or to have accepted in lieu thereof the liability of the other company unless such abandonment and acceptance have been signified by some writing signed by him or by his agent lawfully authorized.

Inspection, &c., of deposited documents. 33 and 34 Vict., c. 61, s. 16.

27. Any person may on payment of such fees as the Governor in Council may direct inspect at the office of the Registrar-General any printed or other document required by this Act to be deposited at such office and procure copies thereof.

Documents to be received in evidence. Ib., s. 18.

28. Every statement abstract or other document deposited with the Registrar-General under this Act shall be receivable in evidence; and every document purporting to be certified by the Registrar-General to be such deposited document and every document purporting to be similarly certified to be a copy of such deposited document shall if produced out of the custody of the Registrar-General be deemed to be such deposited document as aforesaid or a copy thereof, and shall be received in evidence as if it were the original document unless some variation between it and the original document shall be proved.

Penalty for non-compliance with Act. Ib., s. 18.

29. Every company which makes default in complying with the requirements of this Act and shall continue in such default for three days after notice by the Registrar-General or any person interested in the matter of such default shall be liable to a penalty not exceeding Fifty pounds for every day during which the default continues; and in the case of companies registered under "*The Companies Statute* 1864" if default continue for a period of three months after notice of default by the Treasurer, which notice shall be published in one or more newspaper

as the Treasurer may direct, the Court may order the winding up of the company in accordance with "*The Companies Statute* 1864" upon the application of one or more policyholders or shareholders.

Penalty for falsifying statements, &c. Ib., s. 19.

30. If any statement abstract or other document required by this Act is false in any particular to the knowledge of any person who signs the same such person shall be guilty of a misdemeanour, and being convicted thereof shall be liable at the discretion of the Court to be imprisoned for any term not exceeding three years or to a penalty not exceeding One hundred pounds.

Application of penalties. 33 and 34 Vict., c. 61, s. 20.

31. Every penalty imposed by this Act shall be applied in the same manner as penalties imposed by "*The Companies Statute* 1864" are applicable.

Winding up of company. Ib., s. 21.

32. The Court may order the winding up of any company in accordance with "*The Companies Statute* 1864" on the petition of five or more policyholders or shareholders upon its being proved to the satisfaction of the Court that the company is insolvent. In determining whether or not the company is insolvent the Court shall take into account its contingent or prospective liability under policies and annuity and other existing contracts. The Court shall not give a hearing to the petition until security for costs for such amount as a Judge shall think reasonable shall be given and until a *primâ facie* case shall also be established to the satisfaction of the Judge. In the case of a proprietary company having an uncalled capital of an amount sufficient with the future premiums receivable by the company to make up the actual invested assets equal to the amount of the estimated liabilities, the Court shall suspend further proceedings on the petition for a reasonable time (in the discretion of the Court) to enable the uncalled capital or a sufficient part thereof to be called up, and if at the end of the original or any extended time for which the proceedings shall have been suspended such an amount shall not have been realized by means of calls as with the already invested assets shall be equal to the liabilities an order shall be made on the petition as if the company had been proved insolvent.

Winding up of subsidiary company to be ancillary to winding up of principal company. 35 and 36 Vict. c. 41, s. 4.

33. Where the business or any part of the business of a company has, either before or after the passing of this Act, been transferred to another company under an arrangement in pursuance of which such first-mentioned company (in this Act called the subsidiary company) or the creditors thereof has or have claims against the company to which such transfer was made (in this Act called the principal company), then if such principal company is being wound up by or under the supervision of the Court, either at or after the passing of this Act, the Court shall (subject as hereinafter mentioned) order the subsidiary company to be wound up in conjunction with the principal company, and may by the same or any subsequent order appoint the same person to be liquidator for the two companies and make provision for such other matters as may seem to the Court necessary with a view to such companies being wound up as if they were one company, and the commencement of the winding up of the principal company shall save as otherwise ordered by the Court be the commencement of

the winding up of the subsidiary company; the Court nevertheless shall have regard in adjusting the rights and liabilities of the members of the several companies between themselves to the constitution of such companies and to the arrangements entered into between the said companies in the same manner as the Court has regard to the rights and liabilities of different classes of contributories in the case of the winding up of a single company, or as near thereto as circumstances admit.

Where any subsidiary company or company alleged to be subsidiary is not in process of being wound up at the same time as the principal company to which it is subsidiary the Court shall not direct such subsidiary company to be wound up unless after hearing all objections (if any) that may be urged by or on behalf of such company against its being wound up the Court is of opinion that such company is subsidiary to the principal company, and that the winding up of such company in conjunction with the principal company is just and equitable.

An application may be made in relation to the winding up of any subsidiary company in conjunction with a principal company by any creditor or policyholder of or person interested in such principal or subsidiary company.

Where a company stands in the relation of a principal company to one company and in the relation of a subsidiary company to some other company, or where there are several companies standing in the relation of subsidiary companies to one principal company, the Court may deal with any number of such companies together or in separate groups as it thinks most expedient, upon the principles laid down in this section.

Court may reduce contracts. 33 and 34 Vict., c. 61, s. 22.

34. The Court in the case of a company which has been proved to be insolvent may if it thinks fit reduce the amount of the contracts of the company upon such terms and subject to such conditions as the Court thinks just, in place of making a winding-up order.

Valuation of annuities and policies. 35 and 36 Vict., c. 41, s. 5.

35. Where application is being made to the Court to wind up a company or where a company is being wound up by the Court or subject to the supervision of the Court or voluntarily, the value of every life annuity and life policy requiring to be valued shall be estimated in manner provided by the Tenth Schedule to this Act.

Tenth Schedule.

Rules in Tenth and Eleventh Schedules to be rules of Court. Ib., s. 6.

36. The rules in the Tenth and Eleventh Schedules to this Act shall be of the same force as if they were rules made in pursuance of the one hundred and fifty-fourth section of "*The Companies Statute* 1864," and may be altered in manner provided by the said section, and rules may be made under the said section for the purpose of carrying into effect the provisions of this Act with respect to the winding up of companies.

Interest of insured not liable to debts in certain cases.

37. The property and interest of any person to the extent of One thousand pounds in the whole in any policy or policies of assurance on his own life shall not be subject to be seized or taken in execution under the process of any Court, and in the event of the insolvency of such person shall not vest in the assignee or trustee of his estate unless such insolvency occurs within two years after the date of the policy, and on the death of such person

shall not be assets for the payment of his debts; but if he die within two years after the date of the policy a portion of the sum assured equal to the amount of premiums actually paid shall be assets for the payment of his debts. In this section the term "insolvency" includes liquidation by arrangement and composition with creditors under any Act now or hereafter to be in force.

Probate or administration may be dispensed with in certain cases.

38. Upon the death of any holder of a policy upon his own life for a sum not exceeding One hundred pounds if no probate of his will or letter of administration to his estate be taken out within three months after his death the company may pay the amount of such policy to his widow or any adult child of his, and the receipt of such widow or child shall be a valid discharge both at law and in equity for the same.

Assignment of policies. Twelfth Schedule.

39. Every assignment of a policy shall be by memorandum of transfer endorsed upon such policy in the form contained in the Twelfth Schedule to this Act and signed or in the case of a Corporation sealed by the transferror and transferree, and no assignment shall be of any validity until registered as hereinafter provided. Notice of every assignment shall be given to the company for the time being liable upon the policy assigned. Such assignment shall be registered in a book to be provided by the company for that purpose, and the date of such registration shall be inserted in the memorandum of transfer which shall also be signed by the principal officer managing the life assurance business of the company in Victoria, and thereafter the assignee may sue as well at law as in equity in his own name on the policy assigned, and the receipt of such assignee shall be a valid discharge both at law and in equity for all moneys payable thereunder. Every such memorandum of transfer signed as last aforesaid shall be conclusive evidence of the registration thereof and of the date of such registration.

No notice of mortgage or trusts.

40. If any policy be assigned by way of mortgage or upon any trust such mortgage or trust shall be effected by way of defeasance or declaration of trust by some separate instrument and no notice of any such mortgage or trust shall be entered on the memorandum of transfer nor endorsed on the policy, and the company shall not be affected either by express implied or constructive notice of any such mortgage or trust nor be bound or concerned to see to the application of any moneys payable under such policy.

Notices to policyholders. 33 and 34 Vict., c. 61, s. 23.

41. Any notice which is by this Act required to be sent to any policyholder may be addressed and sent to the person to whom notices respecting such policy are usually sent, and any notice so addressed and sent shall be deemed and taken to be notice to the holder of such policy.

Statements, &c., to be laid before Parliament. Ib. s. 24.

42. The Treasurer shall lay annually before Parliament the statements and abstracts of reports deposited with the Registrar-General under this Act during the preceding year.

SCHEDULES.

FIRST SCHEDULE.

Section 4.

THE COMPANY.

We the undersigned hereby make application to register the above-named company under the provisions of "*The Life Assurance Companies Act* 1873" as a company having [*or* not having] secured assets in Victoria:—

1. The name of the company is
2. The head office or principal place of business of the company is at street, London [*or as the case may be*].
3. The head office or principal place of business of the company in Victoria is at
4. The chairman of the company in Victoria is [*insert name in full address and occupation*].
5. The directors [*or* committee of management *or* managing body] of the company in Victoria are [*insert names in full addresses and occupations*].
6. The principal officer [*or* agent] managing the life assurance business of the company in Victoria is
7. The nominal capital of the company [*if any*] is pounds in shares of each.
8. The number of shares subscribed for is and the amount per share paid up to this date is
9. The amount of assets of the company now invested in Victoria and intended to be appropriated as secured assets within the meaning of "*The Life Assurance Companies Act* 1873" is [*in the case of a company to be registered as not having secured assets in Victoria insert* "nil"].

Dated this day of 18 .

A.B., Chairman,
C.D., Secretary [*or* Manager, *or* Agent, *or as the case may be*].

Witness to signatures—
E.F.

We, A.B. and C.D., do severally solemnly and sincerely declare that—

1. We are respectively the chairman and secretary [*or* manager *or* agent *or as the case may be*] of the above-named company.
2. The above statement is, to the best of our knowledge and belief, true in every particular.

And we make this solemn declaration conscientiously believing the same to be true, and by virtue of the provisions of an Act of Parliament of Victoria rendering persons making a false declaration punishable for wilful and corrupt perjury.

Taken before me, &c.,
, J.P.

Section 9.

SECOND SCHEDULE.

THE COMPANY.

INCREASE OF SECURED ASSETS IN VICTORIA.

We the undersigned hereby give notice that the amount of assets of the company now invested in Victoria and intended to be appropriated as secured assets within the meaning of "*The Life Assurance Companies Act* 1878" is pounds, being pounds in addition to the amount of pounds already registered.

A.B., Chairman,

C.D., Secretary [*or* Manager, *or* Agent *or as the case may be*].

Witness to signatures—
E.F.

Section 14.

THIRD SCHEDULE.

REVENUE ACCOUNT of the for the year ending .

18 . [*Date.*]	£ s. d.	18 . [*Date.*]	£ s. d.
Amount of funds at the beginning of the year . . .		Claims under policies (after deduction of sums re-assured) .	
Renewal premiums after deduction of reassurance premiums		Surrenders	
New premiums (after deduction of re-assurance premiums) on new policies assuring £ and yielding an annual revenue of £ after deduction of re-assurances . .		Annuities	
Consideration for annuities granted		Commission	
Interest and dividends . .		Expenses of management . .	
Other receipts (accounts to be specified)		Dividends and bonuses to shareholders (if any) . . .	
		Other payments (accounts to be specified)	
		Amount of funds at the end of the year as per Fourth Schedule	
	£		£

NOTE 1.—Companies having separate accounts for annuities to return the particulars of their annuity business in a separate statement.

NOTE 2.—Items in this and in the accounts in the Fifth and Seventh Schedules should be the net amounts after deduction of the amounts paid and received in respect of re-assurances.

FOURTH SCHEDULE.

Section 14.

BALANCE-SHEET of the on the 18 .

LIABILITIES.	£ s. d.	£ s. d.	ASSETS.	£ s. d.	£ s. d.
Shareholders' capital paid up (if any)			IN VICTORIA—		
Assurance fund			Victorian Government securities .		
Annuity fund (if any) .			Other Government securities (*particulars to be specified*) . .		
Other funds (if any to be specified) . . .			Mortgages		
			Loans on the company's policies .		
Total funds as per Third Schedule .			Loans upon personal security .		
Claims admitted but not paid* .			Railway and other debentures and debenture stocks . . .		
Other sums owing by the company (accounts to be specified)* . .			Railway shares (preferential and ordinary)		
			House property		
			Other investments (to be specified)		
			Agents' balances		
			Outstanding premiums . . .		
			Outstanding interest . . .		
			Cash—		
			On deposit		
			In hand and on current account		
			Other assets (to be specified) . .		
			Total assets in Victoria .		
			ELSEWHERE THAN IN VICTORIA—		
			British Government securities .		
			Indian and Colonial Government securities		
			Foreign Government securities .		
			Mortgages [*stating where*] . .		
			Loans on the company's policies .		
			Loans upon personal security .		
			Railway and other debentures and debenture stocks . . .		
			Railway shares (preferential and ordinary)		
			House property [*stating where*] .		
			Other investments (to be specified)		
			Agents' balances		
			Outstanding premiums . . .		
			Outstanding interest . . .		
			Cash—		
			On deposit		
			In hand and on current account		
			Other assets (to be specified) . .		
			Total assets elsewhere than in Victoria . . .		
		£	Total assets	£	

* These items are included in the corresponding items in the Third Schedule.

Section 15.

FIFTH SCHEDULE.

Revenue Accounts of the for the year ending .

(No. 1.)—Life Assurance Account.

[*Date.*]	£ s. d.	[*Date.*]	£ s. d.
Amount of life assurance fund at the beginning of the year .		Claims under life policies (after deduction of sums reassured)	
Renewal premiums after deduction of reassurance premiums		Surrenders	
New premiums (after deduction of reassurance premiums) on new policies assuring £ and yielding an annual income of £ after deduction of reassurances . .		Annuities	
Consideration for annuities granted		Commission	
Interest and dividends . .		Expenses of management . .	
Other receipts (accounts to be specified)		Other payments (accounts to be specified)	
		Amount of life assurance fund at the end of the year as per Sixth Schedule	
	£		£

Note.—Companies having separate accounts for annuities to return the particulars of their annuity business in a separate statement.

(No. 2.)—Fire Account, and (No. 3.)—Profit and Loss Account.

Are the same as in the British Act.

Section 15.

SIXTH SCHEDULE.

Balance-Sheet of the on the 18 .

Liabilities.	£ s. d.	Assets.	£ s. d.	£ s. d.
Shareholders' capital . . .		(As in the 4th Schedule.)		
General reserve fund (if any) .				
Life assurance fund* . . .				
Annuity fund (if any)* . . .				
Fire fund				
Marine fund				
Profit and loss (if any) . . .				
Other funds (if any to be specified)				
£				
Claims under life policies admitted but not yet paid* . . .				
Outstanding fire losses . . .				
Outstanding marine losses . .				
Other sums owing by the company (accounts to be specified) . .				
	£		£	

* If the life assurance fund is in accordance with section three of this Act, a separate trust fund for the sole security of the life policyholders, a separate balance-sheet for the life branch may be given in the form contained in Fourth Schedule. In other respects the company is to observe the above form. *See also* Note to Fourth Schedule.

SEVENTH SCHEDULE.

POLICIES of the in force on the 18 .

Number.	In Victoria.			Number.	Elsewhere.
	Assurance	for £	.		For £
	Endowment	" £	.		" £
	Annuity (per an.)	" £	.		" £
	Fire	" £	.		" £
	Marine	" £	.		" £
	Other policies (if any)	" £	.		" £

EIGHTH SCHEDULE.

Is the same as the 5th Schedule of the British Act, except that the form for the Summary and Valuation contains, below the totals of the columns, two additional lines, to contain the totals of the results "In Victoria" and "Elsewhere". Section 16

NINTH SCHEDULE.

Is the same as the 6th Schedule of the British Act. Section 18.

TENTH AND ELEVENTH SCHEDULES.

Are the same as the 1st and 2nd Schedules respectively of the British Act of 1872. Sections 35 and 36.

TWELFTH SCHEDULE.

Section 39.

MEMORANDUM OF TRANSFER.

Date of Transfer.	Signature of Transferror.	Witness.	Transferree.			Signature of Transferree.	Witness.	Date of Registration of Transfer.	Signature of principal Officer of Company.
			Name in full.	Address.	Occupation.				

CORRESPONDENCE.

MR. DEUCHAR'S PAPER ON NEGATIVE POLICY VALUES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The employment, in valuations, of negative policy values is a matter of such vital importance to life assurance offices, that the point raised in Mr. Deuchar's paper, that these will not affect an office valuing septennially so materially as one valuing more frequently, requires to be thoroughly proved before it be admitted; and as I think the arguments urged by him in favour of his proposition entirely inadmissible, perhaps you will allow me to point out wherein, in my opinion, the fallacy of his position lies.

I take two offices, A and B, starting together and each doing an annual new business of 1,000 policies of £300 each, age at entry 40,

premium £9. 12*s*., expenses 10 per-cent; both offices to value their gross premiums less 15 per-cent by the Institute tables at 4 per-cent interest, but to allow one third of the premiums paid as a surrender value after payment of three premiums. Then, if the rate of mortality experienced coincide with that to be expected by the Institute tables, and the rate of discontinuance be as assumed in column 2, the experience of the offices will be represented by the following table:—

Year.	Rate of discontinuance.	Entered.	Died.	Discontinued.
1	·060	1,000	10	60
2	·043	930	10	40
3	·039	880	9	34
4	·033	837	9	28
5	·030	800	9	24
6	·026	767	9	20
7	·024	738	10	18
8	·021	710	10	15
9	·019	685	10	13
10	·017	662	10	11
11	·015	641	10	10
12	·014	621	10	9
13	·012	602	11	7
14	·012	584	11	7

Let A value annually, and B septennially. The above-mentioned mode of valuation will have the effect of giving negative values to all policies for the first three years, and of making a reserve insufficient to pay the surrender value for a further period of three years. Now, assuming all the premiums due on, say, 1st January, and the claims all paid during the year, but the lapses and surrenders not to be notified to the office until the 1st January following, if the valuation be made on 31st December, the number of policies to be valued will be equal to the number of entrants of the year, less the deaths therein; and the full effects of taking credit for negative values will be felt, for the office will reckon as assets all the policies of less than three years' standing, some of which will lapse next day, and for many others it will make a reserve too small to pay the surrender value to be allowed on the morrow. The reserve in each year made by A will be as follows:—

Age of policies. Years.	No. of policies valued.	Value. £	Reserve. £
1	990	−13,151	− 13,151
2	920	− 7,877	− 21,028
3	871	− 3,209	− 24,237
4	828	+ 1,103	− 23,134
5	791	5,123	− 18,011
6	758	8,880	− 9,131
7	728	12,381	+ 3,250
8	700	15,658	18,908
9	675	18,780	37,688
10	652	21,752	59,440
11	631	24,622	84,062
12	611	27,378	111,440
13	591	29,973	141,413
14	573	32,491	173,904
		+198,141	
		173,904	
		− 24,237	

And the premiums received, claims, surrenders, and expenses, paid as in the following table:

Year.	Premiums.	Claims.	Surrenders.	Expenses.
*	*	*	*	*
8	63,955	22,800	1,062	6,396
9	70,531	25,800	1,350	7,053
10	76,886	28,800	1,641	7,689
11	83,040	31,800	1,923	8,304
12	89,002	34,800	2,210	8,900
13	94,781	38,100	2,499	9,478
14	100,387	41,400	2,745	10,039

Suppose 4½ per-cent to be the rate of interest at which the average of the funds at the beginning and end of the year is invested, and let us follow the proceedings of both offices from the commencement of the 2nd septennium, both beginning the year with funds in hand amounting to £3,250, to meet a valuation showing positive values of £27,487, and negative values of £24,237, and we shall find the results to be as follows:—

A.

Year.	Funds, 1st Jan.	Funds, 31st Dec.	Reserve.	Amount divided.
	£	£	£	£
8	3,250	37,872	18,908	18,964
9	18,908	56,943	37,688	19,255
10	37,688	79,071	59,440	19,631
11	59,440	104,133	84,062	20,071
12	84,062	132,016	111,440	20,576
13	111,440	162,303	141,413	20,890
14	141,413	195,189	173,904	21,285

B.

Year.	Funds, 1st Jan.	Funds, 31st Dec.
	£	£
8	3,250	37,872
9	37,872	76,780
10	76,780	119,963
11	119,963	167,443
12	167,443	219,235
13	219,235	275,061
14	275,061	334,990

Now, the amount of B's reserve at the end of the period will be the same as A's, and B will therefore distribute as profit (£334,990 less £173,904) £161,086. If, however, we accumulate at 4½ per-cent the various amounts divided by A until the end of the septennium, we shall find that they fall short of the amount now divided by B by £494; and the reason for this apparent deficiency is somewhat curious. The amount that A divides at the end of the 8th year is retained by B and forms part of those funds whose average amount at the beginning and end of the year is invested at 4½ per-cent, so that if m be the amount distributed by A, and n its amount at the end of the year, i being the rate of interest, we have—

$$\frac{m+n}{2}\times i=n-m,$$

$$n\left(1-\frac{i}{2}\right)=m\left(1+\frac{i}{2}\right),$$

whence

$$n=m\frac{1+\frac{i}{2}}{1-\frac{i}{2}}.$$

If $i=\cdot 045$,
we have $n=m\times 1\cdot 046036$;

so that the amounts divided by A must be invested at 4·6036 per-cent to amount to the sum now divided by B, since that is the rate B has made on these sums it has hitherto retained.

Year.	Sum divided by A.	Amount at 4·5 per-cent.	Amount at 4·6036 per-cent.
	£	£	£
8	18,964	24,696	24,844
9	19,255	23,995	24,115
10	19·631	23,411	23,503
11	20,071	22,905	22,973
12	20,576	22,470	22,514
13	20,890	21,830	21,852
14	21,285	21,285	21,285
		160,592	161,086

At first sight it would appear that A would distribute a larger amount than B, since, in each of the seven years, policies in the former office bearing negative values have been discontinued and the surplus shown by their means already distributed in cash; but a little consideration will show that this is not the case. The first year of the septennium A starts with assets of £3,250 in cash and £24,237 in negative values, against a positive liability of £27,487. Certain of these negative values turn out illusory, the policies representing them not being kept up; but as the liability originally set against them remains, there is a deficiency at the end of the year which has to be made good out of any actual profit A may have made during the year on its older policies. Then, in order to balance the account and to declare a bonus, A swells its funds with a fresh amount of negative values on which similar losses are incurred in the ensuing year, and have to be defrayed in like manner. The fundamental error in this argument is in supposing that A can distribute as profit cash in hand, and set negative values against positive liabilities without having in the following year to repay with interest the loss arising out of its negative values before the office proceeds to make another division. B in the early part of the septennium suffers the same losses as A, through taking into account the negative values with which they both begin the period under observation; but as B makes no valuation and no distribution, it does not make the profit which A claims to do, and consequently does not incur the losses that A suffers, but goes on year by year increasing its funds until the end of the septennium, when it has a surplus according to the method of valuation employed, of £136,849 (£334,990 less a positive liability of £198,141); but not satisfied with this B swells its funds—or what comes to the same thing, reduces its liability —by the same amount of negative values, £24,237, that A makes use of in the same year, and divides £161,086, a sum as I have shown above, equal in amount to the several sums divided by A during the septennium.

I am, Sir, your obedient servant,

5 *Whitehall, S.W.*, W. T. GRAY.
2 *Feb.* 1876.

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On Mutual Life Assurance; its Aims and Objects, and the Means of attaining them. By JOHN M. TEMPLETON, *Actuary and Secretary of the National Mutual Life Association of Australasia, Limited, and Fellow of the Institute of Actuaries.*

[Read before the Institute 27 November 1876.]

I MAY say at the outset, that the object of the present paper is not to discuss the merits of Mutual Assurance, as distinguished from the other systems of Life Assurance, but to consider whether there are not special circumstances connected with that system which demand the attention of the professional actuary.

In the discussion of a paper read before the Institute last session, one of the speakers is reported to have said, "There is, of "course, a very marked difference between the mutual and pro- "prietary companies. In the one case the rate of profit has only "to be considered, but in proprietary offices the profit to the "shareholders has to be thought of". I would add to that statement, that the difference is so very marked that, under certain circumstances, they require very different modes of treatment.

The main difference between mutual societies and other life assurance companies is, that in the former there are no proprietors except the policyholders, who themselves conduct the business for

the sake of getting the greatest amount of assurance at the smallest possible cost, while in the latter the proprietors are a distinct body, who conduct the business for the sake of making a profit out of it.

The ultimate aim of a mutual assurance society may be thus expressed,—to grant to its members assurances of a value as nearly as possible equal to that of the premiums which they pay—in other words—to grant assurances at cost price. If the rate of mortality, the rate of interest, and the expenses of management, were matters of absolute certainty, and could be explicitly expressed, the objects of the society would be carried out by calculating tables of premiums under which every member would be charged the exact premium necessary to provide the sum assured at his death and to pay his share of the expenses of management; and in such a case, of course, there never would be any surplus or profit, nor would there be any necessity for periodical valuations of policies. But as these elements are all of them more or less uncertain, and liable to fluctuations, the premiums must be calculated with margins sufficient to provide against the greatest probable adverse fluctuations.

Having calculated a table of the net or pure premiums necessary to cover the risk of assurance, according to the table of mortality which he considers to be the best exponent of the probabilities of the future, and the minimum rate of interest which will most probably be realized during the whole period of the duration of the policies, the actuary is called upon to decide what loading shall be added to these premiums to provide for the expenses of management and for future contingencies. As this loading is intended to provide for contingencies which are unknown and expenses which are uncertain, its amount must be fixed arbitrarily by the actuary, who will be guided by his own experience and foresight—the error in the amount of loading should be on the side of prudence and caution; for the safety and success of the institution depend upon its being ample. But not only should it be ample, but it should also be equitable as between the various members—for every member should contribute his fair share to the fund for expenses and contingencies. The ordinary method is to add a percentage to the risk premiums, but this method appears to me to bear too heavily on the members at the higher ages:—thus, a member entering at age 60 would be compelled to contribute to the fund for expenses and contingencies four times as much annually as the member entering at age 20, simply because he necessarily pays a risk

premium four times as great—yet we know that the expenses incurred in connection with policies do not increase in proportion to the risk premium. Another method, recently recommended by some actuaries, is to add a compound loading, consisting of a percentage on the risk premium and a constant at all ages. This appears to me to be a more equitable method, and is the one I myself should recommend. Before leaving this question of the construction of premiums, I may say, that while I believe that the office annual premium should be uniform throughout life, as being the most convenient to the bulk of insurers, yet I do not think it necessarily follows that its component parts—risk premium and loading—should each be uniform throughout life. I can see no objection to the method of calculating the net premiums, mentioned by Mr. Curtis in the discussion on Mr. Macfadyen's paper, read before the Institute in May 1875, namely, that they may be made small for the first year of assurance and increased afterwards arbitrarily until they become uniform for the remainder of life—the office annual premium being made uniform throughout, by adding a large loading for the first year and decreasing it until it becomes uniform for the remainder of life. This should, of course, be subject to the proviso quoted by Mr. Curtis from the 258th paragraph of "*Jones on Annuities*", that the pure premiums "in "the first instance be sufficient to cover the risk of the term during which they are payable, namely, *not less than the annual premium "for a temporary assurance for the same term*". This method of constructing the office premiums is more consistent with the purposes for which they are intended, and it has this advantage, that it enables the office to pay the heavy expenditure inseparable from the obtaining of new business, without encroaching upon the pure or risk premiums.

I have only referred to the construction of the ordinary whole-life table of premiums, but the same principles can be applied to the construction of tables for the other classes of policies.

All the tables of premiums, then, having been constructed on principles which are fair and equitable, or as nearly so as can be ascertained by actuarial skill, we may assume that, at the outset, the members start on equal terms; and our next enquiry is, as to the adjustments which have to be made from time to time, in order to conserve the rights of all the members, and at the same time to approach as nearly as possible to the ultimate aim of the Society, namely, to make the value of the amounts assured as nearly as possible equal to the value of the members' premiums. This

brings me to the question of the periodical valuations and divisions of profit.

Mr. Sprague, in his paper "On the Proper Method of Estimating "the Liability of a Life Insurance Company under its Policies", says, "the object of the actuary in making a valuation should be, "to equalize the profits of the company; in other words, to spread "the profit derivable from each policy or class of policies, as far as "possible uniformly over their continuance". This may be a very good maxim in the case of a proprietary company, "where the "profit to the shareholders has to be thought of", but it appears to me to be quite inapplicable to the case of a mutual office, in which the aim is to give the members the full value of their contributions. The object of the actuary in making a valuation of a mutual society should be to ascertain the true position of the society, and what adjustments (if any) are necessary in order to carry out its objects. But what method of valuation must be adopted in order to ascertain the true position of the society? The reply to this question by some actuaries would be—make a net premium valuation on the basis of the same table of mortality and rate of interest, as were assumed in the original construction of the premiums, by the well known formula ${}_nV_x = A_{x+n} - P_x(1 + a_{x+n})$, or expressing it in terms of the office premium, ${}_nV_x = A_{x+n} - (\Pi_x - K_x)(1 + a_{x+n})$, in which Π_x represents the office premium, and K_x the loading at age x; and there could be no objection to this method, provided the table of mortality is a true one, and the rate of interest is safe.

But suppose it had been ascertained that the table of mortality originally employed is not a trustworthy measure of the mortality which may be expected among the members of the society; or suppose the society had found it impossible to invest its funds at so high a rate of interest as that originally assumed: how could a valuation made on this method show the true position of the society?

To get true results we must employ true tests; and, therefore, if further investigation of the laws of mortality and the application of more extensive records of experience have produced a table more trustworthy than that originally adopted in calculating the premiums, I think the actuary is bound to use it in his valuation of the policies; and if, at the time of investigation, the average rate of interest at which the funds of the society are invested be lower than that originally assumed, it would be pure recklessness to adhere to the original rate, and I think none would defend it.

It follows, then, that circumstances may arise which necessitate a change in the basis of calculations; but such a change should not be made without very serious consideration, and the most convincing proof that it is necessary in order to produce true results; to change the basis of calculation arbitrarily, or on insufficient grounds, for the purpose of representing the office to be in a better position than is really the case, would simply be dishonesty and trickery of the meanest kind.

We have now to consider what principles should be adopted when it is found necessary to employ data which differ from those previously used by the society.

Many actuaries say that, in such a case, we must altogether ignore the original calculation of the net premiums and the loading, and now re-calculate the net premium for age at entry according to the new data, and value that as the risk premium, reserving as loading that portion of the office premium which remains after deducting the new net premium. Thus, if P'_x represent the net premium originally calculated for age x, and K_x the loading added thereto, the office premium being $P'_x+K_x=\Pi_x$, they say that both P'_x and K_x must be altogether ignored, and the valuation be made by the formula ${}_nV_x=A_{x+n}-P_x(1+a_{x+n})$, in which P_x is derived from the same data as A_{x+n} and a_{x+n}.

The main objection to this method of valuation is, that it is purely hypothetical, and can therefore produce only a hypothetical result. If the premiums payable by the members were not fixed, but subject to variation, according to the basis of calculation adopted by the society, then this hypothetical method would be correct; for the facts would always agree with the hypothesis. But when the contract with the members is to receive from them uniform annual premiums throughout life, it must surely be erroneous to ignore what they pay as an item in the valuation.

Let us test the formula ${}_nV_x=A_{x+n}-P_x(1+a_{x+n})$ by supposing extreme cases, on the principle that what is theoretically true of small quantities is also true of large ones similarly applied. 1st, let $P_x>\Pi_x$, then the formula is manifestly erroneous, for it involves the valuation of an annual premium greater than the member has contracted to pay; 2nd, let $P_x=\dfrac{\Pi_x}{4}$, then we have the anomaly of reserving for future expenses and contingencies an annual amount three times as great as is reserved for the risk itself; and 3rd, let $P_x=\Pi_x$, then we have no provision for future expenses and contingencies at all.

We thus see that whether the value of P_x be increased or diminished by a change in the basis of calculation, when the variation is great it leads to an absurd result, I conclude, therefore, that the risk premium should be constant in all valuations, and should not vary with the data used at each investigation. In other words, that P'_x and K_x are specific amounts, which should be valued and kept separate for the special purposes for which they were intended, and with that view every member's premium should, at the outset, be registered in its component parts. It is true that the K_x, or loading, was originally fixed in an arbitrary manner, but when once fixed upon, it became as much a fact as the office premium itself, and should not be varied unless experience proves it to be necessary.

This brings me to the question—under what circumstances should the loading be varied? To answer this question, we must consider the purposes for which it is intended. These are, to provide for expenses and contingencies of all kinds, which from their nature cannot be foreseen. Among such contingencies may be mentioned possible loss of a portion of the funds from the depreciation of securities; possible inability to realize as high a rate of interest on the invested funds as was originally assumed, and a possible experience of a higher rate of mortality than that provided for in the original construction of the net or risk premium.

When any of these contingencies occur in the form of a temporary variation from the normal experience of the society, and are fully covered by the loading of premiums actually received, the loading on future premiums should be reserved intact. But if the mortality experience proves to be greater than that provided for, and necessitates the adoption for the valuation of a table indicating a higher rate of mortality than that used in the construction of the premiums; or if the interest realised proves to be less than that originally assumed; then it would be perfectly legitimate to include as much of the loading of future premiums as is necessary to make up the deficiency in the net or risk premiums, provided that a sufficient amount of it be left as a reserve for future expenses. Such a variation of the loading is permissible, for against such contingencies as these the loading specifically exists.

If, on the other hand, experience proves that the net premiums are more than sufficient to cover the risk, it proves also—other circumstances being equal—that the loading is more than sufficient. There can, therefore, be no necessity to increase the loading by taking the surplus from the net premium. To illustrate my

meaning, using the same symbols as before, the premiums payable consist of the two parts, P'_x to cover the risk, and K_x to provide for expenses and contingencies, and it is now discovered that P_x, which is less than P'_x, is the correct pure premium to cover the risk. Must we, therefore, value only P_x as risk premium, and increase the loading by $(P'_x - P_x)$? Thus:—

$$_nV_x = A_{x+n} - [\Pi_x - \{K_x + (P'_x - P_x)\}](1 + a_{x+n}),$$

which gives the same result as $_nV_x = A_{x+n} - P_x(1 + a_{x+n})$.

Bearing in mind that the object of the society is to give to its members assurances of a value as nearly as possible equal to that of the premiums which they pay, the only sound reason which could be given for thus increasing the loading would be that the original loading is inadequate, and that the increase is necessary for security. But if it be admitted that K_x is an adequate loading, then I fail to see any sound reason for increasing it by $(P'_x - P_x)$, and I submit that the valuation should be made by the formula

$$_nV_x = A_{x+n} - (\Pi_x - K_x)(1 + a_{x+n}),$$

which will give true results in all cases.

Almost the only objection which can be offered to this method is that, in certain cases, it produces negative values for policies recently issued; and to many this may appear to be a fatal objection. Mr. Sprague, in the paper previously quoted, writes thus strongly on this point:—"It appears, then, that in no case is it permissible "so to value the liabilities of a Life Insurance Company as to lead "to negative policy values. Any method of proceeding which has "this effect, or exhibits any policies as assets instead of liabilities, "is quite inadmissible, and cannot be too strongly condemned". But when I find that the method adopted is one which values the actual facts by true tests, I am bound to believe in the correctness of the results, even although some of them present themselves in the anomalous form of negative values. What? I may be asked,—Do you consider a policy should be an asset? I reply, Certainly not; it should be a liability; but as an actual fact it may be an asset notwithstanding—an unsecured asset, it is true, but still an asset; and the sooner it is made what it ought to be —a liability—the sooner will justice be done to the member. How can a policy which is an asset be made a liability? Certainly not by substituting a mathematical hypothesis for the actual fact, and valuing that. Such a course would treat the policy as if it were a liability, but it would not make it one. To answer the question we must go to the root of the matter, and inquire into

the cause of the anomalous fact that the policy is an asset, and we find the cause to be that the member is paying too much for the benefit he is to receive. Make the member's benefit equivalent in value to his payments by increasing the amount of the benefit, and not only does the policy become what it ought to be—a liability—but substantial justice is done to the member. I feel quite as strongly as anyone the danger of allowing policies to have negative values, and therefore I hold it to be the first duty of the actuary to eliminate them as soon as he discovers them by adjusting that which causes the anomaly; but it should be a real adjustment, and not a make-believe, such as a variation in the method of valuation. The P'_x which the member pays as risk premium is the true measure of the minimum amount of assurance which he should receive: the proper adjustment is, therefore, to increase the amount assured. Thus, if P'_x was charged to the member under the supposition that $P'_x(1+a_x)=A_x$, and it is now found that this equation is not true, but that $P_x(1+a_x)=A_x$, then it follows that $P'_x(1+a_x)=A_x+(P'_x-P_x)(1+a_x)$, and therefore that the amount assured should be increased by $(P'_x-P_x)\dfrac{1+a_x}{A_x}$. Another method of adjustment, which would be perfectly fair and just, would be to reduce the actual premium payable by (P'_x-P_x).

Any additional surplus that may be available for distribution should be divided among the members as nearly as possible in proportion to their contributions thereto. At the first investigation, the total amount of loading paid by every member may be taken as the proportions in which the surplus should be distributed. After the first investigation, the profit arising from interest on investments should be divided separately in proportion to each member's interest in the accumulated fund at the beginning of the period, the remainder of the ascertained surplus being divided, as before, in proportion to the loading paid during the period. This method of division is either the one recommended by Mr. Sprague, or some modification of it.

So far my remarks have applied to the case of a mutual office in which the method of constructing the premiums is known, and is considered to be consistent with the aims and objects of the society. I may, however, be asked, how the principles which I have sketched can be applied to the valuation of an office in which the construction of the premiums is unknown, or in which the office premiums have been calculated by taking an average of the premiums charged by other offices. In such cases we only know

the value of Π_x, the office premium, and it is necessary to determine what portion of this office premium is to be reserved for future expenses and contingencies, and what portion is to be applied to the risk. The actuary must, in such a case, guided by the best tests he can find, arbitrarily divide Π_x into P'_x and K_x, bearing in mind that K_x should be graduated in such a manner that the persons of every age contribute to the fund for future expenses and contingencies, fairly and equitably as between themselves, and that P'_x should be at least adequate to the risk originally undertaken (not including reversionary bonus additions). Having thus arbitrarily fixed their respective amounts, P'_x and K_x will thenceforth be treated as the specific component parts of the office premium, and will be valued by the method previously described.

Having now indicated the method which has commended itself to my mind as the proper one to be adopted for the valuation of policies in a mutual office, I will contrast it with the method which, in my opinion, is applicable to a company having a paid up capital. The formula, ${}_nV_x=A_{x+n}-[\Pi_x-\{K_x+(P'_x-P_x)\}](1+a_{x+n})$, I rejected, because I could see no sound reason for increasing K_x by (P'_x-P_x); but in a proprietary company the case is quite different, for the main object here is to make a permanent profit out of the business for the shareholders who have invested their capital in it, and their interests are best consulted by providing for them regular dividends. The shareholders are entitled to a profit on every policy, and it is this circumstance which makes it desirable to "spread the profit derivable from each policy, or class of policies, "as far as possible uniformly over their continuance". From this point of view, the addition of (P'_x-P_x) to K_x, the loading, is quite allowable, for (P'_x-P_x) is a new source of profit, which the shareholders are entitled to have spread over the continuance of the policy, and this cannot be done better than by the formula now under consideration. I therefore conclude that, although its adoption is inconsistent with the aims and objects of a mutual office, yet, the formula ${}_nV_x=A_{x+n}-[\Pi_x-\{K_x+(P'_x-P_x)\}](1+a_{x+n})$ may properly be used in the case of a company having a paid-up capital, but its adoption is a matter of expediency, not of necessity.

There is this difference between the two cases, namely, that profit is one of the objects of the loading K_x in the proprietary office, while it may be termed an accident of the loading K_x in the mutual office: the former K_x is intended for security and profit,

while the latter K_x is intended for security only, and a surplus results from the fact that only a portion of K_x proves to be necessary for security.

Keeping in view the various objects of the two systems, it appears to me that, in the proprietary office, the standard of measure should be the sum assured, and from that standpoint should be calculated the profit realised from the premiums received on account thereof; but that, in a mutual office, the standard of measure should be the premiums actually payable, and from that standpoint should be calculated what amount of assurance can be given to the member consistent with the most perfect security.

Conscious of its many imperfections, I am led to offer the foregoing paper as a small contribution to the discussion of insurance topics, in the hope that the minds of more experienced and skilful actuaries than I, may thereby be directed to a question which has not, I think, hitherto received the attention which it merits. I do not claim for the method herein advocated that it is perfect, or that it is the only one which should be adopted, but I do claim that it is one which leads to true results, and one by whose means the aims and objects of mutual assurance would be materially advanced.

Discussion.

The President (Mr. J. Hill Williams)—The meeting will probably agree with me that we should thank Mr. Templeton for his paper, and also offer him a cordial welcome in coming amongst us personally from the other end of the world. With respect to the paper, I may say for myself that I have failed to catch the great distinction which Mr. Templeton seems to hold, as to the necessity of applying different standards of measure between proprietary and mutual companies. It may be a question in the appropriation of profits, but when it comes to a question of Mr. Templeton's original valuation, I hold that there cannot be two measures for truly measuring the same thing,—namely, the risk. I think it is of the greatest importance to remember that, when once a policy is issued, it never can be an asset. Notwithstanding all the specious arguments to the contrary, it is, from the very moment it is issued, a liability; and for the purpose of measuring liabilities we cannot use an elastic or a shifting measure, applicable first to one set of circumstances and then to another, according to the caprice of the individual. If once you begin to consider that you can deal with the pure net premium as you like—increase it or lower it as you may think proper—you do not know where it may lead you. It is patent to all of us that the construction of the original table of premiums of most of the offices, must have been

in many cases faulty, in many cases arbitrary, and in many cases very unscientific. But the object of the foundation of this Institute was to apply science instead of caprice to the measurement of these liabilities. I think, therefore, if an actuary has discovered a better table of mortality, representing more accurately the mortality experience which he believes will hereafter affect his own company, he is bound to act upon it, and to consider what is the risk premium according to that better table, and not to shift it, as recommended in this paper.

Mr. A. Baden—Mr. Templeton speaks of having in the first place, in constructing premiums, put on a loading to the risk premiums, which, being fixed at all times and under all circumstances, shall do inevitable justice to the assured. Now, I believe that in the construction of premiums it has hitherto been considered quite a moot question whether strict justice can be done by the addition of any kind of loading; at all events, it certainly is a moot question what that kind of loading is which will do strict justice. If, even when we use a table which is admitted on all hands to be correct, it becomes an impossibility to find any loading which shall operate justly; much more under the altered circumstances supposed in the paper,—namely, valuing by a table different from that on which the premiums are constructed,—does it seem impossible that there can be any loading which, under these two altogether different circumstances, can do equal justice to the assured. Mr. Templeton says that the risk premium may be fixed at any amount you like,—it may be wrong, or it may be right,—but if, when you come to assume for valuation a table different from that upon which your premiums are constructed, you find that the assured is paying more for his risk by the premium that you have entered in your books as the risk premium, than he ought to pay according to the true table you are now using in the valuation,—then, in order to have justice done to him, he should have, in some way or other, an equivalent for this surplus payment. Granted, but he further assumes that by a *valuation* of the risk premium, justice will be done to the assured. Now, it depends not upon the amount you reserve, but upon what you give back to the member; it depends upon the way of distributing the profit you make, and not upon your valuation of the liability, whether you do justice to him or not. If, having valued this premium, which is acknowledged to be a surplus risk premium, and having thrown it into your surplus (as you do by Mr. Templeton's method of valuation)—if then it is divided between all the members of the association alike—it is difficult to see how justice can be done to the individual member whose excess of premium is so valued. It seems to me clear that before Mr. Templeton can prove that justice is done to every individual member, he must show how he divides the profit he ascertains; but he has not done that at all. He seems to think a method of producing a certain reserve will in itself do justice to every member of the society. I confess I am quite unable to see how that can be brought about. But Mr. Templeton has not considered the opposite case. Supposing the risk premium, instead of being in excess, is in deficiency—and that may very well happen. We know, for example, that the Carlisle table and the H^M table cross one another at age 42; at 42 and upwards, if the Carlisle table is used in the construction of the premiums and the H^M table in the valuation,

we shall have risk premiums in deficiency. How will he deal with that? If he is to value only his fixed risk premium, he will indeed get a proper reserve for his policy, but then he will do it at the expense of the other members. This may be his idea of dealing justly with those who are paying in deficiency, but it cannot be justice to the members who are paying more than they ought, and who have to contribute out of their premiums or profits, to the deficiency which the policyholders at the ages above 42 have created. Mr. Templeton puts an extreme case. He says a case may occur of the risk premium being greater by the true table used in the valuation than the whole office premium that stands on the books, including the loading; and he considers that this reduces the theory which is opposed to his own to an absurdity. I do not see that there is any absurdity in the case. The actuary would simply value the premiums actually paid, and take care a proper reserve was made; because, before he had made an alteration in his basis, he would have taken care to ascertain what anomalies there were, and he would have provided for them accordingly. Then I come to the subject of negative values, and Mr. Templeton's treatment of negative values does appear to me to be extraordinary in the extreme, and to amount to a positive demonstration of the falsity of his theory. We have seen several cases where it has been thought advisable to pursue some method of valuation by which negative values do arise, but careful actuaries have thought that the ill-effects of this would in such cases be sufficiently obviated by simply striking out these negative values—that is to say, reducing the values of all assurances which produce them, to zero. But this will not do for Mr. Templeton. He says, "No, we must make positive values of them." And how does he make positive values? By doing what he considers strict justice in these individual cases, so that the fortunate policyholders who happen to come below the line, and whose policies by the method chosen produce negative values, come in for what no other policyholders on the books can get. But those who are above the line of negative values, who are still contributing more than they ought, do not get any equivalent at all; there is no addition made to their policies, and yet the company is to continue to receive the same premium from them as they have paid all along. Why this difference? Why should the fact of your system producing negative values in certain cases, entitle the policyholder whose policy comes out as a negative value, to have justice done to him, whilst those policies just above the line are not entitled to have justice done to them? I cannot see any consistency in a theory which leads to such a result as that. Then, there is another point of view from which it seems to me that this theory may fairly be criticized, and it is this. There are certain members of the society who will surrender their policies, and thus we come to ask whether, under Mr. Templeton's view, a fair surrender value can possibly be given to those who wish to surrender their policies. Well, it is obvious it cannot, and for this reason. It seems to me that, whereas there are two ways of arriving at the reserve value of a policy, there is only one way which is generally kept in mind, and the other is too much lost sight of. The reserve value of a policy may be considered on the one hand *prospectively*, as a function of the relation between the benefit to be received and the contribution

to be paid for it; and on the other *retrospectively*, as a function of the relation between the risk which has actually occurred and the contributions that have actually been paid for it in the past. The appropriate formula of this latter view is

$$(P_x - P^{1}_{x\overline{n}|})\,\frac{N_{x-1}-N_{x+n-1}}{D_{x+n}},$$

that is to say, the difference between the pure annual life premium for the age at entry, and the pure annual premium for the term n, multiplied into the term annuity, and divided by the endowment, represents what is actually in hand from the member's contribution for the number of years the policy has been in force, over and above what was required for the risk. Now, if we look at the value from that retrospective point of view, we shall see the office has actually in hand a certain sum out of the contributions of the member, and is bound to pay back to him a surrender value having the fixed ratio agreed on to the reserve. The value brought out in that way is identical with what I should call the true value,—the premium in both cases being taken according to the true table. But, not only does Mr. Templeton put it out of his power to give back anything like that amount which represents the accumulated reserve, but in his desire to do justice to the assured he has altogether forgotten another accumulation in his hands, to wit

$$(P'_x - P_x)\,\frac{N_{x-1}-N_{x+n-1}}{D_{x+n}},$$

that is to say, the difference between the pure premium according to the tables he has used in constructing his office premiums and the true pure premium, so that that accumulation has to be added to the other. He puts it out of his power to give back the proper ratio of the smaller value, much more to give any proper return for the contribution which has actually been made. It is desirable that, in a young society especially, that method should be adopted which produces an undoubtedly sufficient and strong reserve. The effect of Mr. Templeton's method of valuation will, as the society gets older, inevitably be this,—that having anticipated in the earlier years all this surplus which would otherwise be thrown into the loading, and having thrown it into the profit and divided it, as the business of the society increases and the number of old policies grows larger in proportion to the new business brought in, that source of profit will gradually fail him in increasing ratio; and as the policies will not be able to furnish such a fund towards the surplus as they did in the earlier years of the society, the consequence will be that at a time when the pressure of mortality is telling on the society, instead of having a strong reserve, he will have a weak one, and in addition to this the inducement to new members to come into the society will be diminishing every year, because the profit will be gradually falling off—they will choose other and more vigorous offices—offices which in their earlier years had their business valued upon more sound principles, and they will forsake his.

Mr. A. H. Bailey—I do not wish to discuss the main points of

Mr. Templeton's method, because I have come to learn lately that there is a difference of opinion as to what is meant by the net premium method of valuation, and I think that is a matter that may fittingly occupy the attention of the Institute on some future occasion. I will now, therefore, only remark on one paragraph in Mr. Templeton's paper, in which he seems to think, as Mr. Baden has said, that he has brought the case to a *reductio ad absurdum*. Referring to the formula for the value of a policy after the expiration of n years, he says,—"By supposing an extreme case, first, let P_x be greater than Π_x; that is to say, let the pure risk premium be greater than the office premium." Now, this is a case of occasional occurrence. A familiar example will occur to many of us in the case of children's endowments, which, I believe, have generally been calculated by the Carlisle table. Offices have granted endowments at annual premiums calculated on the assumption that, out of ten children born, eight only survive after the expiration of two years. That, or something very like it, is to be found in the Carlisle table, but in our experience we find the result to be wholly different. I take it that no one who has granted endowments for children under two years of age by the Carlisle table, can doubt that in them the risk premium is greater than the office premium. What does that mean? The office has made a bad bargain, and it must make a larger reserve for it. There will be no profit and no margin in the premium, out of which expenses can be defrayed; but these must be defrayed out of the margins of the other policies. The matter must be looked in the face and is inevitable, but it does not appear to me that it affects the question of principle at all. What should be done in these cases is to substitute for P_x, Π_x, the office premium, because a part cannot be greater than the whole. Another illustration is to be found in the case of deferred annuities. No loading is practicable there, and the competition of the Government, which chooses to grant annuities at prime cost, is such that no office can get a profit out of this business. In deferred annuities, the risk premium and the office premium will probably coincide. In whole-life insurances also, I have in my experience come across cases in which policies have been granted at a premium which I consider below prime cost, so that the office premium is less than the risk premium, and I should in a valuation substitute the one for the other. In Mr. Templeton's other extreme case, we have a reserve for the loading three times as great as the reserve for the risk itself. But here again there is no anomaly. There have been cases, though not many, where enormous extra premiums have been charged, which were not justified by the risk of age. In such a case, care must be taken not to divide this enormous profit at once, for this reason—a very pertinent one, I think—that these policies are likely to be discontinued. I will now say a word or two about a question on which Mr. Templeton touches, although he says it is not the object of his paper, and about which we do not hear much nowadays, as to the distinction between mutual and proprietary insurance offices. It is novel to hear that contracts should be valued on different principles according as they have been entered into by mutual or proprietary societies. A policy has been issued, ten or twenty years have elapsed, and a certain amount is required to be reserved for the liability; and whether it was issued by a mutual or a

proprietary insurance office, surely makes no difference whatever. And even if it did, it is not unimportant to bear in mind that nowadays the distinction between mutual and proprietary insurance societies, as far as the insured is concerned, is very small indeed. By far the larger proportion of the profit in proprietary companies goes to the assured; and therefore I differ with Mr. Templeton when he says that the main object of a proprietary company is to get a dividend for the shareholders. I hold that the main object of proprietary companies is to do the best they can for the insured. The proprietors are no doubt to get a small amount from the surplus, but it is not just to increase that amount to the detriment of the insured.

Mr. MACFADYEN—I do not see that Mr. Templeton has shown that there is any obligation to retain the original net premium in valuations on another basis. Premiums and valuations are quite independent of each other. The rates charged are settled by competition—especially the competition of such offices as occupy the same ground. Besides, the original pure premium is not, even by its computer, always looked on as the equivalent of the risk; for when a 3 per-cent rate is chosen, where 4 per-cent can certainly be made, it is simply a mode of giving additional loading. Mr. Templeton's theory results in a change of the manner of distributing profit. If there be any obligation in such matters to the insured, it lies more in the direction of not changing the bonus system than in retaining the original net premium. Many offices draw attention to the first in their prospectuses, none that I know of refer to the latter. Again, on his own admission, Mr. Templeton's method is not generally applicable. The one case he uses it in, is that of a mutual office with ample loading and excessive net premiums. In such an office, if I understand him, he would in all cases, whether negative values or otherwise, consider the difference between the pure premium originally charged and that now fixed upon, as an annual premium, paid from the commencement of the risk, for an additional insurance sold as at entry age at the new rates. Mr. Templeton does not treat excessive loadings in this way, and yet his logic ought to produce this result. If he had done so, he would have had prominently brought before him a fact conflicting with his theory; for it is a fact that, even in mutual companies, loading is not always intended merely to meet expenses and contingencies, but is often added for the express purpose of making a profit to be returned. Reverting to Mr. Templeton's formula for the additional amount by which the original assurance is to be increased, he gives it as

$$(P'_x - P_x)\,\frac{1+a_x}{A_x}.$$

He does not point out that this formula is erroneous if the policy has already shared in profits distributed on the original basis. Assuming however that there has been no previous allotment, let us study the workings of Mr. Templeton's formula. His paper leaves us to do this for ourselves, as it simply gives the formula for the additional insurance and goes no further—in fact, stops short at the very point where, for my own part, the interest of the paper began. As a step in the direction indicated, I have attempted to compare the reserves under

Mr. Templeton's scheme with those under an ordinary net-premium valuation. The result arrived at was that the total reserve, including loading, under the first, would exceed that under the second when the ratio of the true net premium at the valuation date to the true net premium at entry exceeded 2. Of course this result would be affected if the cash bonuses under the second system were dealt with in such a way as to increase the liability. But, disregarding these bonuses, it appears that in about 15 or 20 years from entry, Mr. Templeton would require to hold against liability under his enhanced insurance amount, a greater sum than would be necessary, under a net-premium valuation, to provide the original insurance amount. The practical effect of Mr. Templeton's scheme is to make his policies non-profit after the change of basis. At that date everything is given away; not as under the net premium method, doled out from time to time. Premature deaths get the same profits as those who live long. Though, possibly, this principal is the most logical in theory, yet in practice, profit-policyholders who live long, undoubtedly consider themselves entitled to, and do look for, special profits on their specially profitable policies. An ordinary net-premium valuation does give a fund for such special bonus. Besides, it is very much safer than Mr. Templeton's method. He converts the whole surplus-producing power—for, to be logical, it ought to be the whole—into liability at the time he changes the basis. The office must stand or fall by the absolute accuracy of his new data, and has no elasticity. I do not think it prudent to so commit a company to any hypothesis, however carefully considered before adoption. Finally I do not agree with Mr. Templeton that it is only in proprietary companies that profits should be equably spread over duration. I have just given reasons why it is quite as expedient to do so in mutual offices. Another reason might be found in the application of bonus to reduce premiums. Too much profit distributed at first might result in the premium being reduced upwards at valuation time—a result far more damaging to insurance companies than a refusal to drain the profit reservoir dry on the first change of basis.

Mr. H. AMBROSE SMITH—Mr Templeton ventures an opinion that the outlay does not increase in proportion to the premium, but he surely forgets that a large item of the outlay is agent's commission, which does bear an exact proportion to the premium; so that is an argument, as far as it goes, for making the loading at the more advanced ages in the same ratio as at the younger ages. If an attempt is to be made to adjust the loading equitably, it should take this shape:—A policy for £1,000 puts the office to no more trouble or expense in its management, setting aside agent's commission, than one for £100; so that, to attempt to arrive at an equitable system of loading, we require a table of premiums for £100, £200, £300, £400, and so on. I can understand that, but I do not understand the alternative suggested by Mr. Templeton. I also, with Mr. Bailey, fail to see the distinction which is drawn between mutual and proprietary offices. I think it is as incumbent to equalize the bonus to be allowed in the one case as the dividend in the other. I do not see any argument which applies to the one which would not apply to the other. It is such an elementary matter that it is almost useless to repeat it—that if a number

of people combine together for mutual insurance, and after the first beginning of the scheme there are no more entrants, and if the premium is in excess of the rate required, the profits accruing yield a bonus which at the first division would be a great one, and would rapidly diminish as the lives die out. If, therefore, you wish to equalize the bonus, it must be done on some mathematical basis; or if new entrants are to be allowed, in justice to them we must not allow the first entrants to get all their own surplus (or its value) and then claim a share in that of the new entrants. As an argument for equalizing the rate, this applies equally to both proprietary and mutual offices.

Mr. H. W. MANLY—I think we have missed the point, or rather the origin, of Mr. Templeton's fallacy. His main objection to the net-premium valuation is, that it is purely hypothetical, and can therefore produce only hypothetical results. Now, I would like to ask, what all his factors are in his valuation? What is A_x, and $(1+a_x)$, or what is the net premium originally calculated for the insurance? Are they not all based upon certain hypotheses? and is not the reason for altering the table of valuation, that we have found by our increased knowledge of science and statistics, that the original hypothesis is wrong? Now what has Mr. Templeton got? He divides his premium into two parts—first, the pure premium, which was originally calculated upon one hypothesis, which he now says was wrong; and he has a balance, which he says is a loading; and in his valuation he takes a table based upon another hypothesis, and consequently values two results based upon two different hypotheses diametrically opposed to each other,—the pure premium charged in the office premium being calculated upon one hypothesis, and the valuation being made upon another. But, then, what is the object of dividing the premium into two parts? He says that the premium originally charged was made up of two portions, the pure premium and the reserve. That simply implies that the pure premium should have been put into one fund to provide for the risk, and that the balance should be put into another fund to provide for the expenses and contingencies. If you carry Mr. Baden's formula in your mind, you will see when the pure premium originally calculated is accumulated, the first P_x in the formula becomes the P_x by the first hypothesis, and the premium for the term assurance is the premium upon the second hypothesis; so that, if the first premium be larger than it would have been upon the hypothesis upon which he subsequently bases his calculations, the fund would be larger than the net-premium valuations. But Mr. Templeton produces this fallacy by his valuation, he says it is smaller! Now, those who defend the net-premium method of valuation, say that it will be exactly the same,—that the fund produced by the net premium, when accumulated, after payment of the claims, shall be exactly the same as the fund produced by a proper valuation. If Mr. Templeton will consider that, I think he may be inclined to alter his views.

Mr. TEMPLETON—I expected to stand pretty well alone, and I find my expectation realized. I must say, however, that the arguments which have been used tonight have not convinced me that I am in error. I am anxious to discover the truth in this matter, and I

thought the best way of doing that, was to come before you, gentlemen, who have had more experience than I have. It appears to me that the great cause of the difference of opinion is that I cannot get you to look at the matter from the same point of view as I do. You will not admit that the member of a mutual office is entitled to get the value of the premiums which he pays; and that lies at the foundation of my method. My view has certainly not been carried out in any mutual office, and that argument has been adduced against me; but to admit the pertinence of such an argument, would be to accept what is, as best, and to oppose all progress. If, in a mutual office, I charge a man £2. 10*s*. a year to secure £100 at death, and 11*s*. 6*d*. a year as loading to provide for expenses and contingencies; and if I afterwards find by better data that £2. 10*s*. a year is the net premium, not for £100, but for £106, I say that the aims of the institution demand that the amount of his policy be increased to £106, seeing that the 11*s*. 6*d*. per annum is still left free for the purposes for which it was intended and is wholly reserved. And, after all, the question whether the loading is sufficient is a question whether its annual amount is sufficient. We ascertain its present value by the best available data and state it in the account; but, when judging of its sufficiency, it is the annual amount we look to and not its present value. Mr. Manly objects to my statement that the ordinary net-premium method leads to hypothetical results, and adds that the whole question is one of hypothesis. Certainly it is on the hypothesis that future experience will coincide with the past, that we ascertain the values; but, having fixed upon our hypothesis, we should value the actual facts by that hypothesis. There is a distinction between valuing an hypothesis and valuing by hypothesis—it is the former to which I object; the facts alone should be valued. I say that the net premium which I charge is an actual fact. You say it is not. Well, supposing it is not, and that the office premium is the only fact; if you are to give the member the value of his premiums, you must value that fact. And then arises the question how to provide for expenses and contingencies. Take off from the office premium what you consider to be sufficient. It cannot be correct to take off 40 or 50 per-cent from the premium of one member and only 3 per-cent from the premium paid by another. That cannot be equitable. It may be perfectly correct to ignore the original calculation of the premiums, and if you think the loading originally added was insufficient, to increase it; but it should be accomplished in such a way that justice will be done to all the members. It has been objected that it is impracticable to do full justice to all. Perhaps it is; but is that any reason why we should not attempt to do it? Should we not be carrying out the objects of the office, if, in attempting to give the insured the full value of his premiums, we approach as near strict justice as we can? Having determined what proportion of the office premiums at all ages should be reserved for expenses and contingencies, the proper method is to take off that proportion at every valuation; but I cannot see that the pure premium which results from the table by which you are valuing, has anything to do with the member's policy. If you adopt that as the pure premium for valuation, you put for the thing to be valued an hypothesis; and herein, I take it, lies the difference between those gentlemen

who have spoken and myself. With regard to Mr. Baden's statement that, according to my method, policies with negative values would receive greater advantages than those with positive values, I have already explained that the adjustment

$$(P'_x - P_x)\frac{1+a_x}{A_x}$$

was intended to be made to all policies. Perhaps this might have been more clearly expressed in my paper, which was prepared somewhat hurriedly for the present meeting. I shall probably write another paper on this subject at some future time which may be read here, if you, gentlemen, think the discussion of sufficient importance. I shall carefully consider all the arguments which have been advanced tonight; and if I find that I am wrong in any respect, I shall not be slow to acknowledge it.

On the Premiums for the Insurance of Recently Selected Lives. By T. B. Sprague, Esq., M.A., *Manager of the Scottish Equitable Life Assurance Society, and a Vice-President of the Institute of Actuaries.*

[Read before the Institute, 18 Dec. 1876.]

THE folloing results wer obtaind in the course ov the years 1870 and 1871, the calculations having been carryd on at intervals, as my professional engagements allowd ov the time ov myself and my assistant being devoted to the matter. It was my intention to carry the investigation still further, by forming graduated mortality tables for entrants ov varios ages and deducing annuity values therefrom; but this intention not having been carried out, I think it better to lay the results in their present state before the readers ov the *Jurnl,* insted ov delaying them any longer in the hope, which may perhaps never be realized, ov making them more complete.

In order to determin the correct premium that shoud be charged to persons insuring their lives at a particular age, it is necessary to determin approximatly the rate of mortality that will prevail among those persons in each ov the succeeding years ov age. It is now, I believe, generally admitted that this rate ov mortality depends, not simply on the age ov the persons under observation, but also, to a very considerabl extent, upon the time which has elapst since they wer examind medically and otherwise, and accepted as good average lives. If therefore we desire extreme accuracy, it will not be sufficient to take the average

rate ov mortality for each age, as given by such a tabl as the Institute H^M Tabl; but the strict method ov solving the problem will be to take the entrants ov each age and trace the mortality among them in every subsequent year ov life until they ar all ded, keeping them throout distinct from entrants at all other ages. The number entering at any one age is, however, too small to giv trustworthy results, and the figurs deduced from them woud be subject to very great irregularities. I hav therefore for the purposes ov this enquiry combined in a single groop the entrants ov each consecutiv five years. Adding together the numbers who enterd at the ages $x-2$, $x-1$, x, $x+1$, and $x+2$, where x is a multiple ov 5, I assume in the first instance that these wer all ov the age x at entry, and ascertain the rate ov mortality among them during each year they wer under observation. In order that ther may be no possibl dout as to the method I hav folloed, I will giv an exampl of the working, from the outset to the final result. Thus, taking $x=65$, we get from pages 160 and 161 ov the *Mortality Experience of Life Assurance Companies, collected by the Institute of Actuaries*, the figurs enterd in the folloing tabl opposit the ages 63,4,5,6,7; and the totals at the foot ar obtaind by addition.

Age at Entry.	Year of Insurance.									
	0		1		2		3		4	
	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Exposed to Risk.	Died.
63	149·5	1	283·5	6	251	3	232·5	9	213·5	9
64	130·	1	248·	5	225	4	210·	9	189·5	11
65	104·5	1	194·5	9	163	5	146·	7	131·5	6
66	79·5	0	150·5	2	137	3	126·	12	106·	3
67	75·	1	137·	5	120	7	109·	7	96·	9
Total	538·5	4	1013·5	27	896	22	823·5	44	736·5	38

In this way wer obtaind the figurs givn in colums 2 and 3 ov the folloing tabl. I call 65 here the "middle" age at entry, rather than "mean" or "average", to show that ther has been no process of calculation gon thro, as is implied when either ov the latter terms is uzed; and I take the age as $65\frac{1}{2}$ rather than 65, for reasons which will be presently explaind.

TABLE (A).

Middle Age at Entry 65½.

Year of Insurance. (1)	Number at Risk. (2)	Deths. (3)	(4)=(2)−(3)	(5)=log (2)	(6)=log (4)	(7)=(6)−(5)
0	538·5	2	536·5	2·73119	2·72957	$\bar{1}$·99838
1	1013·5	27	986·5	3·00582	·99410	·98828
2	896·	22	874·	2·95231	·94151	·98920
3	823·5	44	779·5	·91566	·89182	·97616
4	736·5	38	698·5	·86717	·84417	·97700
						·98892
5	663·	33	630·	·82151	·79934	·97783
						·98891
6	599·5	26	573·5	·77779	·75853	·98074
7	536·5	48	488·5	·72957	·68886	·95929
8	455·5	31	424·5	·65849	·62788	·96939
9	398·5	35	363·5	·60043	·56050	·96007
10	347·	31	316	·54033	·49969	·95936
11	306·5	36	270·5	·48643	·43217	·94574
12	259·5	30	229·5	·41414	·36078	·94664
13	210·	19	191·	·32222	·28103	·95881
14	180·5	26	154·5	·25648	·18893	·93245
15	145·	23	122·	·16137	·08636	·92499
16	117·	15	102·	·06819	·00860	·94041
17	97·5	10	87·5	1·98900	1·94201	·95301
18	81·	14	67·	·90849	·82607	·91758
19	64·5	13	51·5	·80956	·71181	·90225
20	47·	9	38·	·67210	·57978	·90768
21	34·	5	29·	·53148	·46240	·93092
22	27·	6	21·	·43136	·32222	·89086
23	19·	4	15·	·27875	·17609	·89734
24	15·	2	13·	·17609	·11394	·93785
25	12·	4	8·	·07918	0·90309	·82391
26	8·	2	6·	0·90309	·77815	·87506
27	5·	3	2·	·69897	·30103	·60206
28	2·	1	1·	·30103	·00000	·69897
29	1·	...	1·	...	...	...
30	1·	...	1·	...	...	...
31	1·	...	1·	...	...	...
32	1·	...	1·	...	...	...
33	1·	1	...	...	...	...
Total	8643·5	560	8083·5	...	...	...

The figurs in colum (4) ar the differences between those in (2) and (3), and each of them therefore givs the number ov persons remaining alive at the end ov the year ov insurance opposit to which it stands. The figurs in colums (5) and (6) ar the logarithms ov those in colums (2) and (4) respectivly; and those in colum (7) ar obtaind by deducting the logarithm in (5) from that in (6): each difference is therefore the logarithm ov the unadjusted probability ov living thro the year ov insurance opposit which it stands.

It is to be notist, however, that for the deths in the year ov insurance 0, I hav substituted 2 insted ov the figur 4, which is the one givn in the table on p. 96. This alteration has been made for the reason that the lives under observation during the year ov insurance 0, ar on the average under observation only for six months each during that year, and the very low rate ov mortality experienst during the year 0 must therefore be considerd to last only for six months, and not for an entire year. This is a point ov so much importance that it may be desirabl to consider it a littl more minutely. Turning to Mr. Samuel Brown's introduction to the abov mentiond work, we find on page 3 the folloing passage: "As in all cases the office age on entry is the age next birthday, it "was decided that on the whole the current year of age or office age "at the date of assurance, would afford the means of approximating "very closely to the actual age, by the single assumption that the "assured attained that age at the end of the year of entry." The year ov entry here mentiond is clearly the calendar year, which will in most, but not in all, cases agree with the office year ov the companies making the returns. On page 4, we read "The date "of entry and of exit, like the age at entry, it was considered "would in very large numbers be so spread over the whole of "the year, that for all practical purposes the average period of "observation in each may be taken at half a year, and the date of "entry and of exit would consequently correspond with the middle "of the current year of age." On pages 8, 11, and 12, "The "age at entry is to be the actual age next birthday in all cases. "The age of exit is to be found by adding to the age at entry the "difference between the years of entry and exit." The effect ov the abov assumption is shortly this, that all persons under observation ar assumed to hav been born on 31 December, and to hav com under observation on 30 June; so that persons entering at the "current age" 66, ar assumed to attain that age on the average six months after entry, and the age at entry is assumed to be 65½ on the average. Turning next to page 18, we find an instance ov the numerical working ov the process by which the numbers exposed to risk in each year ov insurance folloing the "age at entry 29", ar deduced from the numbers givn in the "Table of Observations, No. 1, Healthy lives, Male", under the current age at entry 30. Here it is to be notist that "the current "age at entry" 30, and the "age at entry" 29 in the H^M table, called by Mr. Brown on page 18 "the actual age past", denote the same age, namely, 29½ on the average at entry; and it would

perhaps hav been better if the age had been givn in both instances as 29½. Disregarding for the present the small number, 75, who discontinued their insurances in the same calendar year in which they enterd, it is assumed that the 5,791 entrants ar each under observation for half a year, thus giving 2,895·5 persons exposed to risk for a full year, ov whom 4 persons died. The inference is that, assuming the mortality to be such as prevaild among these lives during the year 0 ov insurance, out ov 2,895·5 persons exposed to risk for a year, 4 will die; but, bearing in mind that the year 0 ov insurance comprizes only on the average six months, it will be more correct to say that, out ov 5,791 persons exposed to risk for half a year, 4 will die. We hav next to consider the case ov the 75 persons who discontinued their insurances in the same calendar year in which they enterd; and we observ that the manner in which they hav been treated in the abov mentiond calculation, produces the somewhat anomalos result that they hav not been supposed to be under observation at all. They hav in fact been assumed to enter in the midl ov the year, and to discontinue at the same time—the midl of the year; and if insted ov 75 ther had been 750 or any other number in a similar position, they woud not hav made the least difference in the number calculated to be exposed to risk during the year 0 ov insurance (which, in the description is calld the "first year of assurance"). The folloing woud hav been a more correct method ov treating these 75 persons. Let us assume they wer persons who effected their policies at halfyearly premiums, and discontinued them at the end ov the first six months; then it is clear that they must all hav enterd during the first six months ov the year ov entry. Assuming, as we hav don, the whole ov the entries to be uniformly spred over the year, we must suppose 75 other persons to hav enterd in the second half ov the year; and deducting these 150 from the 5,791, we hav 5,641 other persons, whose entries wer also uniformly spred over the year. Then the number ov years ov life will be found as follos:—

Entrants.		Years.
75	persons enterd in the first half ov the year and discontinued their policies at the end ov the first six months; each was under observation for six months . . .	37·5
75	persons enterd in the second half ov the year; but being supposed uniformly distributed over the six months, wer under observation on the average each for three months	18·75
5,641	entrants uniformly distributed over the year, and therefore under observation on the average each for six months	2,820·5
5,791		2,876·75

This result is the same as we shoud hav obtaind if we had assumed the 75 persons to hav been under observation on the average for three months each. It will be seen that the practical difference is so small as to be quite unimportant; and I hav not tho't it necessary to alter the figurs, in deducing the annuity values by the process to be now described. But as an error ov principl, which is unimportant in one case, may produce serios results in another, I hav tho't it better to draw attention to the point. The explanation ov the anomaly appears to be that our fundamental supposition—that the discontinuances ar distributed uniformly over the year, cannot, from the nature ov the case, be applied to the whole ov the year ov entry, but must be restricted to the second half ov that year.

If now we attemt to uze the figurs of Tabl (A) to deduce the values of annuities in the ordinary way, we are met by the difficulty that the year 0 ov insurance comprizes only six months, and the value ov the ordinary annuity can therefore only be obtaind by means ov a trublsom adjustment. Having got then similar results to those givn in Table (A), for the midl ages at entry $20\frac{1}{2}$, $25\frac{1}{2}$, $30\frac{1}{2}$, ... $75\frac{1}{2}$, I hav uzed the unadjusted results to calculate the values ov the annuities by means ov Mr. Woolhouse's very useful formula;—

$$\dot{a}_x = n\mathrm{S} - \frac{n-1}{2} - \frac{n^2-1}{12}(\mu+\delta);$$

where $\mathrm{S} = {}_np_xv^n + {}_{2n}p_xv^{2n} + \text{\&c.},$

(${}_tp_x$ here denoting, as usual, the probability ov the life on which the annuity depends surviving t years,)

μ = force ov mortality at the age x,
δ = force ov discount.

In applying this formula in the present investigation, it is to be observd that $\mu=0$; or, in words, *the force ov mortality at the commencement ov the insurance is zero.* The effect ov the medical examination is to reject all those persons who ar suffering from diseases tending to shorten life, which can be detected by the examiner. If we had the means ov making accurat observations upon a large number ov persons, say the residents in a particular town or the members ov a particular profession, we shoud find that a certain number ov these, perhaps 5 per-cent ov the whole, ar at any particular time suffering under complaints that will, either probably or certainly, proov fatal after a longer or shorter time; and if, after the laps ov, say, a year, we coud examin

carefully the causes ov death ov those persons among them who had died, we shoud find that much more than 5 per-cent, probably even more than 50 per-cent, ov the total deths woud occur among those persons whom we had previosly noted as sickly. A strict medical examination, then, greatly reduces the mortality in the years which immediatly follo it, that is to say, in the erly years ov the insurance. The fewer the number ov years which hav elapst since the medical examination, the greater will be its effect on the mortality; so that, in the first year subsequent to examination, the mortality is very liht; in the first six months after examination, it is still lihter in proportion; while it is an extremely rare thing for a life assured to die within three months ov the medical examination. But in the application ov Mr. Woolhouse's formula, μ is the value ov the force ov mortality at the instant the policy is effected; and we hav therefore to consider the effect ov a medical examination on the mortality, not during the three months, nor even during the month or the week folloing the examination, but during the day or even the hour thereafter. We thus see that, assuming the insurance to be effected immediatly after the life is past by the doctor, all persons laboring under diseases from which it is conceivabl that they shoud die within a day, will be eliminated; and we may almost say that ther is only left the chance ov the life being knokt down, run over, and killd, when leaving the office ov the company. The chance ov this or any other similar caus ov deth, is so extremely minute, that we arrive at the conclusion that, at the instant a policy is effected, $\mu=0$.

As abov explaind, the year 0 ov insurance comprizes on the average only six months for each life, while each ov the succeeding years, 1, 2, 3, &c., comprizes an entire year. It therefore appeard to me convenient to take the quantity n in the abov formula, equal to an integral number ov years $+\frac{1}{2}$. In a well graduated tabl, n may be taken large with litl sacrifice ov accuracy; but the facts to be uzed in the present instance being ungraduated, I tho't it better to take a short interval, and hav therefore taken $n=2\frac{1}{2}$, while, as a chek, the values ov the annuities hav also been obtaind by a second calculation in which $n=3\frac{1}{2}$. Making $n=2\frac{1}{2}$, and $n=3\frac{1}{2}$, successivly, the abov formula becoms

$$\tfrac{5}{2}\left({}_{2\frac{1}{2}}p_x v^{2\frac{1}{2}} + {}_{5}p_x v^{5} + {}_{7\frac{1}{2}}p_x v^{7\frac{1}{2}} + \ldots\ldots\right) + \tfrac{3}{4} - \tfrac{7}{16}\delta,$$

$$\tfrac{7}{2}\left({}_{3\frac{1}{2}}p_x v^{3\frac{1}{2}} + {}_{7}p_x v^{7} + {}_{10\frac{1}{2}}p_x v^{10\frac{1}{2}} + \ldots\ldots\right) + \tfrac{5}{4} - \tfrac{15}{16}\delta;$$

and these ar the working formulas by which the calculations hav been made.

The folloing is a specimen ov the calculation, and shoes how the value ov the annuity at 4 per-cent interest for the midl age at entry 65½, was found.

TABLE (B).

Middle Age at Entry 65½. Interest 4 per-cent.

t (1)	y (2)	$\log ({}_{2\frac{1}{2}}p_y)$ (3)	$\log ({}_{2\frac{1}{2}}p_y v^{\frac{5}{2}})$ (4)	$\log ({}_{t}p_{65\frac{1}{2}} v^t)$ (5)	${}^{t}p_{65\frac{1}{2}} v^t$ (6)
2½	65½	$\bar{1}$·97586	$\bar{1}$·93328	$\bar{1}$·93328	0·85759
5	68	·94208	·89950	·83278	·68043
7½	70½	·92894	·88636	·71914	·52377
10	73	·90914	·86656	·58570	·38521
12½	75½	·87206	·82948	·41518	·26013
15	78	·85376	·81118	·22636	·16841
17½	80½	·85591	·81333	·03969	·10957
20	83	·77367	·73109	$\bar{2}$·77078	·05899
22½	85½	·77562	·73304	·50382	·03190
25	88	·74715	·70457	·20839	·01616
27½	90½	·38907	·34649	$\bar{3}$·55488	·00359
30	93	$\bar{2}$·69897	$\bar{2}$·65639	·21127	·00163

3·09738 = S
2½
6·19476
1·54869
7·74345 = $\frac{5}{2}$S
+ ·75
8·49345
− ·01716

δ = ·03922
∴ $\frac{7}{16}\delta$ = ·01716

$\frac{5}{2}S + \frac{3}{4} - \frac{7}{16}\delta = 8{\cdot}47629 = a_{65\frac{1}{2}}$

The interval n being $= 2\frac{1}{2}$, the ages givn in colum (2) ar those at the commencement ov each period ov 2½ years. The figurs in colum (3) ar the logarithms ov the unadjusted probability that a person ov the age in colum (2) ($=y$, suppose) will liv 2½ years. It is obvios that the first ov these will be found by adding together the three first numbers in colum (7) ov Table (A). To obtain the others it is necessary to halv the figurs in column (7) ov Table (A) opposit the ages 5, 10, 15, ... This will perhaps more clearly appear from the folloing calculation of the four first terms in colum (3).

$\bar{1}$·99838	·97616	·98891	·96939
$\bar{1}$·98828	·97700	·98074	·96007
$\bar{1}$·98920	·98892	·95929	·97968
$\bar{1}$·97586	·94208	·92894	·90914

It will ov cours be understood that the figurs were not actually ritn down as here shoen, but the sums wer obtaind by addition ov

the figurs in colum (7) ov Table (A) after the logarithms ov the probabilities ov living thro the half years in the years ov insurance 5, 10, 15, ... had been previosly inserted, as shoen by the figurs in small type opposit the year ov insurance 5.

The figurs in colum (4) ov the abov tabl (B) ar obtaind by adding $\bar{1}\cdot95742=\log(1\cdot04^{-\frac{5}{2}})$ to each of those in colum (3); and they will therefore be the logarithms ov ${}_{2\frac{1}{2}}p_y v^{\frac{5}{2}}$. The figurs in colum (5) wer then obtaind by the successiv addition ov those in colum (4), and it is eazy to see that they will be the logarithms ov ${}_{2\frac{1}{2}}p_{65\frac{1}{2}}v^{\frac{5}{2}}$, ${}_{5}p_{65\frac{1}{2}}v^{5}$, ${}_{7\frac{1}{2}}p_{65\frac{1}{2}}v^{\frac{15}{2}}$ . . . . The figurs in colum (6) are the anti-logarithms ov those in colum (5), and ar equal to the successiv terms ov S. The remainder ov the work requires no explanation. In this way wer calculated the values shoen in the folloing tabl:—

TABLE (C).

Middle Age at Entry.	3 PER-CENT.			Middle Age at Entry.	4 PER-CENT.		
	$(n=2\frac{1}{2})$	$(n=3\frac{1}{2})$	Mean Value.		$(n=2\frac{1}{2})$	$(n=3\frac{1}{2})$	Mean Value.
$20\frac{1}{2}$	21·484	21·406	21·445	$20\frac{1}{2}$	18·183	18·148	18·165
$25\frac{1}{2}$	20·492	20·480	20·486	$25\frac{1}{2}$	17·547	17·539	17·543
$30\frac{1}{2}$	19·567	19·560	19·564	$30\frac{1}{2}$	16·904	16·900	16·902
$35\frac{1}{2}$	18·443	18·412	18·443	$35\frac{1}{2}$	16·099	16·097	16·098
$40\frac{1}{2}$	17·021	17·022	17·022	$40\frac{1}{2}$	15·019	15·023	15·021
$45\frac{1}{2}$	15·312	15·326	15·319	$45\frac{1}{2}$	13·698	13·710	13·704
$50\frac{1}{2}$	14·017	14·002	14·009	$50\frac{1}{2}$	12·660	12·646	12·653
$55\frac{1}{2}$	12·163	12·173	12·168	$55\frac{1}{2}$	11·115	11·125	11·120
$60\frac{1}{2}$	10·278	10·265	10·271	$60\frac{1}{2}$	9·497	9·499	9·498
$65\frac{1}{2}$	9·079	9·132	9·105	$65\frac{1}{2}$	8·476	8·521	8·498
$70\frac{1}{2}$	7·188	7·239	7·214	$70\frac{1}{2}$	6·784	6·834	6·809
$75\frac{1}{2}$	6·263	6·405	6·334	$75\frac{1}{2}$	5·956	6·094	6·025

It will be notist that the values obtaind by taking $n=2\frac{1}{2}$ and $n=3\frac{1}{2}$ ar in all cases so very close that no dout can remain as to their accuracy, notwithstanding that nothing has been don in the way ov graduation. Comparing the values with each other, we may fairly conclude that they ar correct to the second decimal place, except for the two first and the three last ages in the tabl, and that these ar correct to the first decimal place. Colums (4) and (8) ov the tabl giv the arithmetic means ov the values found by taking the two values ov n.

It will be observd that the results we hav obtaind, ar the values ov ordinary whole-life annuities on the lives ov persons ov the several ages, who hav just been submitted to medical examination; and they will therefore be the proper annuities to enabl us to determin and calculate the premiums that shoud be charged for

insurances on recently selected lives. The ages at entry 20½, 25½, ... not being convenient for the purpos ov comparison with other tabls, it was next necessary to deduce the approximat values ov annuities for the ages 20, 25, ... As the values ov the annuities agree on the whole fairly well with those ov the 17 Offices Experience, I considerd it sufficient to deduce the value ov the necessary correction from the values of the annuities according to that tabl. Thus, the value ov my 3 per-cent annuity at 35½ is 18·443, while the values ov annuities for ages 35 and 36, according to the 17 Offices Experience, ar 18·521 and 18·255, ov which the difference is ·266, giving a difference of ·133 for half a year. I hav added this quantity, ·133, to 18·443, and hav thus got 18·576 as the approximat value ov the annuity at age 35. For the purpos ov finding the correction, the nearest annuity according to the 17 Offices Tabl has been uzed. Thus, for age 75½, my 3 per-cent annuity being 6·334, I hav taken from the 17 Offices Tabl $a_{71}=6·364$, $a_{72}=6·049$, and added the half ov the difference between these, namely ·158, thus getting 6·492 as the approximat value ov the annuity at 75. In this way wer obtaind the values ov annuities for ages at entry 20, 25, 30, ... givn in the colum hedded "First approximation" in the folloing tabl. The annuity values according to the 17 Offices Tabl and the Institute* Tabl ar givn for the purpos ov comparison.

TABLE (D).

Values of Annuities on Insured Lives.

Age.	3 PER-CENT.				Age.	4 PER-CENT.			
	Lives just insured.		As found by the usual method.			Lives just insured.		As found by the usual method.	
	First approximation.	Corrected value.	17 Offices.	Institute.		First approximation.	Corrected value.	17 Offices.	Institute.
20	21·538	21·668	21·797	22·043	20	18·230	18·320	18·451	18·644
25	20·592	20·632	20·842	21·038	25	17·617	17·645	17·803	17·961
30	19·682	19·676	19·754	19·867	30	16·986	16·982	17·010	17·131
35	18·576	18·549	18·521	18·587	35	16·196	16·176	16·144	16·197
40	17·173	17·125	17·123	17·176	40	15·137	15·100	15·093	15·135
45	15·487	15·430	15·540	15·594	45	13·838	13·792	13·857	13·901
50	14·185	14·114	13·820	13·896	50	12·797	12·739	12·470	12·536
55	12·350	12·266	12·021	12·094	55	11·273	11·202	10·978	11·043
60	10·454	10·330	10·188	10·236	60	9·656	9·549	9·415	9·459
65	9·286	9·152	8·382	8·418	65	8·656	8·539	7·835	7·870
70	7·381	7·234	6·685	6·657	70	6·959	6·827	6·317	6·293
75	6·492	6·186	5·148	5·061	75	6·168	5·891	4·915	4·833

* In speaking ov the Institute Mortality Tabl, it seems unnecessary to ad H^M, as this, being the principal tabl, must be the one referd to when no other is indicated.

In the folloing tabl the corresponding annual premiums for an insurance ov 100 ar givn side by side.

TABLE (E).

Premiums for the Insurance ov 100.

Age.	3 PER-CENT.				Age.	4 PER-CENT.			
	Lives just insured.		As found by the usual method.			Lives just insured.		As found by the usual method.	
	First approximation.	Corrected value.	17 Offices.	Institute.		First approximation.	Corrected value.	17 Offices.	Institute.
20	1·526	1·499	1·473	1·427	20	1·354	1·330	1·295	1·245
25	1·719	1·711	1·665	1·625	25	1·525	1·517	1·472	1·428
30	1·923	1·924	1·905	1·880	30	1·714	1·715	1·697	1·669
35	2·195	2·203	2·210	2·193	35	1·969	1·976	1·987	1·969
40	2·590	2·604	2·605	2·589	40	2·350	2·365	2·368	2·352
45	3·152	3·174	3·133	3·114	45	2·893	2·914	2·885	2·865
50	3·673	3·704	3·835	3·801	50	3·401	3·433	3·578	3·542
55	4·578	4·625	4·767	4·725	55	4·302	4·350	4·502	4·458
60	5·818	5·914	6·025	5·987	60	5·539	5·634	5·755	5·715
65	6·809	6·938	7·745	7·705	65	6·510	6·637	7·472	7·427
70	9·020	9·232	10·100	10·148	70	8·719	8·930	9·820	9·866
75	10·435	11·003	13·352	13·585	75	10·106	10·666	13·060	13·299

If we now review the process by which our results hav been obtaind, we soon see that, by assuming the average age of the entrants in each groop to be equal to the midl age, we hav made an assumption that is probably not correct, and that may therefore to som extent vitiate the conclusions we hav arrived at. Let us examin then as to its exact effect. We have assumed, for exampl, that the entrants at the five ages 18½, 19½, 20½, 21½, 22½, ar so distributed over the various ages that their average age at entry was 20½. This woud be correct if the numbers ov entrants at the five ages wer all equal, and in certain other cases which it is unnecessary to specify. The numbers entering at the abov ages ar as follos:—

At age 18½ . .	675
" 19½ . .	1,086
" 20½ . .	1,555
" 21½ . .	2,497
" 22½ . .	3,069
Total . .	8,882

Multiplying each ov these ages by the number ov persons entering at it, and dividing by the total number ov entrants at the five ages, we find the average age ov entry in the abov groop to be 21·20 and not 20·5 as we have assumed. It is perhaps worth while to mention that it is not necessary to multiply each age by the number entering, as the same result will be obtaind by deducting 20½

from each ov the ages, and afterwards adding it to the quotient: thus the actual process will stand as folloes:—

$$\frac{2(3069)+2497-1086-2(675)}{8882}=\cdot 70;$$

and adding this to 20·5 we get the average age at entry 21·2. In this way it was found that the average ages ov the persons in the several groops wer as shoen in the folloing tabl.

TABLE (F).

Groop of Ages.	Number of Entrants.	Average Age.	Difference of age (average age − midl age).
18½ to 22½	8,882	21·20	+·70
23½ „ 27½	22,631	25·69	+·19
28½ „ 32½	26,821	30·48	−·02
33½ „ 37½	23,462	35·40	−·10
38½ „ 42½	17,610	40·34	−·16
43½ „ 47½	12,044	45·33	−·17
48½ „ 52½	7,945	50·30	−·20
53½ „ 57½	4,473	55·27	−·23
58½ „ 62½	2,516	60·16	−·34
63½ „ 67½	1,087	65·13	−·37
68½ „ 72½	333	70·06	−·44
73½ „ 77½	87	74·53	−·97

In the last colum is shoen the amount ov the error committed by assuming that the average age ov the lives in each groop is equal to the midl age. It proceeds with very great regularity, being ov considerable magnitude at the hihest and yungest ages. In the groop ov ages 28½—32½ it is so small that it may for all practical purposes be safely neglected, the average age at entry being there almost exactly equal to the midl age. At yunger ages the average age is greater than the midl age, and at hiher ages the reverse is the case. This is explaind by reference to the summary givn on pages 128,9 ov the Mortality Experience volume. We there find that the number ov entrants is greatest at the "current age" 30, that is, 30 next birthday, or 29½ according to our suppositions. From that age the number ov entrants diminishes almost uninterruptedly, whether we consider the older or the yunger ages, the diminution being much more rapid at the yunger ages. A litl consideration will sho that this law ov diminution must necessarily cause the error to follo the law shoen in the abov tabl. If we had taken 29½ as the midl age, the error in the groop 27½—31½ woud hav been only

$$\frac{2\times 5304+5239-5321-2\times 5289}{5304+5239+5791+5321+5289}=\frac{-52}{26944}=-\cdot 002.$$

Applying a correction in the manner abov explaind, we finally get the values ov the annuities and premiums as shoen in the colums headed "corrected value" in tabls (D) and (E). It will be notist that except at the ages 30–45, these final values ar intermediat between the values we previosly obtaind and the "Institute" annuities deduced by the ordinary process. We therefore lern that any method ov investigation which thros together the entrants at five consecutiv ages and considers the midl age as the common age ov all the groop, will giv an over-estimat ov the effect ov selection, both at the ages below 30 and at those abov 45.

Our general conclusion is that when proper allowance is made for the effect ov selection, the premiums for 50 upwards ar reduced, and those for 45 downwards, increast. At the yungest ages, 20, 25, the increase is about 5 per-cent; from 30 to 60 the difference is so small as to be ov no practical consequence; but for 65 and higher ages the true premiums ar greatly less than those obtaind from the mortality tabl formd in the usual way.

Discussion.

The President (Mr. J. Hill Williams) wished to express, not for the first time, the satisfaction which the council and the members generally must feel, to find so many proofs of the value of the great work of the Institute—the Mortality Experience. Mr. Sprague has investigated the very interesting question of finding out the values of the annuity, and the proper insurance premium, for recently selected lives. Looking at his valuable summary of the results at which he has arrived, it is satisfactory to find that the approximate method which has been usually adopted,—of dealing with the whole tables of mortality and not confining our attention to recently selected lives,—is not very far from the truth, and that for all practical purposes we are perfectly safe in using the general tables.

Mr. W. S. B. Woolhouse said that the subject chosen by Mr. Sprague in this paper was one of a peculiar kind, and out of the usual beaten track. In the working out of the calculations he did not see any point to criticize, except, perhaps, the assumption that, after the medical examination, the force of mortality commences at zero. No doubt the medical examination very greatly reduces the tendency to death; but that it entirely neutralizes it, seems to be an extreme view of the case.

Mr. Walford thought that, if the mortality is so much reduced by selection as Mr. Sprague seems to think, the mortality of short-term lives should be less than the ordinary rate of mortality; and yet it is generally considered that the mortality on term policies has been larger than on whole-life policies. In the report of the Mutual Society

of New York, published this year, Professor Bartlett has given much attention to the subject treated in this paper. He has not only shown the influence of selection at different ages, but has brought out the fact that the duration of life in America, contrary to the general belief, is longer than it is on this side of the water.

Mr. G. KING stated that he had prepared tables of annuities based upon the results given in his paper which appeared in the July number of the *Journal*, and they very closely approximate to Mr. Sprague's figures. He proceeded: In my treatment of the year of insurance O I somewhat differ from him; and, although I may take an opportunity of making some adjustment for that year later on, when I shall speak of the annuity values, still, without having been in any way corrected, my figures agree very closely with Mr. Sprague's, as the following examples will show:—

Annuities at Date of Assurance—3 per-cent.

Age at Entry.	By Mr. Sprague's table.	By my own table.
20	21·668	21·250
30	19·676	19·660
40	17·125	17·255
50	14·114	14·255
60	10·330	10·874

My values are on the whole a little higher than his, except at age 60, where I am considerably in excess, and I do not quite understand how the difference arises. With regard to the premiums, Mr. Sprague has shown very clearly that the premiums of the H^M table at the young ages of entry are too low, whilst those at the older ages are too high. That arises from the heavy mortality of the young ages at entry, after the lives have been sometime assured, being mixed with the lesser mortality of the older entrants who have been comparatively recently selected. It is interesting to note that the Carlisle table gives very nearly the true premium, according to Mr. Sprague's figures.

Age.	Mr. Sprague.	Institute.	Carlisle.
20	1·499	1·427	1·494
30	1·924	1·880	1·952
40	2·604	2·589	2·599

So that for the principal assuring ages, the Carlisle premium seems to be very close to the true premium—closer than that of the Institute. This fact may be of some importance; for instance, in dividing profits it is fairer to distribute the surplus in proportion to the loading on the Carlisle net premium than in proportion to that on the H^M, although for most purposes the H^M table is superior to the Carlisle. But again, even Mr. Sprague's premiums at the younger ages at entry are still too low for practical purposes, because so many policies lapse

during the first twenty years of insurance, when the mortality is heavy, that few are left on the books to give the company the benefit of the lighter mortality which occurs later on. The premium—calculated in a way which does not take account of lapses,—is reduced by the lighter mortality experienced after the policy has been a great many years in force, and from which the office will not derive a proportionate advantage. It will be found that at the younger ages at entry, the term-premium, for a term say of twenty years, calculated by a table based on the mortality followed out for each age at entry, will be still more in excess of the H^M term-premium than is Mr. Sprague's whole-life premium in excess of the H^M whole-life premium. A great many offices now charge extra for female lives, although it is well known that female life, taken throughout, is better than male; and the reason for this extra charge is that during the assuring ages the mortality is heavier. So it will be with the younger ages at entry. There is a heavier mortality during the time they are on the company's books; and when but comparatively few remain, the mortality is lighter again. If we want to charge a fair premium it should be derived from a net premium even higher than Mr. Sprague's, which is itself higher than the H^M.

Mr. G. W. Berridge said that the annuity values for the moment of entry given in his paper which appeared in the *Journal* for last April, came very close to Mr. Sprague's, taking his corrected value for comparison.

Age. (1)	Institute. (2)	Mr. Sprague's corrected value. (3)	My own value. (4)	Difference (4)−(3).
25	21·038	20·632	20·657	+·025
30	19·867	19·676	19·619	−·057
40	17·176	17·125	17·176	+·051
50	13·896	14·114	14·065	−·049

He was not quite happy in his mind as to that peculiar correction, which he had never seen before, but which Mr. Sprague had made on account of the centre age not being the real average age at entry. The effect has been very curiously to bring the annuity nearer to the Institute table, and *pro tanto* to diminish apparently the effect of selection—that is, the true effect of selection is less than it appears from the first calculation. With regard to Mr. Woolhouse's remark as to taking the force of mortality zero at the moment of selection, he thought that would only affect the annuity values at the beginning, for something like the first half-year of age. He had himself taken 0 as the force of mortality and only applied it to the first $2\frac{1}{2}$ years of the currency of annuity, in deducing the value of the annuity at the instant of the completion of the transaction, from the value of the annuity $2\frac{1}{2}$ years older, and his values came exceedingly near to Mr. Sprague's.

Mr. W. Sutton remarked that Mr. Sprague, at the end of his paper, comes to the conclusion that from ages at entry 30 to 60,—in other words, the ages at which, speaking generally, lives become

assured,—there is no difference between the premiums as calculated by him and the ordinary premiums; from age 65 upwards, at which, generally speaking, lives do not enter, there is a considerable difference; so that for practical purposes we are just as we were before. With regard to the year 0 of insurance, he had hitherto seen no reason to find fault with the method adopted in the Institute book, and he considered any theoretical departures, like this, to be practically of little value. As to putting the force of mortality at entry at 0, he agreed with Mr. Woolhouse, and was not certain that even theoretically it is correct to do so. When he considered that this force of mortality is often taken to be made up of two parts, one part generally increasing with the age, and the other constant throughout life, he was not at all prepared to admit that their values are both zero at the instant a life is pronounced insurable at the ordinary rate by the company's medical officer. However this may be, the whole point is almost immaterial with regard to the effect upon the annuity values, as it only slightly affects the second decimal place. Moreover, Mr. Sprague seems to have made a far rougher approximation himself than would be made by employing the ordinary Institute table, taking the year 0, and considering the first six months of the calendar year to give the same mortality results as the next six months; namely, when he deduces the annuity value for age 75 from that for 75½. He had himself pointed out—in a letter which appeared in the *Journal* of the Institute,* five or six years ago—that the discontinuances in year 0 have been so treated in the Institute experience as to have no effect whatever upon the rate of mortality. He thought that the paper, although it might not advance our knowledge of the effect of selection, was still in many respects extremely valuable, particularly because it forms a sort of model investigation for the younger members of the Institute to follow when they hereafter see their way to take up similar investigations. There are many useful practical hints throughout the paper, some of which are the results of great experience; and younger members of the Institute, at any rate, are greatly indebted to Mr. Sprague for coming forward and disclosing all his processes and methods.

Mr. A. H. BAILEY said, that it was felt by the committee which took in hand the working out of the Institute experience, that there was not sufficient material available to work out the monetary tables, as published, on the principles laid down in this paper—namely, that the ages should be kept separate for each year of entry; and the small difference in the results when this was done, seemed to prove that the decision was a right one. The manner of treating this unfortunate year 0, which has been the subject of so much criticism, was a matter of necessity, and not of choice; inquiries were made at the offices of the older insurance companies, and it was found that, as a rule, by the way in which the register was kept in old times, the date of birth was not recorded, but merely the age at entry. It was a question, therefore, of throwing away the most valuable part of the experience altogether, and it was decided that it was wiser to take what could be got, which was the age at entry and not the date of birth; and, having got the age at entry only, some such assumption

* See *Journal*, vol. 16, p. 75.

as was made was inevitable. Mr. Galloway, many years ago, assumed for the Amicable experience, that the lives were half a year younger than the office age at entry, and he found that on the average instead of six months, they were about four months younger. In such a mass of materials as this, the difference between four months and six months is scarcely worth considering, and does not very materially influence the practical results.

Mr. Sprague's Reply.

Mr. King argues that my premiums are too low at the younger ages, "because they are calculated in a way which does not take account of lapses." This, I think, is a mistake. Mr. King will no doubt admit that, if the policies are permanently kept in force till death, my premiums are the correct ones to charge. But if a policy is not kept up, the proper correction will be made by adjusting the surrender value; so that, in either case, my premiums are correct. Mr. Woolhouse and others doubt whether I am theoretically correct in assuming the force of mortality to be zero immediately after the medical examination. This is a point well deserving further consideration, and will no doubt receive it. At present the only argument brought against my view is Mr. Sutton's, that the force of mortality is made up of two parts, one constant and the other increasing with the age, and that my assumption practically amounts to considering each of these parts to be zero. But the above representation of the force of mortality applies only to a mixed body of lives, and not to a body consisting wholly of lives which have just passed a medical examination. If we take a body of lives, all of the same age, but otherwise mixed, good and bad, such as we find among the general population, and divide them into two groups, of which one contains the lives which would be pronounced by a competent medical examiner to be insurable at the ordinary rate, and the other contains the remaining lives, which are all more or less damaged, it would be clearly improper to suppose that the force of mortality is the same in the two groups or is capable of being represented by the same formula. It appears to me the more reasonable supposition that the instantaneous mortality of the latter group will be the same as that of the whole body, and consequently that the instantaneous rate of mortality, or the force of mortality in the former group will be zero. Strictly speaking, the assumption that the force of mortality is zero for a body of lives that have been just medically examined, only amounts to this, that none of them will die in the instant (or, practically, say in the day) after the medical examination. Lastly, I do not agree with Mr. Bailey that the method of treating the year of insurance 0 was a matter of necessity and not of choice. If it had been decided to classify the facts according to the years of insurance instead of the calendar years, no difficulty would have arisen as to the year 0. It would have been necessary to make some supposition as to the exact age at entry, but that is quite a different matter. In order to classify the facts according to the years of insurance, it is not necessary, as Mr. Bailey implies, to know the date of birth. All that is required is such information as

the following:—Out of a certain number of persons insuring at a particular office age, say 30 next birthday, so many died within a year from the date of insurance, so many lapsed their policies at the end of the 1st year; so many died in the 2nd year of insurance, so many lapsed at the end of the 2nd year; and so on. The reason that the Institute mortality experience gives no exact information on these points is, that the calendar year of entry only was asked for, and not the exact date; and similarly the calendar year of death, or exit by lapse or surrender, instead of the exact date of death, lapse, or surrender. The Institute experience therefore gives us no exact information as to the rate of mortality among insured lives in the year following the date of insurance, and would have given us no more information on this point even if the date of birth of all the lives had been given with the greatest accuracy. Moreover, the above information could have been perfectly well given by offices that could not state the exact date of birth, and there was therefore no question of throwing away any part of the experience. In conclusion I may remark, that in investigating the rate of mortality in the 1st, 2nd . . . . years of insurance, it is needless to estimate the age of entry with any minute accuracy, as differences of even some years in the age at entry will have a very trifling effect on the mortality in the early years of insurance.

NOTICES OF NEW BOOKS.

Encyclopædia Britannica—Ninth Edition—Article, Annuities.

It would be difficult to name a subject more important, from a financial point of view, than that of annuities. They may for all practical purposes be divided into two kinds—those depending on, and those independent of the duration of human life. The latter, which are known to actuaries as annuities certain, and are by financiers often called terminable annuities, admit of purely mathematical treatment, whereas the former are in the first instance based on the results deduced from observations on the average duration of human life. Rent-charges under the Lands Improvement Acts, Government loans, corporation loans, and leaseholds, are familiar instances of annuities certain, but the same principles are carried still further into the business of life. The famous "Three Years' System" is now applied to many other things than the purchase of pianos, and numberless commercial transactions now involve the application of the theory of annuities certain. On the other hand, the subject of life annuities is one which meets most of us face to face every day of our lives. Life interests are only too common, and it is every professional man's duty to bear in mind that his income is an annuity ceasing with his life,—if not sooner. Indeed, annuities of one kind or the other are constantly coming under our notice, and it is therefore nothing more than we should expect to find an article on the subject in a work of such pretensions as the *Encyclopædia Britannica*.

This article is from the pen of Mr. T. B. Sprague, whose writings in this *Journal* are well known to all its readers. Many years have elapsed since the article by Mr. Milne in former editions of the Encyclopædia was first published, and it is throwing no discredit on its writer to say that, while extremely valuable at the time it was written, it is now completely out of date, the subject of life contingencies having, like most other things, made great progress in the interval. Not only have many additions been made to the science, but it is now much more widely studied than formerly, and it is no exaggeration to say that there are many actuaries now living, with a far wider grasp of the subject in all its ramifications than was possessed by any practising actuary in Milne's time. The greatest stride of all has been made in the application of the methods of the calculus to obtaining a formula which shall approximately represent what is termed the law of mortality, and to the calculation of the values of annuities based on such law. One inestimable advantage of the extended knowledge of the theory of life contingencies, has been the promulgation of a uniform system of notation (for which, by the bye, we are in great measure indebted to Mr. Sprague); and this article will serve a very useful purpose indeed in this respect, the notation throughout being in accordance with the system recommended for adoption by the Institute of Actuaries some time ago.

As regards annuities certain there is little to be said that has not already been said, and, in fact, there is nothing in the subject but what a fairly intelligent man could easily acquire. Mr. Sprague's article therefore, as may be expected, does not deal with this part of the subject at any great length. He gives the usual formulas in the theory of compound interest, and points out their application to the theory of annuities certain. Special points, too, such as the separation of the periodical payment into interest and repayment of capital, the question of sinking funds (that bugbear to many people), also the question of annuities payable more often than once a year, are considered. In connection with this part of the subject, there is given a very complete account of most of the tables published in England, and at the same time many valuable suggestions for their improvement. Considerable attention is given to Smart's *Interest Tables*, and it is carefully pointed out that although these tables are practically useless for the odd half years, yet they are theoretically correct, and cannot be called "wrong", as Mr. Milne has described them.

Thus, for instance, the value of 1 due $n+\frac{1}{2}$ years hence, is given as $(1+i)^{-(n+\frac{1}{2})}$, and the value of an annuity certain for $n+\frac{1}{2}$ years is given as $\frac{1-(1+i)^{-(n+\frac{1}{2})}}{i}$. As regards the former, all that can be said is that it is not in accordance with practice, but as regards the latter it is difficult to say what it stands for. The author then proceeds to show how "tables calculated at the rate of interest $\frac{i}{m}$ may be used to find the amounts and present values at the rate i convertible m times a year", and makes the excellent suggestion that instead of using "years" in the headings we should use "terms". As a model form of heading for Interest Tables, Mr. Sprague gives the following:—

Tables of Amounts, Present Values, &c., at 5 per-cent Interest.

$i = \cdot 05.$

n	$(1+i)^n$	$(1+i)^{-n}$	$\frac{(1+i)^n-1}{i}$	$\frac{1-(1+i)^{-n}}{i}$	$\frac{i}{1-(1+i)^{-n}}$	$\frac{i}{(1+i)^n-1}$
1	1·05000000	·95238095	1·00000000	·95238095	1·05000000	1·0000000
2	1·10250000	·90702948	2·05000000	1·85941043	·53780488	·4878049
3	1·15762500	·86383760	3·15250000	2·72324803	·36720856	·3172085
4	1·21550625	·82270247	4·31012500	3·54595050	·28201183	·2320118
5	1·27628156	·78352616	5·52563125	4·32947667	·23097480	·1809748

While strongly approving of the above generally, we would suggest whether it is really necessary to give both the last two columns, the difference between them being necessarily i in all cases. In passing we would remark that, considering the author's familiarity with continental writers, we were rather surprised to find no mention of the tables published in France and elsewhere.

It is, however, in the part relating to life annuities that most of our readers will be directly interested; and we can confidently state that the article contains, in the space of a few pages, the clearest and completest account of the subject that has hitherto been published. After explaining the nature of a mortality table, and showing how the values of annuities are derived therefrom, there follows a capital sketch of the history of life annuities, including the vexed question of the first publication of the method now known as the commutation method.

After a well digested review of the whole evidence as to Barrett's share in the matter, the author concludes as follows:—"We cannot help thinking that Barrett's merits in the matter have been somewhat exaggerated. The first idea of a commutation table was not due to him, but (leaving Tetens out of view) to Dale and Morgan; and it is certain that he was familiar with the latter's treatise. The change he introduced into the arrangement of the table, namely, multiplying by a power of $(1+i)$ instead of by a power of v, is the reverse of an improvement; and accordingly his form of table has never been in practical use by any person but himself, excepting only Babbage. It is of course not to be denied that great credit is due to him, as a self-educated man, for perceiving more clearly than his predecessors the great usefulness of the commutation table; but in our opinion he does not stand sufficiently out from those who preceded and followed him, to justify the attempt to attach his name to the columnar method of calculating the values of annuities and assurances." This, we are bound to confess, exactly represents our own views, and, we believe, those of many others who have considered the matter.

Mr. Sprague is, apparently, under the impression that no joint life commutation tables, based on De Morgan's form $\left(D_{xy}=l_x l_y v^{\frac{x+y}{2}}\right)$, have ever been published. If we mistake not, this form has been adopted in Dr. Farr's *English Life Table*. Considering, however, how persistently the work in question ignores the well established usages of the actuarial profession, it is by no means a matter for surprise that Dr. Farr should find his labours to some extent unknown among

actuaries. The same formula for D_{xy} is also used in Mr. Neison's *Contributions to Vital Statistics.* The list of D and N tables will be found very useful, and includes nearly all of any value that have been published in this country. We notice, however, that no mention is made of the D and N tables, based on the Institute H^{MF} Table, which were communicated to the 16th volume of this *Journal* by the late Mr. Samuel Brown.

The space at Mr. Sprague's command being very limited, many questions could only be treated in very general terms; but we may safely state that every matter of importance is, in some way or other, referred to.

Thus for instance, the late Sir John Lubbock's formula for the calculation of an annuity is given with that very useful adjunct, a practical example worked out *in extenso.* Mr. Woolhouse's method of obtaining the same value, and which is, as stated by Mr. Sprague, an improved form of Sir John Lubbock's, is also treated in the same manner. The writer of this notice well remembers being struck with the similarity of the two methods, and the reader will find in vol. xv, p. 307, of this *Journal*, a demonstration showing how Mr. Woolhouse's formula can be deduced directly from Sir John Lubbock's. "Comparing the two processes," the author says, "we see that when we have the values of μ and δ already computed, Woolhouse's is decidedly the shorter. On the other hand, it is easy to see that Lubbock's formula applies not only to annuities but to other benefits; and that it will be applicable to find the values of such quantities as contingent annuities, the values of which cannot be found exactly except by a very long series of calculations."

Again, in reference to annuities payable at shorter periods than a year, the author says:—"When an annuity is payable half-yearly, the common rule for finding its value is to add ·25, or a quarter of a year's purchase, to the value of the annuity payable yearly. When it is payable quarterly, ·375 is added; and when by instalments at n equal periods throughout the year (or by $\frac{1}{n}$thly instalments), the addition is $\frac{n+1}{2n}$. The values thus found are sufficiently correct for most purposes. More correct methods of finding the values of annuities payable half-yearly, quarterly, &c., are investigated in papers in the *Assurance Magazine*, by Woolhouse, xi. 327, and by Sprague, xiii. 188, 201, 305. Some authors have assumed that when an annuity is payable half-yearly, interest is also convertible half-yearly, overlooking the circumstance that the true rate of interest is thereby changed, as we have explained in the earlier part of this article. In fact, as we showed, 5 per-cent interest convertible half-yearly, is equivalent to a true rate of interest £5. 1*s.* 3*d.* per-cent. If, then, we have found the value of an annuity when payable yearly at 5 per-cent interest, and require, perhaps, in the course of the same investigation, the value of an annuity payable half-yearly, it is clear that that value should be computed, not at £5. 1*s.* 3*d.* per-cent interest, but at 5 per-cent; or if we prefer the rate £5. 1*s.* 3*d.*, then the value of the annuity payable yearly should also be calculated at that rate."

Mr. Sprague, as would naturally be expected, deals very lucidly

with the subject of Gompertz's law and Makeham's modification thereof, and his remarks are so much to the point that we need offer no apology for reprinting them at length.

"In dealing with annuities in which three lives are involved, we are met by the difficulty that no tables exist in which the values of such annuities are given to the extent required in practice. Such tables as those computed for the Carlisle 3 per cent. table by Herschel Filipowski are of too limited extent to be of any practical utility; for the values being given only for certain ages differing by multiples of five years, a considerable amount of labour is required to deduce the values for other ages. When, therefore, we desire to find the value of an annuity on the joint lives of say x, y, and z, it is usual to take the two oldest of the lives, say x and y, and find the value of a_{xy}, then to look in the table of single life annuities for the annuity which is nearest in value to this,—a_w suppose,—and lastly, to find the value of a_{wz}, and use it as an approximation to that of a_{xyz}. De Morgan, in a paper written for the *Philosophical Magazine* for November 1839, and reprinted in the *Ass. Mag.*, x. 27, proved that the value of a_{xyz} thus found would be strictly accurate, if the mortality followed the law known as Gompertz's; that is to say, if the number of persons living according to the mortality table at any age, x, could be represented by means of the formula dg^{q^x}. Gompertz proved, in the *Philosophical Transactions* for 1825, that by giving suitable values to the constants, the above formula might be made to represent correctly the number living during a considerable portion of life, say from age 10 to 60; but in order to represent by the same formula the numbers living at higher ages, it is necessary to give fresh values to the constants; and the discontinuity thence resulting has always been a fatal obstacle to the practical use of the formula. It has, however, from its theoretical interest, attracted a great deal of attention from actuaries; and numerous papers on the subject will be found in the *Assurance Magazine*. A claim to the independent (if not prior) discovery of the formula has been put forward by Mr. T. R. Edmonds; but this claim, respecting which many communications will be found in the *Assurance Magazine*, is generally repudiated by competent judges. De Morgan further showed (*Ass. Mag.*, viii. 181) that if the above property holds good, or $a_{xyz}=a_{wz}$, then the mortality must follow Gompertz's law; and Woolhouse gave independently a simple algebraical demonstration of the same property, x. 121. Makeham removed the above mentioned objection to Gompertz's formula, by introducing another factor, and showed (*Ass. Mag.*, xii. 315) that the formula $dg^{q^x}s^x$ will correctly represent the number living at any age x from about the age of 15 upwards to the extremity of life; and this formula has been found very serviceable for certain purposes.

"The fact that Gompertz's law does not correctly represent the mortality throughout the whole of life, proves that the above-described practical method of finding the value of an annuity on three joint lives is accurate only in certain cases. Makeham has shown (*Ass. Mag.*, ix. 361, and xiii. 355) that when the mortality follows the law indicated by his modification of Gompertz's formula, the value of an annuity on two, three, or any number of joint lives, can be readily found by means of tables of very moderate extent. Thus the value

of an annuity on any two joint lives can be deduced from the value of an annuity at the same rate of interest on two joint lives of equal ages; the value of an annuity on any three joint lives, by means of a table of the values of annuities on three joint lives of equal ages; and so on; and Woolhouse has shown (vol. xv. p. 401) how the values of annuities on any number of joint lives, at any required rate of interest, can be found by means of tables of the values of annuities on a single life at various rates of interest. These methods, we believe, have not hitherto been practically employed to any extent by actuaries, and it would perhaps be premature to say which of them is preferable.

"As the reader will have observed, neither Gompertz's nor Makeham's formula represents correctly the rate of mortality for very young ages. Various formulas have been given which are capable of representing with sufficient accuracy the number living at any age from birth to extreme old age, but they are all so complicated that they are of little more than theoretical interest They are, however, likely to prove of increasing value in the problem of adjusting (or graduating) a table of mortality deduced from observations,—an important subject, which does not fall within the scope of this article. We may mention in particular those given by Lazarus in his *Mortalitäts-verhältnisse und ihre Ursache* (*Rates of Mortality and their Causes*), 1867, of which a translation is given by Sprague in the eighteenth volume of the *Assurance Magazine*, namely, $CK^{q^x}h^xH^{p^x}$; and by Gompertz (see *Ass. Mag.*, xvi. 329),

$$l_x = \text{const. } A^{\sigma^x} B^{f^x \cdot x} C^{q^x} D^P, \text{ where } P = \theta\omega^{\pi^x(x-u)}.$$

"If l_x represents the number living at any age in the mortality table, the force of mortality, or the instantaneous rate of mortality, is equal to $-\frac{d}{dx}\log_e l_x$. Hence, in Gompertz's original law the force of mortality at any age x is proportional to q^x, or is equal to aq^x, where a is a constant; in Makeham's law the force of mortality is equal to $aq^x + b$, where a and b are constants; and in Lazarus's law the force of mortality is equal to $aq^x + b + cp^x$, where a, b, and c are constants, or to $a\epsilon^{-mx} + b + c\epsilon^{nx}$. Dr. Thiele has shown (see *Ass. Mag.*, xvi. 313) how to graduate a mortality table, by assuming the formula for the force of mortality, $a_1\epsilon^{-b_1x} + a_2\epsilon^{-\frac{1}{2}b_2^2(x-c)^2} + a_3\epsilon^{b_3x}$; and Makeham has explained (*Ass. Mag.*, xvi. 344) a very convenient practical method for adjustment, which results in assuming that the number living at any age x can be accurately represented by the sum of three terms of the form $dg^{q^x}s^x$.

"The employment of formulas such as those given in the last paragraph, and the application of the differential calculus to the theory of life contingencies, have naturally led to an improvement in the theory which is probably destined to become of very great importance —we refer to the introduction of the idea of "continuous" annuities and assurances. If the intervals at which an annuity is payable are supposed to become more and more frequent, until we come to the limit when each payment of the annuity is made momently as it accrues, the annuity is called continuous. Strictly speaking, of course, this is an impossible supposition as regards actual practice; but if

an annuity were payable by daily instalments, its value would not differ appreciably from that of a continuous annuity; and if the annuity be paid weekly, the difference will be so small that it may be always safely neglected. The theory of continuous annuities has been fully developed by Woolhouse (*Ass. Mag.*, xv. 95). Assuming the number living in the mortality table at any age x to be represented by l_x, the value of a continuous annuity on a nominee of the age x is $\frac{1}{l_x}\int_x^\infty l_x v^x dx = \frac{1}{l_x}\int_x^\infty l_x \epsilon^{-\delta x} dx$, putting $\delta = \log_\epsilon (1+i)$. From the nature of the case, l_x must be a function that is never negative for positive values of x; and as x becomes larger, l_x must continually diminish, and must vanish when x becomes infinite. It will be noticed here that the superior limit of the integral is ∞. This is necessary if l_x is a continuous mathematical function; for in that case, however large x be taken, l_x will never become absolutely zero. Makeham has shown (*Ass. Mag.*, xvii. 305) that when the number living, l_x, can be correctly represented by the formula $cg^{q^x}e^{-ax}$, the value of a continuous annuity is equal to $\frac{1}{\log q} \cdot \frac{1}{10^{-10^z}.\epsilon^{-nz}} \int_z^\infty 10^{-10^z}.\epsilon^{-nz}.dz$, where $n = \frac{a+\delta}{\log_{10} q}$, and $z = x \log_{10} q + \log_{10}^2 \frac{1}{g}$; and he has given (pp. 312–327) a table, by means of which the value of the annuity can be found when the values of n and z are known. This table requires a double interpolation, and is therefore rather troublesome to use. Mr. Emory M'Clintock has shown in the eighteenth volume of the *Assurance Magazine*, how the value of an annuity may be found by means of the ordinary tables of the gamma-function. As Lazarus has pointed out in his above-mentioned paper, when mortality tables are given in the ordinary form, it is difficult to compare them and define precisely their differences; but if they can be accurately represented by a formula containing only a few constants, it becomes easy to show wherein one table differs from another; and the methods of Makeham and M'Clintock enable us to compare the values of annuities, for any ages desired, according to different tables as determined by such constants, without the labour of computing the mortality tables in the usual form. They can therefore scarcely fail to grow in popularity as they become better known."

There is one other point to notice. *Apropos* of the numerous references to contributions which have appeared in this *Journal*, the writer gives a brief history of the Institute of Actuaries, and under the circumstances we trust our readers will pardon us for quoting the passage at length.

The student who wishes to pursue the subject more thoroughly, and to become acquainted with all the improvements in the theory of annuities that have been introduced of late years, should carefully study the various articles contributed to the *Journal of the Institute of Actuaries*, particularly those of Woolhouse and Makeham. The Institute was founded in the year 1848, the first sessional meeting being held in January 1849. Its establishment has contributed in various ways to promote the study of the theory of life contingencies. Among these may be specified the following:—Before it was formed, students of the subject worked for the most part alone, and without any concert; and when any person had made an improvement in the theory, it had little chance of becoming publicly known

unless he wrote a formal treatise on the whole subject. But the formation of the Institute led to much greater interchange of opinion among actuaries, and afforded them a ready means of making known to their professional associates any improvements, real or supposed, that they thought they had made. Again, the discussions which follow the reading of papers before the Institute have often served, first, to bring out into bold relief differences of opinion that were previously unsuspected, and afterwards to soften down those differences,—to correct extreme opinions in every direction, and to bring about a greater agreement of opinion on many important subjects. In no way, probably, have the objects of the Institute been so effectually advanced as by the publication of its *Journal.* The first number of this work, which was originally called the *Assurance Magazine*, appeared in September 1850, and it has been continued quarterly down to the present time. It was originated by the public spirit of two well-known actuaries (Mr Charles Jellicoe and Mr. Samuel Brown), and was carried on by them for two years,—we believe, at a considerable loss. It was adopted as the organ of the Institute of Actuaries in the year 1852, and called the *Assurance Magazine and Journal of the Institute of Actuaries*, Mr. Jellicoe continuing to be the editor,—a post he held until the year 1867, when he was succeeded by Mr Sprague. The name was again changed in 1866, the words "*Assurance Magazine*" being dropped; but in the following year it was considered desirable to resume these, for the purpose of showing the continuity of the publication, and it is now called the *Journal of the Institute of Actuaries and Assurance Magazine.* This work contains not only the papers read before the Institute (to which have been appended of late years short abstracts of the discussions on them), and many original papers which were unsuitable for reading, together with correspondence, but also reprints of many papers published elsewhere, which from various causes had become difficult of access to the ordinary reader, among which may be specified various papers which originally appeared in the *Philosophical Transactions*, the *Philosophical Magazine*, the *Mechanics' Magazine*, and the *Companion to the Almanac;* also translations of various papers from the French, German, and Danish. Among the useful objects which the continuous publication of the *Journal* of the Institute has served, we may specify in particular two:—that any supposed improvement in the theory was effectually submitted to the criticisms of the whole actuarial profession, and its real value speedily discovered; and that any real improvement, whether great or small, being placed on record, successive writers have been able, one after the other, to take it up and develop it, each commencing where the previous one had left off. The result has been, as stated above, that great advances have lately been made in the theory. It may be truly said that the recent advances and improvements in the theory of life contingencies have rendered all the existing text-books antiquated; and until a new one shall be produced, bringing the treatment of the subject down to the present time, a complete knowledge of it can only be gained by a diligent study of the *Journal of the Institute of Actuaries and Assurance Magazine.*

It may interest our readers to know that the Council of the Institute contemplate the publication of such a text-book as that here described, and that the necessary preliminary steps in the matter have already been taken.

W. S.

HOME AND FOREIGN INTELLIGENCE.

Germany. Gotha Mutual Life Insurance Office. Extracts from the Report for 1874.

Although several important events affecting the development of the German Empire occurred in 1874, commercial affairs remained in the same stagnant condition as in the previous year, and continued to suffer from the effects of the late severe and widespread crisis. The

harvest generally was ample, and yet there was scarcely any diminution in the cost of the necessaries of life. Still the accounts of the principal life insurance companies seem favorable, and the experience of the *Gotha* office may be said to be good in every respect, showing that the unsatisfactory state of business generally cannot have been materially injurious to this particular branch. In a country like Germany, where only a small percentage of the population look upon life insurance in the light of a duty, the growth of the companies will not be affected to any considerable extent by commercial fluctuations, whether favorable or unfavorable; the same exertions will from year to year increase, if only to a trifling extent, the number of insurances; and though it cannot be denied that great calamities and events which convulse the world, and exclusively occupy men's thoughts for a long time, will occasionally check even these small additions, yet, on the other hand, to all provident persons, such events act as stern and effective reminders of the danger of neglecting life insurance.

The following report of the operations of the *Gotha* office will fully confirm the statement made above, that the progress of the Society during the past year has been satisfactory in every respect. The amount of the new insurances was greater than in any former year: the net increase in the total insurances after deduction of surrenders and lapses, was £815,986*; this exceeded by £18,100, the amount for 1873, and was only £73,443 less than the corresponding sum for 1872, which was the largest ever recorded.

The whole expenditure of the past year having been met or provided for, and a proper reserve made for the liabilities in respect of the policies in force, there remains a net surplus of £163,420. This is the largest ever made by the office in one year; and if the rules of the Society did not require it to be amalgamated with the surpluses of the two adjacent years, it would be sufficient to provide a bonus of 38·3 per-cent upon the premiums received in the year. This gratifying result was obtained at an expenditure (including Agents' Commission) of only 5·31 per-cent of the year's income.

We now subjoin details of the year's transactions:—

NEW INSURANCES.

	Proposals.	Amounts.
Policies issued on new lives	3,387	£1,018,729
" " on lives already insured .	561	169,729
	3,948	£1,188,458
Proposals declined by the office . . .	535	180,614
" withdrawn by the proposers .	64	33,486
" in progress at the end of the year	96	25,772
Total new proposals . . .	4,643	£1,428,330

The 3,948 policies issued were upon 3,890 lives, whose ages were as follows:—

* 7 thalers are taken as equal to £1.

From 15 to 20 years	28	lives	insured	for	£8,743
„ 21 „ 25 „	267	„	„		49,814
„ 26 „ 30 „	742	„	„		227,029
„ 31 „ 35 „	958	„	„		319,072
„ 36 „ 40 „	806	„	„		262,772
„ 41 „ 45 „	475	„	„		142,443
„ 46 „ 50 „	345	„	„		110,986
„ 51 „ 55 „	178	„	„		44,943
„ 56 „ 60 „	84	„	„		19,014
„ 61 „ 65 „	11	„	„		3,071
„ 66 „ 67 „	1	„	„		571
Total . .	3,890				£1,188,458

The greatest number of new insurances was effected at age 34, namely, 212; at this age also the amount insured was greatest, namely, £73,586. Of the 3,890 persons, 3,721 were males, insured for £1,147,586, and 169 were females, insured for £40,872.

TOTAL INSURANCES IN FORCE.

	Lives.	Sums Insured.	Lives.	Sums Insured.
Insurances in force on 1 January 1874 .			42,522	£11,999,514
Add New insurances during the year .			3,387	1,188,458
			45,909	£13,187,972
Deduct Cancelled:—				
By deaths occurring in 1872 and 1873 .	2	£215		
„ do. in 1874 . .	917	246,800		
„ short-term policies expired . .	11	3,557		
„ policies become payable in lifetime .	12	2,300		
„ policies issued but not taken up .	76	27,829		
„ surrenders and lapses	247	91,771		
			1,265	372,472
Remaining in force at the end of 1874 .			44,644	£12,815,500

The net increase was therefore £815,986, insured on 2,122 lives. The insurances may be classified as follows:—

(*a*) According to the nature of the insurances:—

	Lives.	Sums Insured.	An. Prems.
Payable at death or on attaining the age of 90	41,491	£11,908,572	£394,201
Payable at death or at ages less than 90 .	2,859	829,614	37,958
Survivorships . .	235	60,400	1,524
For terms of years .	59	16,914	322
	44,644	£12,815,500	£434,005

(*b*) **According to the sex of the insured:—**

	Lives.	Sums Insured.
Males	42,493	£12,372,271
Females	2,151	443,229
	44,644	£12,815,500

(*c*) According to the ages of the insured and amount of insurances:—

Sums Insured.	Number of Lives at Ages undermentioned.								Total.	
	15-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	Lives.	Sums Insured.
										£
£15 to £150	36	1,673	6,174	7,122	5,593	3,021	1,036	112	24,767	2,604,286
151 „ 300	7	619	2,484	2,689	2,078	1,138	416	45	9,476	2,398,700
301 „ 450	...	180	940	1,057	765	419	161	13	3,535	1,453,400
451 „ 600	2	93	540	642	446	212	93	10	2,038	1,144,229
601 „ 750	...	101	589	586	359	159	49	6	1,849	1,313,957
751 „ 1,000	...	53	347	405	271	102	40	3	1,221	1,059,957
1,001 „ 2,000	1	70	431	451	287	115	45	1	1,401	1,932,043
2,001 „ 3,000	...	21	128	123	71	12	2	...	357	908,928
	46	2,810	11,633	13,075	9,870	5,178	1,842	190	44,644	12,815,500

The youngest age of the lives insured was 15, at which age there were 3, insured for £500. The oldest age was 89, at which age there were also 3 lives insured for sums together amounting to £1,643. Between these limits there were insurances at all ages, the greatest number (1,452) at age 41. In consequence of the large accession of young lives, the average age of all the lives was further reduced, being 47 years and 9 months in 1873, and 47 years and 7 months in 1874. The average amount insured upon each life, which was £282 at the beginning of the year, rose to £287 at the end of the year. The sum at which the greatest number of insurances (11,832) was effected was £143 (=1,000 thalers).

CLAIMS.

(From 1872.) 13 claims remained for adjustment in 1874, including one (£72) which had not been previously reported. In one case the sum insured (£429) was forfeited; the Declaration having been proved by judicial decision to be untrue. Two claims (sums insured £286) were again postponed: in one case the claim was disputed by the office, on the ground that the life insured had committed suicide; in the other the policy was lost, and the probationary term of two years not having expired until December last, a legal discharge could not be obtained before the end of the year. The remaining 10 (for £914) were paid.

(From 1873.) 107 claims were left for adjustment in 1874, including one (£143) which had not been previously reported. Of these 98 were paid (£24,171) and 9 (£1,571) were postponed because

the policies were lost and the two years probationary delay had not expired.

In 1874, 917 claims occurred, of which

899	(£242,057)	were admitted.
3	(357)	temporarily postponed on technical grounds.
15	(4,386)	being deaths by suicide, the claims on the sums insured were forfeited.
917	(£246,800)	

The cases in which payment of the full sum insured could not be allowed have all been settled: the holders of the policies have surrendered them to the office on receiving the compensation appropriated to them out of the reserve together with the bonuses.

The difference between the claims expected, according to an estimate founded upon the mortality tables, and the actual claims, was as follows:—

Expected claims . .	£271,990	on	1,012·03	deaths.
Actual " . .	242,414	"	902	"
Difference . .	£29,576	"	110·03	"

The general healthiness of the lives was the sole cause of the claims falling short of the expectation; for the mortality was not greater among the small insurances than among those for larger sums, and the amount of each of the actual claims was, on the average, precisely the same as that of the expected claims, namely, £268.

In order to determine the actual profit gained by the office through favorable conditions of mortality, we must take into account the reserve which was made in respect of both the expected and actual mortality. The proportionate reserve, having regard to the ages, that belongs to the expected claims (£271,990) amounts to £92,598: for the actual claims (£242,414) there was a reserve of £96,172 in hand. The reserve for the larger amount was therefore less than that for the smaller. This is simply because a large proportion of the actual deaths occurred at high ages, at which of course the reserves were greater. If we diminish the expected claims (£271,990) by the estimated reserve pertaining thereto (£92,598) and the actual claims (£242,414) by the actual amount reserved in respect of them (£96,172), and subtract the latter result from the former, we have a sum of £33,150, which is the profit derived from the low rate of mortality.

The following table shows the proportion which the actual mortality among the lives insured in 1874 bore to the expected mortality. The deaths by suicide, &c., are included in this table. The ages are taken as at the dates when the premiums fell due in 1874.

Ages.	Insurances in force during 1874.	Deaths.		Difference.		Number of Lives exposed to Risk for a whole Year.	Mortality per-cent.
		Expected.	Actual.	More.	Less.		
15–25	592	3·98	1	...	2·98	435	·23
26–30	2,359	19·91	9	...	10·91	1,970	·46
31–35	5,156	51·25	29	...	22·25	4,688	·62
36–40	6,657	75·50	30	...	45·50	6,281	·48
41–45	6,831	83·59	47	...	36·59	6,628	·71
46–50	6,435	98·80	78	...	20·80	6,291	1·24
51–55	5,696	114·14	97	...	17·14	5,624	1·72
56–60	4,414	113·11	107	...	6·11	4,381	2·14
61–65	3,164	104·66	123	18·34	...	3,156	3·90
66–70	2,283	103·18	127	23·82	...	2,278	5·58
71–75	1,425	100·74	135	34·26	...	1,420	9·51
76–80	651	83·22	85	1·78	...	649	13·10
81–85	209	46·41	40	...	6·41	208	19·23
86–90	37	13·54	9	...	4·54	34	26·47
	45,909	1012·03	917	78·20	173·23	44,043	2·08 on the average.

The average age at entry of those who died was 40 years and 4 months (the first insurance only being regarded when there were more than one on the same life), and at death 61 years and 6 months. The average duration of these insurances was therefore 21 years and 2 months. The distribution of deaths as regards sex was 860 males and 57 females; and as there were 43,673 males and 2,236 females insured, the mortality of the former was 1·97 per-cent, and of the latter 2·55 per-cent. The mortality of the females was therefore, as usual, somewhat greater than that of the males, although their average ages were nearly the same.

In the Appendix are given tables showing the causes of death, the occupations of those who died, the months of the year in which the deaths took place, the ages at death, and the duration and amount of each policy; males being in all cases distinguished from females. The substitution of these tables for the lists of claims hitherto appended to the Report will, it is hoped, add considerably to its value. [We subjoin abstracts of two of these tables, see p. 127.]

INSURANCES WHICH BECAME PAYABLE IN LIFETIME.

Last year, 5 of the insured reached 90 years of age, and consequently received in their lifetime the sums insured (£829), although they had only paid the ordinary premiums. Besides these, 7 who had paid additional premiums to acquire this advantage at younger ages, received the sums insured (£1,471) on attaining those ages.

CASH BONUSES.

Of the profit made in 1868 and divisible in 1873, there was, at the end of 1873, a balance in hand of . .	£1,176
Of this, the sum paid to the participants, up to 8 December 1874, was	712
Leaving a balance of	£464

which, according to the Rules, is forfeited to the Society and is included in the receipts.

The bonus payable in 1874 was derived from the surplus of 1869, and amounted to 37 per-cent of the premiums, or	£118,013
Of this, there was paid to the insured, partly by deductions from the premiums and partly in cash . .	116,722
Leaving	£1,291

which may be received by the parties concerned, provided application be made not later than 8 December 1875.

PAYMENTS FOR SURRENDERS.

The sum paid last year out of the Reserve Fund for surrendered policies amounted to £6,943, of which, £739 was paid in respect of deaths from suicide.

The difference between the payments and receipts of the year was £192,062, raising the funds at the end of the year to £3,155,007. This increase of the funds is however not the real surplus of the year 1874. In order to find this, the value on 31 December 1874 of all the liabilities of the Society must be ascertained. These were as follows:—

Claims and bonuses now due but not yet paid .	£35,463
Deposits received in cash to be hereafter accounted for	20,980
The reserve, or the value of all policies in force on 31 December 1874, irrespective of the premiums paid in advance for periods after that date .	2,224,969
Premiums and parts of premiums paid in advance for periods after 31 December 1874	214,032
	£2,495,444
Deducting this value of the liabilities on 31 December 1874 from the above mentioned funds .	3,155,007
We find the net surplus to be . .	£659,563
This serves in the first instance as a Guarantee Fund, to meet extraordinary contingencies; but a proportion of it is divided from year to year.	
Deducting from this amount the net surpluses of 1870 to 1873, which are included in it (see below)	496,143
We have	£163,420

for the amount of net profit earned in 1874.

Last year the sum of £118,013 was taken from the Guarantee Fund for paying cash bonuses. The Fund is now composed as follows: —

Residue of surplus of 1870 . .	£62,244
Surplus of 1871	126,781
„ 1872	144,216
„ 1873	162,902
„ 1874	163,420
Total	£659,563

The surplus belonging to the insurance year 1870–71 will be returned in the course of 1875 to those entitled to profits, in proportion to the full yearly premiums paid in 1870. It comprises

1. The residue of the surplus of 1870	£62,244
2. A sum which bears to the surplus of 1871 the proportion which the premiums carried over to 1871 from 1870 bear to the total premiums of 1871. This sum will be found to be . . .	60,614
3. To avoid fractions in the rate of bonus, there has been taken from the surplus of 1871 a further sum of	811
Total	£123,169

The ordinary premiums paid in 1870 for whole life and survivorship insurances being £332,889, the above sum of £123,169 gives a bonus of 37 per-cent, payable in the course of 1875—on all existing policies by way of reduction of premiums; and on all policies cancelled by death or surrender, by payment in cash on bonds which are issued at the extinction of the policies.

From the above it will appear that there still remains of the surplus of 1871 a balance of £65,856 towards next year's bonus.

The total assets of the Society amount to £3,155,007, the principal part of which is invested on loans and mortgages as follows:—

£2,707,115	first mortgages on land of at least double the value.
11,357	first mortgages on house property of at least double the value, and insured against fire.
153,000	in bonds of public loan societies based on land securities, particularly mortgage bonds of Prussian provinces, Prussian and Bavarian revenue bonds.
1,878	in bonds of public corporations which are under supervision of the State.
3,686	on mortgages of German stocks or bonds, whose value exceeds the loan by at least 10 per-cent.
£2,877,036	

The average rate of interest upon these investments is 4·83 per-cent.

Among the above mentioned securities there is a sum of £3,000 for an estate in West Prussia, in regard of which a loss of £1,571 has been written off in the first quarter of the current year. This is the estate which was valued by public appraisers at £10,575, and upon which the office advanced £3,000. Reference to it has already been made in the reports for the years 1868–71, and it was stated in the report for 1871 that after holding the estate for several years the office had sold it at a price which completely covered the amount of the original loan. The loss has been caused by delay in payment of the purchase money. It was hoped that the purchaser, who was described as an intelligent and industrious farmer, would be able to pay the purchase money by instalments out of the profits of the farm, which contained 2,043 acres; but in this hope the office was deceived.

In order to save further and greater sacrifices, it was necessary to get rid of the estate entirely, which could only be done at a loss of the sum above mentioned. The loss will be entered in the accounts of the current year, and will therefore appear in next year's Report.

The interest due on loans is stated in the accounts at £34,277; this however includes £31,179 for accrued interest, payable in 1875 or at the end of 1874: the balance (£3,098) is the amount due and unpaid at the end of 1874, and has since been received, with the exception of three sums together amounting to £178, which are in dispute.

The agents' balances amount to £32,681, and are less than the total premiums due in December: the majority of these, on account of the four weeks' grace allowed, had not been received at the end of the year, and therefore could not be remitted by the agents. Nearly the whole amount has since been received.

. . . . .

Deaths in 1874.

	Cause of Death.																							
Classed:— (1) According to Occupation.	Fever.	Influenza.	Cholera.	Eruptive Diseases.	Inflammation.	Hæmorrhage.	Apoplexy.	Rheumatism.	Gout.	Dropsy.	Diseases of the Brain, and Insanity.	Diseases of the Spine.	Chronic Diseases of the Respiratory Organs.	Diseases of the Heart.	Diseases of the Abdomen (except Cancer).	Spasms.	General Dyscrasy, especially Cancer.	External Injuries and Malignant Ulcers	Poisoning.	Trichinosis.	Old Age.	Suicide.	Accidents.	Total.
Without occupation (56 of these were females)	3	1	1	..	12	..	13	..	1	1	5	..	5	4	2	..	7	1	..	..	14	1	..	71
Public Officials	14	1	2	2	82	5	52	2	..	9	21	4	47	28	38	..	20	5	1	..	47	5	5	390
Medical Men	..	..	..	..	5	1	5	..	..	..	2	..	3	5	4	..	3	..	1	..	2	1	..	32
Private Teachers	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	1
Artists	1	..	..	..	3	..	2	..	..	1	1	1	4	3	2	..	..	..	..	..	1	..	..	19
Servants	..	1	..	..	..	..	1	..	..	..	..	..	3	..	..	..	..	..	..	..	..	..	..	5
Farmers	3	..	..	..	5	..	3	..	..	1	..	..	4	1	4	..	1	..	1	..	2	2	..	27
Forest Guardians	1	..	..	..	3	1	6	1	..	1	1	..	6	1	4	..	8	1	..	..	4	1	1	35
Miners	..	..	1	..	5	..	..	..	..	2	..	..	..	..	..	..	..	..	..	..	1	..	..	9
Tradesmen	5	..	..	..	28	1	20	..	..	3	6	1	17	21	13	..	12	4	..	1	4	1	1	138
Transport Service	4	..	..	..	4	..	6	..	..	1	3	..	10	3	4	..	1	..	..	..	2	3	..	41
Engineers and Surveyors	..	..	..	..	..	..	1	..	..	..	..	..	..	1	1	..	1	..	..	..	..	..	1	5
Manufacturers	8	..	1	1	18	..	11	4	1	3	2	1	24	10	11	1	10	2	..	..	7	2	2	119
Military Men	..	..	..	..	3	1	1	..	..	1	1	..	..	1	..	..	..	..	..	..	4	1	..	13
Various	..	..	..	..	3	..	1	..	..	..	..	..	3	..	2	..	2	..	..	..	1	..	..	12
Total	39	3	5	3	171	9	122	7	2	23	42	7	127	78	85	1	60	13	3	1	89	17	10	917
(2) According to Age at Death.																								
15 to 20	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
21 " 30	1	..	..	..	1	..	..	..	..	..	..	..	8	..	..	..	1	..	..	..	..	..	..	11
31 " 40	12	..	..	1	8	..	2	3	..	..	2	1	16	2	3	..	1	2	..	1	..	4	3	61
41 " 50	5	..	1	..	24	1	7	1	..	2	8	2	29	13	12	1	6	2	1	..	..	6	3	124
51 " 60	8	..	2	..	37	2	27	2	1	5	11	..	31	20	24	..	16	4	1	..	..	6	2	199
61 " 70	10	3	2	2	58	3	33	1	..	8	12	2	26	31	25	..	29	3	1	..	8	1	..	258
71 " 80	3	..	..	..	40	2	42	..	..	7	9	2	15	11	19	..	7	2	..	..	56	..	1	216
81 " 90	..	..	..	..	3	1	11	..	1	1	..	..	2	1	2	..	..	..	..	..	25	..	1	48
Total	39	3	5	3	171	9	122	7	2	23	42	7	127	78	85	1	60	13	3	1	89	17	10	917

REVENUE ACCOUNT *for the Year* 1874.

RECEIPTS.

Amount of Funds at end of 1873		£2,962,945
New premiums for 1874	36,680	
Renewal premiums	390,296	
Total premiums		426,976
Extra premiums and commuted premiums		12,666
Premiums paid in advance		110
Interest on loans		140,770
Bonuses of 1868 unclaimed		464
Profit from exchange and discounts		285
House rent, deducting charges		86
Extra receipts for deposits, &c.		5,795
Total		£3,550,097

PAYMENTS.

377 Claims by death	£235,185
12 do. fallen due in lifetime	2,300
Bonuses to the insured	117,434
Do. unclaimed (as per contra)	464
Surrenders	6,943
Commission	14,379
Expenses of management	16,801
Repayments of deposits, &c.	1,584
Amount of funds at end of 1874	3,155,007
Total	£3,550,097

BALANCE SHEET *for* 31 *December* 1874.

LIABILITIES.

Claims unsettled (144)		£34,172
Unclaimed bonuses of the year 1869		1,291
Deposits received		20,980
Reserve (value of policies in force at the end of the year)		2,224,969
Premiums and proportions of premiums paid for periods after 31 Decr. 1874	£219,799	
Less Agents' commission already paid thereon	5,767	
		214,032
Guarantee Fund:—		
Surplus of 1870–73	£496,143	
Surplus of 1874	163,420	
		659,563
Total		£3,155,007

ASSETS.

Cash and bills in hand		£11,825
Loans on mortgage, &c.		2,877,036
Loans on policies		142,175
Half-premiums unpaid		29,111
Interest due on loans	£34,277	
Do. from Bankers and others	22,994	
		57,271
Agents' balances		82,681
Value of the Society's house		5,408
Total		£3,155,007

D. A. B.

NATIONAL LIFE ASSURANCE SOCIETY.

Established 1830.

EXTRACTS FROM THE REPORT OF THE ACTUARY (CHAS. ANSELL, JR.) UPON THE QUINQUENNIAL VALUATION UP TO 31 DEC. 1875.

The first quinquennial investigation of the affairs of the Society made in accordance with the provisions of the Deed of Settlement as altered in the years 1870 and 1873, being now completed, I beg to submit to you the results and my report thereon.

Prior to the year 1871 a statement of the affairs of the Society was required to be made as up to the 25th March, every year. The business of the Society was until then confined to two classes, namely, first, Members' assurances, which were entitled to participate in the profits; and secondly, Non-Members' assurances, which were not entitled so to participate. Hence one statement only, embracing the whole of the business, was sufficient.

The profits were applied in reducing for the then current year, by an equal percentage, the premiums upon all Members' assurances which had been in force for not less than five years.

The effect of the alterations alluded to in the Deed of Settlement was to require a valuation to be made only at the end of every fifth year, instead of annually as hitherto, and at the same time to constitute four distinct classes of assurances; namely:—

CLASS A, comprising all assurances entitled to participation in profits, effected prior to 1871.

CLASS B, comprising all assurances, other than endowments, effected in or after 1871, and entitled to participation in profits.

CLASS D, comprising all endowments, entitled to participation in profits; and

CLASS C, comprising all assurances not entitled to participation in profits.

It was also provided that the profits in classes A, B, and D respectively should belong exclusively to the members of each of those

classes, and that the profits of Class C should at each quinquennial period be divided among the other three classes in proportion to the respective amounts of premiums received in those classes, in such period. [The return under the Fifth Schedule states that, should it happen that the funds of any class are insufficient to meet the claims upon it, the funds of the other classes are liable to make good the deficiency.] It is therefore necessary to keep separate accounts of the four classes, and to prepare distinct statements for each of them at the quinquennial investigations.

The period embraced in the present investigation is the five years ending 31st December 1875.

The respective accounts of the four classes are appended to this report.

CLASS C.

The surplus of assets over liabilities, in other words the amount of profit realized in the quinquennial period, is £9,836. 17s. 2d., which, in accordance with the provisions of the deed of settlement of the Society, has been apportioned as follows among classes A, B, and D, namely:—

To Class A	£8,766	4	7
To Class B	1,037	2	5
To Class D	33	10	2

The amount of profit here shown is equal to nearly 15 per-cent upon the total amount of premiums received during the quinquennial period.

CLASS A.

It will be seen from the statement, that after reserving upwards of 14 per cent of the premiums to meet charges of management in future years, (which is considerably above the present rate of expenditure of the Society for that purpose,) there is a surplus sufficient to provide for a reduction of 60 per-cent upon the premiums in the present and future years; and as all existing policies in this class have now been in force at least five years, they are all entitled to the benefit of that reduction immediately, or, in the cases of assurers who have so elected [at the time of effecting the policy], to an equivalent reversionary bonus generally, until the year 1881, when a fresh valuation will be made.

The great increase in the rate of reduction from 45 per-cent to 60 per-cent in so short a period as five years, is attributable to three main causes:—(1), the most important of them, is the separation of the business of the Society into classes in the year 1871, the effect of that measure being that the large accumulation of undivided profits which then existed was reserved for the assurers in this class, from whose payments exclusively they had arisen; (2), the mortality in the class during the quinquennial period has been considerably under the expectation; and (3), the rate of interest realized on the investments has been much in excess of that upon which the valuation is based.

CLASS B.

The surplus of assets over liabilities, that is to say, the amount of profit realized during the quinquennial period is, including the proportion of surplus transferred from Class C, £4,742. 10s. 10d., which is equal to about 18¼ per-cent of the total amount of premiums received in the class.

The share of the surplus allotted to each policy will be applied in purchasing a fixed reduction in the future premiums; which reduction will commence at once in the cases of policies effected prior to 1st January 1874, and in the cases of policies effected more recently, as soon as three years' premiums shall have been paid on them.

[We learn from the return under the Fifth Schedule that the Deed of Settlement provides that the surplus, or such part thereof as the Directors may appoint, shall be apportioned among the assurers in the following manner, namely, There shall as regards each policy issued in the preceding quinquennial period be calculated the amount of premiums paid thereon accumulated at 3 per-cent compound interest up to the date at which the account is made up, and also the reserved value of the assurance at that date. The proportion of the differences between the accumulated premiums and the value of the assurance so determined for each policy respectively, to the total of such differences for all the policies collectively, shall denote the proportion of surplus to be apportioned to each policy; and such share of the surplus so assigned shall, if, or as soon as, three annual premiums shall have been paid on the policy be applied as the purchase money for an annuity on the life assured, according to his age at the date of making up the account; and such annuity shall be the amount of reduction in his annual premium from that time during the remainder of life.]

The percentage of the reduction of premium so obtained will vary with the age of the life assured, and with the term that the policy has been in force; but it may be stated that for the policies effected in the year 1871 the reduction of premium will average about 10 per-cent.

It is scarcely necessary to remark that the respective reductions in Classes A and B are not directly comparable one with the other, inasmuch as while in the former the reductions at successive quinquennial periods are in lieu of, not in addition to, preceding ones, in the latter they are cumulative.

Apart from a systematic valuation there is a simple though valuable test of the soundness of an Assurance Office, which is of general application, and gives approximately correct indications in all cases where the business is of the usual character. The test alluded to is the proportion that the accumulated funds, minus the present value of the reversionary bonuses which have been allotted, if any, bear to the total amount of the premiums that have been received upon existing policies.

It was given in evidence before a Committee of the House of Commons, some years ago, by the then leading actuaries of the day, that if such proportion amounted to, or exceeded 50 per-cent, the office might be considered to be in a safe position.

It will be seen from the subjoined statement, that in the case of our own Society, the proportion is nearly 78 per-cent.

Class.	Amount of the premiums (exclusive of temporary extra premiums for foreign risks) that have been received upon policies which were in force 31st December 1875, the premiums paid upon reassurances being deducted.	Amount of the accumulated fund upon 31st December 1875.	Proportion of the accumulated fund to the total amount of premiums received upon existing policies.
	£ s. d.	£ s. d.	
A	715,806 12 0	563,872 7 6	78·77 per-cent.
B	23,332 4 9	18,958 17 6	81·25 „
C	110,469 6 11	78,649 4 0	71·20 „
D	719 13 5	736 9 4	102·36 „
Total	£850,327 17 1	£662,216 18 4	77·88 Average.

In Class C, the amounts given are exclusive of Annuity receipts and liabilities.

In Class A, the value of the reversionary bonuses, namely, £9,201. 4s. 2d., is deducted from the accumulated fund; and the amount of premiums stated as received is the amount that has nominally become due, not taking into account the annual cash reductions of premiums that really have been allowed to assurers in that class; those cash reductions amount to £213,549, so that the proportion of funds to the amount of premiums actually received in that class is 112·27 per-cent, and for the four classes collectively 104 per-cent.

With regard to Class A, which is now closed to new comers, there is reasonable ground for believing that at the next quinquennial investigation a further reduction in the premiums, over and above the present one of 60 per-cent, may be made.

With regard to Class B, that in which all new assurances participating in profits are now issued, it has already been mentioned that the rate of reduction on policies effected in the year 1871 will average about 10 per-cent; but it may be proper to add, that if the profits in the next five years should be in the same ratio as they were in the five years just ended, the average further reduction in 1881 on those policies, which will then have been between nine and ten years in force, will be considerably above 10 per-cent, so that there is reason to believe that the total reduction may then materially exceed 20 per-cent, and that it may not improbably reach a much higher rate.

It may be remembered that when the separation of classes was made five years ago, an estimate was formed based upon the past experience of the Society in regard to bonus returns, that the premiums of assurers of average age in Class B, might be expected, by the operation of successive quinquennial reductions, to be exempted from any further payment of premium in about twenty years from the time their policies were effected; and it may be desirable here to state that the experience of that class for the five years that have since elapsed tends to confirm the correctness of the estimate then made.

[After the premium on any policy is extinguished, the profits subsequently assigned to it will be employed in making bonus additions to the sum assured.]

A Statement of the Assets and Liabilities on the 31st December 1875, of the Several Classes into which the business is divided.

CLASS A.

Dr.		£	s.	d.
Present value of £1,236,694 the net amount assured after deducting reassurances		672,846	1	8
Present value of £18,415. 18s. 5d. the net amount of reversionary bonuses after deducting reassurances		9,201	4	2
Reserve for future bonuses on special policies	£595 13 9			
Do. do. on reversionary bonuses	5,047 5 4			
		5,642	19	1
Reserve for reduction of premiums in future years at the rate of 60 per-cent		265,639	15	7
Reserve for charges of management and contingencies in future years at the rate of 14·112 per-cent upon the premiums		62,476	10	6
		£1,015,806	11	0

Cr.	£	s.	d.
Amount of Assurance Fund	564,307	7	1
Proportion of Surplus in Class C to be transferred	8,766	4	7
Present value of £38,428. 14s. 3d. per annum, the amount of annual premiums after deducting reassurances	442,732	19	4
	£1,015,806	11	0

CLASS B.

Dr.	£	s.	d.
Present value of assurances after deducting reassurances, £259,872	14,216	6	8
Surplus	4,742	10	10
	£18,958	17	6

Cr.	£	s.	d.
Amount of Assurance Fund	17,921	15	1
Proportion of the Surplus in Class C transferred	1,037	2	5
	£18,958	17	6

[In this Class the sum of £476. 13*s.* 7*d.* was apportioned among 276 policy-holders assured for £283,472.]

CLASS C.

Dr.		£	s.	d.
Present value of assurances after deducting the value of reassurances, £406,811		78,649	3	10
Present value of annuities £850 per annum		4,465	1	4
Surplus, to be transferred to Classes A, B and D in accordance with the provisions of the Deed of Settlement of the Society, viz. to:—				
Class A	£8,766 4 7			
Class B	1,037 2 5			
Class D	33 10 2			
		9,836	17	2
		£92,951	2	4

Cr.	£	s.	d.
Amount of Assurance Fund	92,951	2	4
	£92,951	2	4

CLASS D.

Dr.	£	s.	d.
Present value of endowments, £10,050	721	17	8
Surplus	14	11	8
	£736	9	4

Cr.	£	s.	d.
Amount of Endowment Fund	702	19	2
Proportion of the Surplus in Class C to be transferred	33	10	2
	£736	9	4

[In this Class no profits were divided.]

The tables of mortality used in the valuation, are

As regards Class A, the mortality table calculated by the late Mr. Griffith Davies from the experience of the Equitable Assurance Society.

As regards Class B, a mortality table corresponding at 3 per-cent interest with the premiums charged in that Class.

As regards Class C, for assurances on single lives, a mortality table corresponding at 3 per-cent interest with the premiums charged in that Class; and for assurances on two or more lives, a mortality table corresponding at 2½ per-cent interest with the premiums charged.

As regards Class D, the mortality table from which the premiums charged in that Class are calculated.

The rates of interest assumed in the calculations, are

As regards Class A 3¾ per-cent.

As regards Classes B and D 3 per-cent.

As regards Class C, for assurances on single lives, 3 per-cent; for assurances on two or more lives 2½ per-cent; and for endowments not involving any contingency of life, rates varying from 3 per-cent to 3½ per-cent, according as the term of the endowment is long or short.

In Class A, comprising about two thirds of the total business of the Society, there is reserved as a provision for expenses in future years, a sum equal to somewhat more than 14 per-cent, and for future profits a sum equal to somewhat more than 61 per-cent on the present value of all the future premiums. The total proportion of the annual premium income of this Class thus reserved as a provision for future expenses and profits is about 75¼ per-cent.

In Classes B and C the reserve made is that arising from the valuation being based upon the assumption of rates of interest lower, and of rates of mortality higher, than those which will probably prevail, and is consequently not any uniform percentage of the premium income.

In Class D the reserve made is partly that arising from the valuation being based upon rates of interest and mortality lower than those which will probably prevail; but there is also made a special reserve for expenses of 6 per-cent upon the annual premium income.

Average Rate of Interest yielded by the Funds of the Society.

Date.		Funds actually invested.		Entire funds, including cash at bankers and other unproductive amounts.	
		£ s. d.		£ s. d.	
On 31st December	1871	4 13 6	per-cent.	4 10 11	per-cent.
Do.	1872	4 17 0	"	4 14 7	"
Do.	1873	4 15 11	"	4 13 7	"
Do.	1874	4 14 3	"	4 12 3	"
Do.	1875	4 11 4	"	4 9 1	"

Comparative Statement of Business.

Year.	Amount of the Assurance Fund (net) on 31st December.	Income arising from funds as invested on 31st December.	New Assurances less reinsurances and substituted policies.	Gross amount of assurances in force on 31st December.	Gross annual premiums thereon.	Gross annual income from premiums and interest.	Cash bonus or abatement of premiums.	Ratio of management expenses to net income.	Year.
	£	£	£	£	£	£	£	£ s. d.	
1866	533,651	26,046	156,303	2,012,512	66,290	92,336	15,038	7 17 6	1866
1867	543,904	26,585	110,245	2,047,797	67,888	94,473	14,533	7 10 5	1867
1868	552,177	25,525	123,920	2,071,978	67,329	92,854	15,584	7 14 7	1868
1869	574,730	26,341	137,380	2,165,683	70,283	96,624	14,379	8 5 3	1869
1870	588,449	28,364	99,264	2,136,723	66,400	94,764	14,628	7 17 2	1870
1871	602,098	28,001	91,250	2,112,848	65,638	93,639	14,629	8 16 0	1871
1872	625,422	30,302	71,332	2,112,451	65,539	95,841	15,272	6 18 3	1872
1873	633,782	30,254	90,738	2,114,986	65,121	95,375	15,770	7 3 8	1873
1874	664,591	31,692	100,180	2,100,520	66,442	98,134	16,821	7 2 0	1874
1875	675,883	30,745	126,486	2,132,527	68,583	99,328	16,615	6 16 11	1875

[The report states that no commission has been paid for the introduction of new business since 1870.]

In printing the following statement we hope we may be excused if we depart for once from our usual rule, of making no comment whatever on the accounts we reprint, either as to their form or the results they exhibit. It appears to us that the form of this Statement of Property and Income leaves nothing to be desired, and that it might with great advantage, in any future amendment of the Life Assurance Companies Act, be required to be given by all the Life Offices.—Ed. *J. I. A.*

Statement of Property belonging to the Society and of the Income arising therefr
as on 31*st December* 1875.

	Value.	Total Value.			Income.	
	£ s. d.	£	s.	d.	£	s.
Mortgages on Property within the United Kingdom	...	119,221	14	2	6,086	2
Loans on the Society's Policies (within their value)	...	35,000	13	3	1,750	0
Loans on Life Interests and Reversions . . .	...	211,177	17	7	11,461	19
Loans upon Personal Security	...	912	10	0	45	12
Loans upon County and Borough Rates . . .	...	5,120	0	0	224	19
British Government Securities:—						
£35,000 0 0 New Three per Cents . . .	31,850 0 0					
65,000 0 0 Consols	59,800 0 0					
		91,650	0	0	3,000	0
Indian and Colonial Government Securities:—						
£11,760 Great Indian Peninsula Railway Stock .	...	13,171	4	0	588	0
Railway Debenture Stocks:—						
£11,000 Great Eastern Debenture Stock, A. .	12,980 0 0					
9,000 Do. Do. B. .	10,440 0 0					
10,000 London, Chatham and Dover 4½ per-cent Arbitration Debenture Stock . .	10,600 0 0					
10,000 Midland Railway Debenture Stock. .	10,100 0 0					
		44,120	0	0	1,850	0
Railway Guaranteed and Preference Stocks:—						
6,000 North-Western 4 per-cent Preference Stock	5,760 0 0					
10,000 Great Western 5 Do. Do.	11,500 0 0					
5,120 South Eastern Preferred Stock. . .	6,730 0 0					
20,000 Brighton 5 per-cent Preference, No. 6 .	22,200 0 0					
		46,190	0	0	2,047	4
Dock Preference Stock:—						
£20,000 Southampton Dock	...	20,000	0	0	1,000	0
Temporary Loan on North Eastern Railway Stock .	...	40,000	0	0	1,000	0
House Property—Freehold, No. 2 King William-street	...	25,000	0	0	552	0
Reversions	...	10,289	14	9	617	7
Rent Charges, £996. 17*s*. 3*d*. per annum . .	...	11,428	9	7	522	4
INVESTED ASSETS	...	£673,282	3	4	£30,745	11
UNPRODUCTIVE ASSETS:— Agents' Balances, Outstanding Premiums, Interest due and accruing, and Cash . .	...	16,697	16	4		
GROSS ASSETS	...	£689,979	19	8	£30,745	11

INSTITUTE OF ACTUARIES.

FIRST YEAR'S EXAMINATION, 1876.

Examiners.—Messrs. C. D. Higham, W. Sutton, and J. Whitcher.

I.

1. State the rule for contracted multiplication; and give an algebraical explanation of the same. Find to six places of decimals the value of

$$272947 \times \cdot 0032999.$$

2. Find under what conditions a vulgar fraction can be expressed by a finite decimal.

Given that it is found by long division that

$$\frac{1}{29} = \cdot 03448 \text{ with a remainder } 8,$$

so that $\frac{1}{29} = \cdot 03448\frac{8}{29}$,

show without long division that a further approximation to the value of $\frac{1}{29}$ is

$$\cdot 0344827586206896551 7\frac{7}{29}.$$

3. Reduce 3 wks. 4 days 5 hrs. 6 min. 7 sec. to the decimal of a calendar month of 31 days.

4. Write down the square root of

$$\frac{a^2c}{b} - cf + 2ac\sqrt{-\frac{f}{b}}.$$

5. If x is small compared with a, show that

$$\frac{(a+x)^3}{a^3} = \frac{a+3x}{a} \text{ approximately.}$$

Hence deduce the following rule for approximating to the cube root, viz.:—If N^3 be the number whose root is required, and a^3 the nearest rational cube to N^3, then

$$N = a\frac{a^3+2N^3}{N^3+2a^3}.$$

6. A certain sum is $\cdot a\dot{b}$ of £1. If to this be added two guineas, three half-crowns, and twice $\cdot 0a\dot{b}$ of £1, the sum is 5·35 of ten shillings. Find a and b.

7. Explain the difference between an equation and an identity. Show that a quadratic equation can have only two roots.

8. Transform the equation $ax^2+bx+c=0$ into an equation not involving the first power of x. As an example take the equation

$$8x^2+12x-5=0.$$

9. Solve the equations

$$\text{(i.)} \quad \sqrt{x}+\sqrt{2a+x}=\frac{4a}{\sqrt{2a+x}};$$

$$\text{(ii.)} \quad a^{1-\frac{r}{x}}-b^{qx}c^{3p-x}=0;$$

$$\text{(iii.)} \quad \left.\begin{array}{l} \frac{x}{5}+\frac{5}{y}+\frac{13}{3}, \\ \frac{y}{5}+\frac{5}{x}+\frac{13}{4}. \end{array}\right\}$$

10. A merchant sold goods for £39, gaining as much per-cent as the goods cost him; what was the price of the goods, and what is the meaning of the negative root in this case?

11. Define series in arithmetical and geometrical progression, and show how to sum them.

A number of persons bought a field for £345, the youngest paying a certain sum, the next £5 more, and so on in arithmetical progression. The younger half took a part of the field proportional to the sum they had subscribed, and this they agreed to divide equally, by making their contributions £22 each. How many persons were there in all?

12. The sum of n terms of any arithmetical progression whose common difference is equal to the least term, will be equal to the sum of $n+1$ magnitudes, each of which is half the greatest term of the progression. Prove this.

13. If S, a, r, n, be respectively the sum, first term, common ratio, and number of terms of a geometrical progression: find the sum of the series

$$(S+a)+(S+a+ar)+(S+a+ar+ar^2)+\ \&c.$$

II.

14. One root of the equation $x^3-11x^2+37x-35=0$ is $3+\sqrt{2}$. Find the remaining roots.

15. Sum the series

$$\frac{1}{\sqrt{2}(1+\sqrt{2})}+\frac{1}{(1+\sqrt{2})(2+\sqrt{2})}+\frac{1}{(2+\sqrt{2})(2\sqrt{2}+3)}+\ \&c., \textit{ ad infin.}$$

16. What does $n.\overline{n-1}\ldots.\overline{n-r+1}$ express? Give your demonstration in full.

How many different permutations can be made of the letters in the words INSTITUTE OF ACTUARIES taken all at a time?

17. Show that the total number of combinations of n things taken

1, 2, 3 .... n together, of which p are of one sort, q of another, r of another, &c., is $\overline{p+1}.\overline{q+1}.\overline{r+1}\ldots.-1$.

What proportion will the number of combinations of the letters in the words INSTITUTE OF ACTUARIES taken five at a time bear to the number taken six at a time?

18. State the Binomial Theorem, and give Euler's demonstration of it.

Show that $$(a+b)^n=a^n+na^n\cdot\frac{b}{a+b}+\frac{n.\overline{n+1}}{1.2}a^n\cdot\frac{b^2}{(a+b)^2}$$
$$+\frac{n.\overline{n+1}.\overline{n+2}}{1.2.3}a^n\cdot\frac{b^3}{(a+b)^3}+\&c.$$

Hence sum the series

$$\frac{1}{3}+\frac{3}{2}\cdot\frac{1}{3^2}+\frac{3.4}{2.3}\cdot\frac{1}{3^3}+\frac{3.4.5}{2.3.4}\cdot\frac{1}{3^4}+\&c., \textit{ ad infinitum.}$$

19. Define a logarithm, and *from the definition* show that

(1.) $\log a^b=b.\log a$.

(2.) $\log ab=\log a+\log b$.

(3.) $\log\frac{a}{b}=\log a-\log b$.

(4.) $\log_a b\times\log_b a=1$.

Given the *mantissa* of log 98714 $=\cdot9943788$,
" of log 245932 $=\cdot3908150$,
" of log 1224903 $=\cdot0881018$,

find the value of $(987\cdot14)^{11}\times(\cdot245932)^{40}$.

20. Expand $\log(1+x)$ in a series of powers of x, and state whether the series is convergent. State and prove the rule for proportional parts.

21. A and B are playing with two dice, each having staked £1, the highest throw to win. A has thrown 6. What is B's expectation?

22. The prize at a raffle is to be given to the drawer of the only marked ball in a bag containing as many balls as there are drawers. Which of the drawers has the best chance? Prove your answer.

23. If $\Delta x=a$ and $\Delta u_x=u_{x+a}-u_x$, prove that

$$u_{x+n}=u_x+\frac{n}{a}\Delta u_x+\frac{n.\overline{n-a}}{2a^2}\Delta^2u_x+\frac{n.\overline{n-a}.\overline{n-2a}}{2.3.a^3}\Delta^3u_x+\&c.$$

Find log 512, given that

log 510=2·70757018,
log 511=2·70842090,
log 513=2·71011737,
log 514=2·71096312,

24. Given every nth term of a series of values, *i.e.*, u_x, u_{x+n},

u_{x+2n}, &c., show at length how the intermediate terms u_{x+1}, u_{x+2}, &c., may be obtained by interpolation.

Given that in the series u_x, u_{x+1}, u_{x+2}, ...

$$\begin{aligned} u_x &= 9936675\cdot4, \\ \delta_1 &= +12767\cdot62, \\ \delta_2 &= -3013\cdot725, \\ \delta_3 &= +422\cdot8247, \\ \delta_4 &= -34\cdot72847, \\ \delta_5 &= +1\cdot254221, \end{aligned}$$

you are required to construct the series as far as the term u_{x+10}. What assumption is necessary?

25. Describe the various books of account generally in use, stating definitely what purposes each is intended to serve. What is meant by a trial balance?

SECOND YEAR'S EXAMINATION, 1876.

Examiners.—Messrs. G. W. BERRIDGE, T. G. C. BROWNE, D. A. BUMSTED, and G. KING.

I.

1. If p_x be the probability that a person aged x will live one year, find expressions for the following probabilities:—

That out of 1000 persons aged x

a, Exactly 20 will die in a year.

b, Not more than 20 will die in a year.

c, 20 designated individuals, and no more, will die in a year.

d, 20 designated individuals, at least, will die in a year.

2. Determine the present value of an annuity certain of £1 per annum for n years, which is to pay during its continuance a given rate of interest on the purchase money, and to replace the purchase money at the expiration of the term at a different rate of interest.

3. When interest is convertible momently, investigate the relation which exists between the nominal rate and the effective rate. Write down, in terms of the "force of discount" and the continuous annuity, the value of an assurance of 1 on x, payable at the instant of death.

4. The following is taken from a diary for 1877:

How to value leaseholds.—Divide the proposed purchase money by the number of years to run before the lease expires; the quotient, deducted from the net rental, gives the interest on the purchase money.

Is this correct? If not, show how the true rate of interest is to be found.

5. Having given a complete table of p_x, accurately representing the probabilities of life at all ages, show how, from the deaths taking place in one year, to calculate approximately the total numbers living in a stationary population where there is no disturbance from immigration or emigration.

6. Explain under what conditions ${}_nV_x < {}_{n-1}V_{x+1}$, and give an example from some known table of mortality where the anomaly occurs.

7. Investigate the relation which subsists between A_x and a_x, finding equations in terms of v, d, and a_∞ respectively.

8. Give a general formula for the class of be efits, the single premiums for which can be derived from Orchard's Tables, and explain it verbally.

9. Find the present value of an annuity on x (payable half-yearly), commencing at £1, the payments to be doubled every ten years.

10. Find a formula for the annual premium for an assurance payable in the event of A dying before B or within t years after.

11. An annuity of £m is to be paid half-yearly to x's wife after his death; but shou'd x survive his wife, a sum of £n is to be paid at his death. Required the annual premium (payable during the joint lives) to secure these benefits.

12. Prove the following expression,—

$$|_nA_x=1-d(1+a_x)-\frac{D_{x+n}}{D_x}\{_nV_x+P_x(1+a_{x+n})\}.$$

13. Explain the construction of the ordinary commutation columns, and very briefly illustrate their use.

II.

14. Mention the two methods as regards interest upon which the D and N columns for joint lives have been constructed. Under what circumstances is the one preferable to the other?

15. Show how to construct a table, from which may be found the annual premium corresponding to any given single premium on a whole-life insurance.

16. Deduce a formula for finding the value of the next n presentations to a living.

17. Required the net single premium for an insurance upon the life of x, of £1000 increasing at compound interest during the first five years at the rate of 4 per-cent per annum.

18. Find the value of a temporary life annuity which will yield a certain rate of interest on the capital employed, and provide for a premium for an assurance to replace the capital.

19. Give a verbal explanation of the formulas

$$A_x=v-da_x,$$
$$A_x=v(1+a_x)-a_x.$$

20. Express the value of a policy

(*a*) in terms of the premiums and the rate of interest,

(*b*) in terms of the assurance at entry and at valuation.

21. Demonstrate a general formula for deriving the value of a benefit at age x from the value at age $x+1$. Given a table of the values of P_x (the premium), show how to derive p_x (the probability).

22. To obtain the office premium, the net premium is loaded k per unit. Find the annual premium for an assurance of 1 on x, where all the premiums paid, with simple interest at the rate i, are to be returned along with the sum assured.

23. Having given the usual commutation tables at the rate of

interest i, investigate a formula for the value of an annuity at a lower rate of interest i'.

24. The actual claims for the year in an office exceed the expected amount,—Does the difference represent the loss from mortality during the year? Give the reasons for your answer.

25. Give a form for a book in which to record the mortality experience of a company from year to year.

THIRD YEAR'S EXAMINATION, 1876.

Examiners.—Messrs. C. J. BUNYON, A. J. FINLAISON, R. P. HARDY, and G. HUMPHREYS.

I.

1. Describe briefly the practical process of investigating the mortality experience of a life assurance company. What additional facts would you think it necessary to bring out, in order to show the true bearing of the results arrived at?

2. Show how you would exhibit the measure of the effect of medical selection amongst the lives assured.

3. (1.) Give the *rationale* of Mr. Woolhouse's method of graduation, and state whether it involves any assumption of an inherent law of mortality.

(2.) Explain the modification of Mr. Woolhouse's method, proposed by Mr. Charles Ansell, Jun.

4. What is your opinion of the merits and applicability of the English Life Table (No. 3) for the purposes of a life assurance company?

5. Describe the statistical records that should be kept, in order that the financial position of a life assurance company can be ascertained with as little delay as possible.

6. Sketch a form of "Valuation-Sheet" which will combine the materials for—(1) The valuation of ordinary whole-term policies; (2) Ascertaining the ratio for bonus distribution; (3) Apportioning the surplus and converting it into its several equivalents.

7. If the premiums of an assurance office have been formed by the addition of a constant percentage to the Carlisle rates, and if it was desired to make a valuation according to the "Institute of Actuaries" (H^M) 3 per cent, show:

(1) How you would proceed to arrive approximately at the amount of loading to be taken off the office premiums, in order to make what is called a "net-premium valuation".

(2) What would be the financial effect of valuing the Carlisle premiums and liabilities by the "Institute of Actuaries" (H^M) 3 per cent Table, and what assumptions are made thereby?

8. The premium rates of a mutual life assurance society were founded upon a special table, supposed to represent the exceptionally heavy rate of mortality of the district from which the business was to

be drawn, and some addition was made thereto by way of margin. The mortality experienced has been greatly below the rates shown in the table on which the premiums were based. The periodical valuations have been made, by the net method, upon the original mortality table, but with interest closely approximating to that obtained upon the society's investments, and the whole of the surplus has been allotted by way of reversionary additions.

What will be the probable effect of the above-mentioned course upon the future prosperity of the society?

9. Discuss briefly the effect *inter partes* of apportioning the divisible surplus of an assurance office

1. In proportion to the premiums paid in the period.
2. " value of each policy.
3. " difference between the premiums paid from the commencement of the assurance (with compound interest) and the values of the policies.
4. " "loading", assuming the latter to be constant at all ages.

10. Can a life assurance company be now established in any other way than under the Companies Act, 1862 (25 Vict. c. 89)? State the provisions in this respect in the 4th section of that Act.

11. Explain the terms "Covenant", "Freehold", "Tenant in tail with possibility of issue extinct". What is the distinction in effect between a Warranty and a Representation? What is the *rationale* of the distinction?

12. The Supreme Court of Judicature Act, 1873, provides, section 5, that "any absolute assignment by writing (not purporting to be by way of charge only) of any debt or other legal chose in action, of which express notice in writing shall have been given &c., shall be effectual at law (subject to equities entitled to priority &c.) to pass and transfer the legal right to such debt or chose in action, and all legal and other remedies." What effect will this provision have upon voluntary assignments of policies of life assurance, or what questions in equity arising upon them appears, for the future, to be set at rest by this enactment?

13. What are the main points you would bring out in a confidential report to a Board of Directors, showing the extension of a life assurance company's business during the past year, and the most important financial results of the year bearing upon the accruing surplus.

II.

14. Amalgamate the two accounts in the published weekly statement of the Bank of England, so as to show the assets and liabilities of the Corporation in one classified enumeration. Do the accounts afford any indication of the amount of the obligations of the Government which would be available for an emergency?

15. In what way does the price at which the Government of India is enabled to sell its bills, indicate the course of the transaction between this country and the East Indies?

16. State what you consider would be the effect of a currency £1 notes. What is your opinion of having such a currency?

17. Explain how you would treat the investments of a life assurance company in reversionary securities, both in the annual accounts and in the valuations.

18. How would you deal with a large investment in Foreign Government securities, both in the annual accounts and in the valuation?

(1.) When the market had risen just before the period of account.

(2.) When the market had fallen just before the period of account.

19. Draft the following minutes:

(1.) Authorizing three of the existing Directors of an assurance company to give a discharge for a reassurance policy effected in the names of three Directors since deceased, and indemnifying the paying company.

(2.) Appointing an Inspector of Agents, and setting forth the terms of his engagement.

(3.) Receiving and adopting the Actuary's report upon the valuation, and appropriating the surplus in terms of the Deed of Settlement.

(4.) Authorizing and directing the resistance of a claim.

20. State in what respects you think the extra premiums and conditions of foreign residence could be lessened without impairing the general security of a life assurance company.

21. How would you value an absolute reversion?

(1) For legacy duty.

(2) For sale in the market; and state how you would value the invested fund, and what circumstances sometimes affect market values.

(3) As a security for a loan; and state what margin should be reserved, and why.

22. Real estates are settled to the use of A for life (in precarious health), remainder to B (his sister's son) for life, with remainder to B's male issue in tail, with remainders over, but with a power of appointment over the fee exercisable by A and B jointly. The settlement also contains a "name and arms" clause. B has heavily charged his reversionary life interest by way of annuity and his creditors threaten to sell his entire remaining interest; he has male issue, and his life is now insurable only at an advanced premium. What arrangements could be suggested to extricate B from his difficulties, without diminishing A's income—

(1) Supposing the charge on the reversionary life estate could be redeemed on fair terms.

(2) Supposing the charge cannot be redeemed, except on very onerous terms.

23. A (60) and B (60) are in partnership in an old-established business, yielding £5,000 a-year, taking the income in equal shares during their joint lives, and the survivor becoming entitled to the whole business and its profits. A is childless, but B has a son, C (30). It is proposed to introduce C as a partner, with the special provision that the whole income is still to be divided, as heretofore, between A and B during their joint lives; but, that on the death of

either (but not otherwise), C is to succeed to the share and interest of the deceased partner. What sum should be paid to A, in order that he may consent to the proposed arrangement?

24. Real property, yielding £1000 a-year net, has been taken by a Railway Company, under compulsory powers, at the price of £40,000. The property, subject to a charge of £5,000 on the fee at 4 per cent interest, and to a life annuity of £100 a-year to C, a female (70), is settled to the use of A, a male (65), for life, with remainder absolutely to B, a male (35). Apportion the proceeds of the purchase money between the several parties interested therein.

25. A fund of £100,000 Consols is settled to the use of a married woman, A (30), for life, with remainder to her husband, B (40), for life, with remainder to the issue of the marriage (of which there are four children) in equal shares; with remainder absolutely to B. B is bankrupt; and the liquidators desire to know (1) What B's reversionary interest is worth; and the family desire to know (2) At what sum they may be advised to buy back B's interest.

Estimate the required values, and state the grounds of your opinion in each case.

CORRESPONDENCE.

ON THE LOADING OF ASSURANCE PREMIUMS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Considering that every actuary has occasion to deal frequently with this question, and looking to its important bearing on the interests of his company, I am rather surprised that it has not obtained a more prominent place in our discussions.

In the great majority of published tables, the computer seems to have been content to increase the net or pure premiums by an arbitrarily fixed ratio, while, supported by the high authority of Mr. Jellicoe (see *Assurance Magazine*, volume x, page 336), a modified method appears to be now frequently acted on, which consists in making two additions, one of them a percentage of the net premium in respect of the outlay for agents' commission and of profit to be made by the transaction, and the other a percentage on the sum assured, which is the same at all ages, to cover the "going" office expenses incurred in the collection and investment of the premiums, and in its continued general management. This is obviously but an imperfect substitute for the older method, seeing it but changes the *measure* of the charge, from the premium to the sum assured; and (assuming that the fixed charge should be the same on each policy) it could only be corrected by tabulating a graduated scale of premiums for the assurance for £100, £200, £300, &c., at each age. Apart from the last objection, however, I have been led on examination to doubt whether, on any reasonable hypothesis of the ratio of expense usually incurred, the theory of a fixed addition is tenable.

In late discussions on the rate of expenditure of well-conducted offices, it has been found that an outlay of 50 per-cent of the first year's

premium revenue, and 7 per-cent of the renewals, is a fair indication of the disbursement for management. To find therefore a premium which, when lessened by this charge, and computed to meet a benefit increased by a proportion for office profit, let P be such a premium. Its value, when the foregoing deductions are made, will be

$$P(1+a)-P(\cdot 5+\cdot 07a)=P(\cdot 93a+\cdot 5);$$

and if q per unit be the loading for profit, we shall have

$$P(\cdot 93a+\cdot 5)=A\ (1+q),$$

whence

$$P=\frac{A(1+q)}{\cdot 93a+\cdot 5}.$$

Taking $q=\cdot 1$, by the four per-cent H^M table we obtain the following results:—the premiums being those chargeable at the ages indicated, and the third column showing the proportion of "loading" requisite to be added to the net premium to produce the same result:—

Age.	Premium per-cent.	Loading.
20	1·5075	21·1 per-cent.
30	2·0261	21·4 "
40	2·8636	21·8 "
50	4·3371	22·5 "
60	7·0723	23·8 "

It will be seen that the ratio of loading increases with the age, while a premium computed on the method of making a fixed extra at all ages would exhibit from its nature a diminishing ratio.

The latter method may be expressed by the equation

$$\varpi x+y=P,$$

where, P being ascertained, it is required to determine the numbers x and y which would (founded on the net premium) produce it. This is amenable to the treatment so ingeniously made available by Mr. Sprague in his late paper* on office expenses; and, following his application of the doctrine of least squares for arriving at a correct average of the five rates above quoted, we find

$$x=1\cdot 306 \text{ and } y=-\cdot 2318,$$

leaving the fixed amount to be *subtracted* from the product of x and the net premium. The application of these numbers to the net premium gives the following results:—

Age.	Premium per-cent.	Loading per-cent.
20	1·3935	12·0
30	1·9483	16·7
40	2·8395	20·7
50	4·3934	24·1
60	7·2322	26·6

The weight of evidence consequently goes to prove that this method of loading can hardly be an improvement on the older one,

* The reference here is to a letter in the *Insurance Record* for 10 March 1876, the substance of which we hope to place before our readers at some future date. —ED. *J. I. A.*

and I hope the present contribution to the general question will elicit something that may be found more useful.

An opinion, for which I think we were first indebted to a German writer*, was ventilated lately at a discussion in the Institute, to the effect that at any valuation of a current policy made professedly on a "net premium" system, it would be right to deal with the value of the premium as an increasing one, although the actual rate paid by the assured does not alter. Thus in the case herein explained, it may be assumed that the *net* premium paid for the first year's assurance should do no more than cover the risk for the term, the excess of the actual payment ranking as "loading." To compensate for this, the after premiums should be estimated as greater than the ordinary net premiums for the whole of life, such increased premiums being taken credit for at a valuation.

The argument is not without interest, and it may be well to exhibit the following figures in illustration of it.

The total premiums assumed are those resulting from the formula

$$P_x = \frac{100A_x}{\cdot 93a_x + \cdot 5}$$

by the H^M table at three per-cent without farther addition.

Age. (1)	First Year's Net Premium. (2)	Future Annual Net Premium. (3)	Whole of Life Net Premium. (4)	Percentage of Increase credited. (5)
20	·7835	1·4572	1·4272	2·10
30	1·0334	1·9221	1·8795	2·27
40	1·4283	2·6567	2·5891	2·61
50	2·1087	3·9222	3·8005	3·20
60	3·3572	6·2444	5·9874	4·29

Opposite the age at entry are given—

In Column (2). The net premium received for the first year's assurance (by hypothesis one half of the entire payment made by the assured).

" " (3). The net premium paid thereafter, being the whole payment less 7 per-cent spent in management.

" " (4). The ordinary or net tabulated premium for the whole of life assurance.

Column (3) exhibits the net premium for which credit would be taken under such a system of valuation, in contradistinction to the amount in Column (4). The last column exhibits the ratio of *increase* thus valued, which it will be seen is considerable.

Had the premium payable by the assured been of the usual form of an ordinary net premium for the whole of life, its alteration by the above system would have exhibited a greater discrepancy when the equivalent for the half of the first year's payment was added to it.

Something may be said for the adoption of the practice so long

* See *Journal* xv, 420.

as the smaller premium reserved for the first year's assurance is in any measure in excess of the value of the risk run by the company during that year, but it is, to say the least, disingenuous to adopt the popular definition and to describe such a valuation as a "net premium" one.

I am, Sir,
Your most obedient servant,
London. H. AMBROSE SMITH.

*** As the latter part of the above letter deals to some extent with the same subject as Mr. McCandlish's paper, published in the last number of the *Journal*, Mr. Smith wishes us to mention that it was in type before he had heard of Mr. McCandlish's essay.—ED. *J. I. A.*

MR. DEUCHAR'S PAPER ON NEGATIVE POLICY-VALUES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The question discussed by Mr. W. T. Gray, in his interesting letter in the Oct. Number of the *Journal* (p. 73), is one of importance. He has shown by figures that, at any rate in one particular case, it is a matter of indifference to a company which so treats its policies as to convert many of them into assets instead of liabilities, whether it value at long or short intervals of time. But an arithmetical demonstration, however useful in illustration of one more general, is not altogether satisfactory by itself; and on this account, as well as because Mr. Gray appears to have fallen in one respect into a little confusion, I venture to trouble you with a brief mathematical proof.

Let us suppose two companies, A and B, identical, to start with, in every respect, doing an identical amount and description of business, and making the same rate of interest, say i, on their funds; and let them value on the same basis, and so that all policies have negative values for t years. Let, however, A divide its profits annually, and B septennially. At the beginning of any given septennium, the two companies will be identical in position. At the end of the first year of that septennium A will distribute a certain surplus, say m_1; at the end of the second year another, m_2; and so on; and at the end of the seventh year, m_7. At the end of the first year let B also make a valuation for the private information of its manager, but let it retain in hand the discovered surplus, which will also be m_1, and carry it to a separate account in its ledger. Except for a book entry, it is precisely as it would have been had it not made an investigation, and its life funds proper are of the same amount as those of A at the same instant of time, only they are supplemented by the sum m_1 in the suspense account. At the end of the second year let the same process be repeated. In addition to the first surplus, now grown to $m_1(1+i)$, it brings out another, which must be m_2, and treats it in a similar manner, and so on to the end of the septennium, when the total surplus—now to be distributed—in its funds will be—

$$m_1(1+i)^6+m_2(1+i)^5+\ \ldots\ldots\ +m_6(1+i)+m_7,$$

or exactly equal to the profits declared by A, accumulated at interest.

All through the septennium, B, so far as the outside world is concerned, has been an office valuing every seven years, although it has made an annual calculation of which none but its own staff were cognisant, and opened a suspense account in its ledger. But it has not any superiority of position, either as regards reserves or profits, over A, which would not have arisen under the most rigid system of valuation, bringing out the largest possible reserve for every policy, young or old. In fact, after the supposition at the outset of negative values, it has been unnecessary to make mention of them throughout the argument, and the supposition might have been entirely omitted. It is true that, during the currency of the septennium, the financial position of A remained stationary, while that of B was gradually strengthening through the accumulation of undivided profits; but this was not by virtue of negative policy values. A similar result would have appeared with any system of valuation, and would have been the more marked the severer the test applied.

It will be observed that we have not made any supposition as to the nature of the business transacted by the companies. We have only stipulated that an amount equal to the profits divided at the end of each year by A, be immediately and specially invested by B, and allowed to accumulate at compound interest to the end of the septennium: and the result is, that the amount which B then has available for distribution is *exactly* equal to the profits declared by A, accumulated at interest. With this result Mr. Gray's figures do not, at first sight, quite coincide, for he finds that the profits of A, accumulated at interest, fall short of the available surplus of B: and his conclusion, which the above reasoning shows to be erroneous, he attempts to justify by algebra, in a way which is easily proved to be fallacious. He says, speaking of the second septennium of the two companies:—

"The amount that A divides at the end of the 8th year is retained " by B, and forms part of those funds whose average amount at the " beginning and end of the year is invested at $4\frac{1}{2}$ per-cent, so that if " m be the amount distributed by A, and n its amount at the end of " the year, i being the rate of interest, we have—

$$\text{“}\frac{m+n}{2}\times i=n-m,$$

$$\text{“}n\left(1-\frac{i}{2}\right)=m\left(1+\frac{i}{2}\right),$$

" whence
$$n=m\frac{1+\frac{i}{2}}{1-\frac{i}{2}}.$$

" If $i=\cdot045$,

" we have $n=m\times 1\cdot046036$; so that the amounts divided by A must " be invested at $4\cdot6036$ per-cent to amount to the sum now divided " by B, since that is the rate B has made on these sums it has hitherto " retained."

But surely it is self-evident that the equation from which Mr. Gray starts to find n, is not a legitimate one; and it naturally leads to a

reductio ad absurdum, for n, the amount of m at the end of the year, comes out actually greater than $m(1+i)$, a result which no manipulation of investments could possibly produce. It would seem that he has confounded the fixed quantity, m, with the fluctuating funds of the company, and made an unnecessary attempt to find its amount at the end of the year; an attempt which has led him to change the rate of interest, 4·5 per-cent, with which he started, to 4·6036 per-cent. This changed rate of interest he has then employed for the accumulation of the funds of the two companies. It therefore, instead of the original 4·5 per-cent, becomes the rate symbolically written i, and on its basis he has correctly worked out the successive positions of the two companies, and proved that if they both realize that rate on their funds, the profits divided annually by A, if accumulated at that rate to the end of the septennium, will exactly equal the sum then available for division by B. The same conclusion would have been arrived at had he adhered in his calculations to the rate, 4·5 per-cent, which he at first selected; but it only introduces confusion to go back for comparison to the rate 4·5 per-cent, when the figures have been worked out at 4·6036 per-cent.

I am, Sir,

Your obedient servant,

GEORGE KING.

1 *Bartholomew Lane,*
London, E.C.

*** Mr. Gray, to whom we forwarded a copy of the above letter, requests us to insert the following reply.—ED. *J.I.A.*

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The only point of difference between Mr. King and myself on this subject appears to turn on the question of "two offices making the same rate of interest on their funds."

Mr. King, in his mathematical demonstration, very conveniently assumes the rate i, which he applies to A's surpluses alone, and leaves out of consideration the actual working of the funds of the two companies. The only practical answer to the question that I have hitherto come across is in Messrs. Malcolm & Hamilton's Report to the Board of Trade, where the rate realized by the various offices is obtained by taking the mean amount of the assurance fund at the beginning and end of the year, and dividing it into the total amount of interest received. This method of obtaining the rate is well adapt d to the working of a model office; for it at once solves the difficulty attending the daily fluctuation of the assurance fund, from premiums and interest received, and claims, surrenders, and expenses, paid away. It was for these two reasons that I adopted the rate as given in the letter to which Mr. King refers; but it is evident that, as the funds at the end of the year are made up, *inter alia*, of interest received during the year, the rate quoted ($4\frac{1}{2}$ per-cent) can only be a nominal rate, and must not be confounded with a rate of $4\frac{1}{2}$ per-cent on the amount at the beginning of the year. Each time I referred to

it, I stated the conditions attending its use; but these Mr. King has disregarded throughout, and, as a necessary consequence, has been led to the conclusion that the equation he quotes is not a legitimate one, and further that I have used two different rates of interest.

In speaking of the accumulation of the annual surpluses at $4\frac{1}{2}$ per-cent, I had in mind the ordinary formula for that purpose (in fact the one used by Mr. King), $m(1+i)^t$; but it is clear that this formula is inapplicable to a nominal rate of interest, and that it must be replaced by its equivalent effective rate, payable annually at the end of the year, if that formula be employed. The accumulation however might equally well be made, without the intervention of the effective rate, by means of the formula $m\left(\dfrac{1+\frac{i}{2}}{1-\frac{i}{2}}\right)^t$, i being the nominal rate previously assumed.

It would seem that Mr. King is of opinion that a book entry with reference to the annual surpluses, would have the effect of separating them from the fluctuating funds of office B; but this I cannot admit, unless special securities be set aside for the purpose, in which case the accumulation would depend on the rate realized by these particular securities, and not on the "disputed" common rate realized by both offices on all their funds.

Thanking you for your courtesy in allowing me this early opportunity of replying to Mr. King's remarks,

I am, Sir,

Your very obedient Servant,

W. T. GRAY.

5 Whitehall, S.W.

Insurances against Issue.

THE following particulars of the Issue Insurances effected with British Life Offices, are extracted from the returns made to the Board of Trade. They will, no doubt, be interesting to many of our readers, as giving a fair indication of the total amount of the transactions of the kind entered into in this country and still subsisting, and of the premiums paid thereon and the estimated outstanding risk. We believe that, in consequence of the advanced age of the lives upon which such insurances are usually effected, the risks run off much more rapidly than in ordinary insurances, and the aggregate amount of the profit made by the companies must be considerable. We have only heard of one single insurance against issue becoming a claim.

contract is at an end.* This being premised, the formula first given above being perfectly general, may be applied to any or all of the assurances on the books of an office, and it may be laid down that

$$(\Sigma V+\Sigma P)(1+i)=\Sigma\,{}_{+1}V+q(\Sigma S-\Sigma\,{}_{+1}V),$$

where S represents the sums assured, P the risk premiums payable, and ${}_{+1}V$ the value at one year of policy age older than is shown by V, q being, of course, the probability of death in the year, and Σ, as usual, the symbol of summation. It may be seen that the increased value is provided for the whole bulk of the assurances, and, in addition, the balance between sum assured and value for the claims; the fund $q(\Sigma S-\Sigma\,{}_{+1}V)$ being the true amount which, according to the mortality table employed, should be used to supply the difference between sum assured and value in the case of the policies which fall in. If it were possible that the influence of death could be entirely removed, the profit† from this cause would be not, of course, $q\Sigma S$, as the usual estimate of mortality would imply, but $q(\Sigma S-\Sigma\,{}_{+1}V)$, for the increased value would have to be reserved for every one of the policies. If, on the other hand, every life were to drop, the loss would be $p(\Sigma S-\Sigma\,{}_{+1}V)$, for ${}_{+1}V$ is in hand for the whole amount assured, and $(S-{}_{+1}V)$ for

* At Mr. Hardy's suggestion, a table is appended in further illustration, S being the sum assured at the age, and $L=p_{x+n}S$, $D=q_{x+n}S$, so that $S=L+D$.

	Sum Assured.	Accumulation at interest of the Reserve at commencement of year, and the Risk Premium.	Application of Accumulation. Reserve required for Assurances remaining in force.	Reserve required in part payment of Claims.	Contribution to the year's claims, and applied in part payment thereof.
Assurances to be continued.	L	$L({}_nV_x+\varpi_x)(1+i)=$	$L\,{}_{n+1}V_x$	...	$+q_{x+n}L(1-{}_{n+1}V_x)$
Assurances become claims.	D	$D({}_nV_x+\varpi_x)(1+i)=$	...	$D\,{}_{n+1}V_x$	$+q_{x+n}D(1-{}_{n+1}V_x)$
Total . .	S	$S({}_nV_x+\varpi_x)(1+i)=$	$L\,{}_{n+1}V_x$	$+D\,{}_{n+1}V_x$	$+q_{x+n}S(1-{}_{n+1}V_x)$
		$=$	$L\,{}_{n+1}V_x$	$+D\,{}_{n+1}V_x$	$+D(1-{}_{n+1}V_x)$
		$=$	$L\,{}_{n+1}V_x$	$+D.$	

† By the use of the words "profit" and "loss", as being more convenient than valuation surplus or deficiency, it is not intended to be implied that the one is necessarily to be divided or the other to cause alarm, seeing that attention must always be given to the data of the calculations and the special circumstances in each case, such as the influence, for instance, to be attributed to selection on one side or the other.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On the True Measure of the Death Strain on the Funds of a Life Assurance Society. By CHARLES D. HIGHAM, F.I.A., F.S.S., *Assistant Actuary to the Metropolitan Life Assurance Society.*

[Read before the Institute, 29 Jan. 1877.]

THE practice of making a comparison between the mortality actually experienced by a life assurance society during a given period of time, and that which might have been expected (to use the term in its professional sense) according to the mortality table on which the periodical valuations are based, has now become so general, that no apology is needed for drawing further attention to the subject, if it can be shown that the method usually employed at present is not altogether satisfactory. We are accustomed to see in the reports of companies particulars set forth as to the number of policies which have become claims, the amounts assured and bonus additions, and occasionally the premiums payable thereon, and for purposes of comparison a statement is made giving similar particulars as to the claims which the mortality table used would warrant. But when it is remembered that for every policy on the books of an office some reserve value of greater or less amount is in hand, it is apparent that no system of comparison of actual and expected mortality can be correct which does not take into account the extent to which the claims are and should be

met by the reserve values held against them. The actual cost to the office of the death of an assured life is not the sum assured, but the difference between that amount and the reserve value retained to meet such a payment, and it is this balance which should be the measure in any attempt to ascertain the strain of the mortality on the funds. As from the gross sum assured on the office books a certain proportion is detached, as it were, as the amount which may be expected to be paid away in claims; so from the gross liability shown by a valuation should that part be set aside, which is considered to be the reserve value of such an amount. The difference between these two segregated quantities is the true standard by which the strain of the deaths on the funds is to be measured, and according as the balance between sum assured and reserve value of the claims actually paid falls short of or exceeds this sum, is gain or loss to be attributed to the mortality experience.

In the theoretical consideration of a policy value, it is perfectly well understood that if to the value of a policy at the end of any year the annual premium be added, and the sum accumulated for a year at the proper rate of interest, there will result a fund sufficient in amount both to pay in full that portion of the sum assured which is shown by the mortality table used to be the normal death claim, and also to provide the increased reserve value for the remainder. Seeing however the stress which is laid in this paper on the fact that every risk has its reserve, it is better slightly to vary the terms of the above proposition, and to say that the accumulation of value and premium will furnish the increased reserve value for the whole sum assured, and in addition the balance between such increased reserved value and the full sum assured for that portion thereof which may be expected to drop. The modification of the usual proof is as follows:—

$$
\begin{aligned}
({}_nV_x+\varpi_x)(1+i) &= \left\{1-\frac{1+a_{x+n}}{1+a_x}+\frac{1}{1+a_x}-d\right\}(1+i) \\
&= \left\{1-d-\frac{a_{x+n}}{1+a_x}\right\}(1+i) \\
&= \left\{v-\frac{vp_{x+n}(1+a_{x+n+1})}{1+a_x}\right\}(1+i) \\
&= 1-\frac{(1-q_{x+n})(1+a_{x+n+1})}{1+a_x} \\
&= 1-\frac{1+a_{x+n+1}}{1+a_x}+q_{x+n}\left[1-\left(1-\frac{1+a_{x+n+1}}{1+a_x}\right)\right] \\
&= {}_{n+1}V_x+q_{x+n}\left[1-{}_{n+1}V_x\right],
\end{aligned}
$$

which is, of course, the equivalent of $p_{x+n}\,{}_{n+1}V_x + q_{x+n}$ as generally given.

The value of a policy used in this formula $[{}_nV_x = A_{x+n} - \varpi_x(1 + a_{x+n})]$ assumes the payment of a premium to be immediately due, whereas the formula more often found in official practice considers that an interval of half a year will be required to elapse first. $[A_{x+n} - \varpi_x(\frac{1}{2} + a_{x+n}) = \text{(say)}\ {}_nV'_x]$. No substantial variation however is rendered necessary in the proposition above stated, but it may be shown that there will be a deficiency in the funds at the end of the year to the extent of six months' interest on the premium on half the amount of claim, if it be taken for granted that the premium on a moiety of the claim will be paid, and that in the case of the other moiety death will occur before payment becomes due.

$$\begin{aligned}{}_nV'_x(1+i) + \left(p_{x+n} + \frac{q_{x+n}}{2}\right)\varpi_x\left(1+\frac{i}{2}\right) &= \left({}_nV_x + \frac{\varpi_x}{2}\right)(1+i) + \left(1 - \frac{q_{x+n}}{2}\right)\varpi_x\left(1+\frac{i}{2}\right)\\ &= ({}_nV_x + \varpi_x)(1+i) + \frac{\varpi_x}{2} - \frac{q_{x+n}}{2}\varpi_x\left(1+\frac{i}{2}\right)\\ &= {}_{n+1}V_x + q_{x+n}[1 - {}_{n+1}V_x] + \frac{\varpi_x}{2} - \frac{q_{x+n}}{2}\varpi_x\left(1+\frac{i}{2}\right)\\ &= {}_{n+1}V'_x + q_{x+n}\left[1 - \left({}_{n+1}V'_x - \frac{\varpi_x}{2}\right)\right] - \frac{q_{x+n}}{2}\varpi_x\left(1+\frac{i}{2}\right)\\ &= {}_{n+1}V'_x + q_{x+n}[1 - {}_{n+1}V'_x] - \frac{q_{x+n}}{2}\varpi_x\frac{i}{2},\end{aligned}$$

which may be expressed also in the form of

$$p_{x+n}\,{}_{n+1}V'_x + q_{x+n} - \frac{q_{x+n}}{2}\varpi_x\frac{i}{2}.$$

This deficiency being of relatively very small amount need not enter into the considerations of this paper, but it will be, perhaps, more convenient to make use of the annuity-due formula, when a reference to the value of a policy may be hereafter necessary.

Putting aside, as at present irrelevant, any question of surrender, lapse, or forfeiture, one of two contingencies must befall every policy at the end of any theoretical year—it must either be continued or be a claim. In the one case ${}_{n+1}V_x$ is held back as the reserve for the future, and the balance $q_{x+n}(1 - {}_{n+1}V_x)$ thrown into hotchpot with similar balances from other similar assurances, to form the fund from which the difference between sum assured and value will be supplied for the q_{x+n} to be claimed. In the other case, the same $q_{x+n}(1 - {}_{n+1}V_x)$ goes into hotchpot, while $(1 - {}_{n+1}V_x)$ is drawn out and paid with ${}_{n+1}V_x$ to the assured, and the particular

that although the sum assured fallen in exactly corresponds therewith, yet there is a variation to the extent of

$$-\{a({}_nV_x-{}_1V_{x+n-1})+b({}_{n-1}V_{x+1}-{}_2V_{x+n-2})+\&c.\}$$

in the strain, and this amount is therefore the gain or loss from the mortality experience of the year, according as it is negative or positive as a whole, for it must be remembered that its terms may have varying signs. Seeing, then, that a surplus or deficiency represented by $-\{a({}_nV_x-{}_1V_{x+n-1})+b({}_{n-1}V_{x+1}-{}_2V_{x+n-2})+\&c.\}$ has arisen through an abnormal distribution of claims agreeing in amount of sum assured with the estimate, it follows that the amount of sum assured paid away may be increased without loss or diminished without gain, the variation being limited to that amount, of which the balance between sum assured and value is equal to the surplus or deficiency.

Generally, therefore, if there be gross assurances amounting to ΣS, of which the value is ΣV, the measure of the death strain, as defined above, being $q(\Sigma S-\Sigma V)$, and if the actually claimed sum assured be $(q+y)\Sigma S$, with a reserve value of $(q+z)\Sigma V$, where y and z may be positive or negative, but neither less than $-q$, then the actual strain on the assurance fund is $(q+y)\Sigma S-(q+z)\Sigma V$ $=q(\Sigma S-\Sigma V)+y\Sigma S-z\Sigma V$, and therefore the quantity $y\Sigma S-z\Sigma V$, according as it is negative or positive as a whole, is gain from favourable or loss from unfavourable mortality. If the claims are less but the reserve greater than the expected, that is, if y be negative and z positive, the quantity $y\Sigma S-z\Sigma V$ will be negative, implying profit; if, on the other hand, the contrary be the case, and y be positive and z negative, the quantity will be positive, showing loss: or the claims may be excessive and their value also, when y and z will be positive, or both may be under the estimate, and y and z negative, in either of which cases the ultimate sign of the quantity depends upon the comparative magnitude of its component parts. In other words, the actual strain is less than, equal to, or greater than the expected, that is, there is profit, the tabular strain, or loss, according as

$$q(\Sigma S-\Sigma V)+y\Sigma S-z\Sigma V <=> q(\Sigma S-\Sigma V),$$

or

$$y\Sigma S <=> z\Sigma V,$$

that is, according as the variation in the sum assured fallen in is less than, equal to, or greater than the variation in reserve value, regard being had to the signs of the different sides of the inequality or equation.

The following numerical example may not be out of place, though it will be seen that the figures have been chosen rather for convenience than for resemblance to official experience.

q of it, so that the sum to be made up would be that balance for the remainder, or $p(\Sigma S - \Sigma\,_{+1}V)$. Between these two limits any result is possible. The strain on the assets may be just the expected, and yet the fund $q(\Sigma S - \Sigma\,_{+1}V)$ may be either split up into many small sums, and applied in supplying the balance to many policies with a large reserve value, in which case the sum assured paid in claims would exceed the amount estimated; or it may be divided into parts, smaller in number but greater in amount, and used for the balance of assurances with a small value, in which case the claimed sum assured would fall short of the expected sum. Or the fund may not be all used, and there will be gain, or more may be required and there is loss, and in either case the sum assured paid in claims may exactly coincide with that set forth in the estimate of mortality. But whatever the actual results may be, no real comparison can be made with the expected unless the amount of this fund is known, since it only is the true measure of the death strain.

In illustration, assume a case of n assurances, each for 1, taken out $n, n-1, \ldots.$ 1 years ago respectively, at the respective ages of x, $x+1 \ldots. x+n-1$, so that the present age of all the lives is now $x+n$, and the sum of the values of the policies $_nV_x + {}_{n-1}V_{x+1} + \&c. + {}_2V_{x+n-2} + {}_1V_{x+n-1}$, the total sum assured being n. Then the amount which, according to the table, is due for payment for the mortality of the past year is $q_{x+n-1}(1+1+\&c.+1+1)$ $= q_{x+n-1}n$, which amount should be supplied to the extent of

$$q_{x+n-1}({}_nV_x + {}_{n-1}V_{x+1} + \&c. + {}_2V_{x+n-2} + {}_1V_{x+n-1})$$

by the reserve value, the strain on the assets being

$$q_{x+n-1}\{n - ({}_nV_x + {}_{n-1}V_{x+1} + \&c. + {}_2V_{x+n-2} + {}_1V_{x+n-1})\}.$$

If, however, the actual claims, although amounting to qn in the aggregate (omitting the subscript to q), are found to be not evenly distributed among the different assurances, but otherwise as $(q+a)+(q+b)+\&c.+(q-b)+(q-a)$, where a, b, c, &c., may be any fractions not exceeding q, and either positive or negative, then the contribution from reserve is

$$\begin{aligned}&(q+a)_nV_x + (q+b)_{n-1}V_{x+1} + \&c. + (q-b)_2V_{x+n-2} + (q-a)_1V_{x+n-1}\\ &= q({}_nV_x + {}_{n-1}V_{x+1} + \&c. + {}_2V_{x+n-2} + {}_1V_{x+n-1}) + a(V_x - {}_1V_{x+n-1})\\ &\qquad + b({}_{n-1}V_{x+1} - {}_2V_{x+n-2}) + \&c.\end{aligned}$$

and the strain on the assets is, consequently,

$$\begin{aligned}&q\{n - ({}_nV_x + {}_{n-1}V_{x+1} + \&c. + {}_2V_{x+n-2} + {}_1V_{x+n-1})\}\\ &\qquad - \{a({}_nV_x - {}_1V_{x+n-1}) + b({}_{n-1}V_{x+1} - {}_2V_{x+n-2}) + \&c.\}\end{aligned}$$

On comparison of the actual results with the estimate, it appears

that although the sum assured fallen in exactly corresponds therewith, yet there is a variation to the extent of

$$-\ \{a({}_nV_x-{}_1V_{x+n-1})+b({}_{n-1}V_{x+1}-{}_2V_{x+n-2})+\ \&c.\}$$

in the strain, and this amount is therefore the gain or loss from the mortality experience of the year, according as it is negative or positive as a whole, for it must be remembered that its terms may have varying signs. Seeing, then, that a surplus or deficiency represented by $-\{a({}_nV_x-{}_1V_{x+n-1})+b({}_{n-1}V_{x+1}-{}_2V_{x+n-2})+\ \&c.\}$ has arisen through an abnormal distribution of claims agreeing in amount of sum assured with the estimate, it follows that the amount of sum assured paid away may be increased without loss or diminished without gain, the variation being limited to that amount, of which the balance between sum assured and value is equal to the surplus or deficiency.

Generally, therefore, if there be gross assurances amounting to ΣS, of which the value is ΣV, the measure of the death strain, as defined above, being $q(\Sigma S-\Sigma V)$, and if the actually claimed sum assured be $(q+y)\Sigma S$, with a reserve value of $(q+z)\Sigma V$, where y and z may be positive or negative, but neither less than $-q$, then the actual strain on the assurance fund is $(q+y)\Sigma S-(q+z)\Sigma V$ $=q(\Sigma S-\Sigma V)+y\Sigma S-z\Sigma V$, and therefore the quantity $y\Sigma S-z\Sigma V$, according as it is negative or positive as a whole, is gain from favourable or loss from unfavourable mortality. If the claims are less but the reserve greater than the expected, that is, if y be negative and z positive, the quantity $y\Sigma S-z\Sigma V$ will be negative, implying profit; if, on the other hand, the contrary be the case, and y be positive and z negative, the quantity will be positive, showing loss: or the claims may be excessive and their value also, when y and z will be positive, or both may be under the estimate, and y and z negative, in either of which cases the ultimate sign of the quantity depends upon the comparative magnitude of its component parts. In other words, the actual strain is less than, equal to, or greater than the expected, that is, there is profit, the tabular strain, or loss, according as

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that is, according as the variation in the sum assured fallen in is less than, equal to, or greater than the variation in reserve value, regard being had to the signs of the different sides of the inequality or equation.

The following numerical example may not be out of place, though it will be seen that the figures have been chosen rather for convenience than for resemblance to official experience.

H^M 4 per-cent. Present Office Age, 70.

Office Age at Entry.	Total Amounts existing at present age, before payment of the claims of the year just passed.					Estimate of Claims for year just passed.				Actual Claims entirely on Oldest Policies.			Actual Claims entirely on Newest Policies.			Actual Claims entirely on Oldest Policies.			Actual Claims entirely on Newest Policies.		
	Sum Assured.	Risk Premium.	Reserve Value at present age.	Difference between Sum Assured and Value, being Amount really at risk.	Percentage of Sum Assured at risk.	Sum Assured.	Risk Premium.	Reserve Value.	Difference between Sum Assured and Value, or Strain.	Sum Assured.	Risk Premium.	Reserve Value.	Sum Assured.	Risk Premium.	Reserve Value.	Sum Assured.	Risk Premium.	Reserve Value.	Sum Assured.	Risk Premium.	Reserve Value.
15	5000	53	3214	1786	35·7	287	3	184	103	5000	53	3214	...	...	...	5000	53	3214	...	...	...
20	10000	125	6287	3713	37·1	573	7	361	212	9908	123	6229	...	...	...	10000	125	6287	...	...	...
25	20000	286	12307	7693	38·5	1147	16	706	441	...	...	...	...	...	...	4155	59	2557	...	...	...
30	30000	501	17933	12067	40·2	1720	29	1028	692												
35	40000	788	23036	16964	42·4	2294	45	1321	973												
40	50000	1176	27400	22600	45·2	2867	67	1571	1296												
45	40000	1146	20422	19578	49·0	2294	66	1170	1124												
50	30000	1063	13836	16164	53·9	1720	61	794	926												
55	20000	892	7888	12112	60·5	1147	51	452	695												
60	10000	571	3027	6973	69·7	573	33	174	399	...	...	...	9908	567	2998	...	...	...	4282	245	1296
65	5000	371	889	4111	82·2	286	21	50	236	...	...	...	5000	371	889	...	...	...	5000	371	889
	260000	6972	136239	123761	47·6	14908	399	7811	7097	14908	176	9443	14908	938	3887	19155	237	12058	9282	616	2185
	43750· = q_{69}		43750· = q_{69}							9443 = Reserve Value			3887 = Reserve Value			12058 = Reserve Value.			2185 = Reserve Value.		

Total Amounts (Sum Assured):
13000
1820
78
10

14908 = {Estimate of Claims as to Sum Assured.
7811 = {Estimate of Claims as to Reserve Value.

7097 = Difference, being Measure of Strain.

Total Amounts (Reserve Value at present age):
6812
953
41
5

7811 = {Estimate of Claims as to Reserve Value.

Actual Claims entirely on Oldest Policies:
5465 = Actual Strain.
7097 = {Measure of Strain.

1632 = Profit.

Actual Strain, 77 per-cent of estimated.

Actual Claims entirely on Newest Policies:
11021 = Actual Strain.
7097 = {Measure of Strain.

3924 = Loss.

Actual Strain, 155 per-cent of estimated.

Actual Claims entirely on Oldest Policies:
7097 = Actual Strain.
= {Measure of Strain.

Sum Assured, 128 per-cent of Estimate.

Actual Claims entirely on Newest Policies:
7097 = Actual Strain.
= {Measure of Strain.

Sum Assured, 62 per-cent of Estimate.

Assume that an office has assurances to the extent of £260,000 existing at age 70, the risk premiums payable being £6972, and consequently the reserve value by the H^M 4 per-cent Table £136,239, leaving £123,761 as the amount really at risk. Then by the table, a sum of £14,908 is payable for the claims of the past year, and if the mortality is evenly distributed over the whole of the assurances, £7811 thereof will be provided from reserve value, leaving the difference of £7097 as the strain on the assets, this difference being the true measure defined above. But if the deaths are found to have actually fallen entirely on the oldest policies, though the claims amount to precisely the tabular £14,908, yet the strain on the assets will only be £5465 (or 77 per-cent of the expected), by reason of the large proportion of value in hand to meet them, and there is consequently a profit of £1632: if, on the other hand, only the lives most recently assured have dropped, claims to the extent of the same sum of £14,908 will cause a strain equivalent to 155 per-cent of the estimated, and will entail a loss of £3924. It is evident, then, that in spite of the payment of exactly the estimated amount of sum assured, there may be a profit of £1632, or loss of £3924, or any intervening result—in other words, in the actual strain there may be a range of 78 per-cent of the estimated. But further, the profit of £1632 may be applied to meet the increased strain caused by an augmentation of the claim sum assured, and if this additional mortality should also fall entirely on the oldest policies, the profit will permit of an extension by £4247 of the claims, and therefore £19,155, equivalent to 128 per-cent of the estimate, may be paid away without any deviation from the tabular strain. Similarly the loss of £3924 may be avoided, if the claims should be reduced to £9282, or 62 per-cent of the expected. Here then it may be seen that the sum assured payable in death claims may range between 128 and 62 per-cent of the estimate, without causing loss or bringing profit to the office. In spite of the extreme character of this example, its evidence is clear: it cannot be expected that each set of assurances will exactly conform to the mortality law, but the balance of the variations will be the profit or loss at the age, and the balance of these profits and losses the gross profit or loss on the general transactions of the year.

Nothing has yet been said about bonus, but there is an inherent difference between the reserve value of the reversionary bonuses at any age and that of the policies proper, in that reversionary bonus being a fixed sum payable without any future benefit to be received,

the same value will have to be provided for every 1 thereof, while the value of the policies varies according to the premiums payable thereon. Taking the assurances at any given age, as far as the contract policy moneys only are concerned, it may be roughly stated, that the more recent is the assurance which has become a claim, the more severe is the strain. Looking only to reversionary bonus, however, it would be a matter of indifference where the mortality fell, if the bonus were evenly distributed among the different policies; but seeing that it is usual for the preponderance to attach to the older ones, it is evident that, in the consideration of the bonus by itself, the more recent the assurances which fall in, the less is the strain on the funds, since a less amount of bonus is payable. For sum assured and reversionary bonus, then, two different forces are working, and it may well happen that their opposition may have the effect of counteracting excessive profit or loss, especially in the case of offices where there is any tendency to an undue increase in the reversionary bonus appropriated to the policies of standing.

As has been proved above in the case of a policy value, it may also be shown of a simple reversion, that the accumulation at interest for a year of its value at any time, provides the increased value at the end of the year of the whole reversion, together with the balance between such increased value and the reversion in respect of that portion which by the mortality table should fall in.

$$\begin{aligned}
\mathrm{A}_{x+n}(1+i) &= 1-ia_{x+n} \\
&= 1-ivp_{x+n}(1+a_{x+n+1}) \\
&= 1-d(1+a_{x+n+1})+q_{x+n}d(1+a_{x+n+1}) \\
&= 1-d(1+a_{x+n+1})+q_{x+n}-q_{x+n}\{1-d(1+a_{x+n+1})\} \\
&= \mathrm{A}_{x+n+1}+q_{x+n}(1-\mathrm{A}_{x+n+1}),
\end{aligned}$$

which may also be carried a step farther into

$$p_{x+n}\mathrm{A}_{x+n+1}+q_{x+n}.$$

As before, it will be seen that the profit resulting from a total absence of claim would be $q_{x+n}(1-\mathrm{A}_{x+n+1})$, and that a loss of $p_{x+n}(1-\mathrm{A}_{x+n+1})$ would be caused by an absolute failure of life. It is apparent that the measure of the death strain here should be the same as for the contract policy moneys treated of above, and a repetition of arguments is unnecessary. Still less will one be called for when it is remembered that the usual practice in an office valuation is, to treat reversionary bonus as if it were additional sum assured without premium, and the only importance attaching

to the question is from the modifying influences of which such additions may be the cause.

Nor is it necessary to give special consideration to bonus in reduction of premium, save only to remember that the substitution of a smaller premium, whether for a term or for life, or the extinction of the annual charge, increases reserve, and consequently diminishes strain. Any preponderance, then, of appropriation to the older policies intensifies rather than counteracts the advantage of the deaths falling among such assurances.

It remains to be seen, what general rules can be laid down in the application of the foregoing theories to practical use. Seeing that both sum assured (including reversionary bonus) and reserve value are subject to certain multiplications, it is needless to point out that advantage may be taken of the system of generalization given by the Class Book in the grouping of amounts to which the same multiplier is appropriate; but it must not be forgotten that the individual assurances thereby amalgamated have all an individual influence on the mortality experience, depending on the difference in the proportions of the sums assured which represent the real cost to the office.

Since allowance must be made for the time during which no risk is run under certain policies which are only in force for a portion of the period, say the year, for which the estimate is wanted, no accurate computation can be made until its expiry. When the end of the year has come, and a valuation of all the policies in force has been made, a detailed statement giving the sum assured and reserve value at each age, the policies which, for various reasons, have become extinct during the year, will have been omitted, while others will have been included which have not been in existence during the whole period, and some adjustment is necessary. The totals therefore of the claims by death, the policies otherwise terminated, and the new assurances must be obtained from the Class Book at each age, and, except in the case of the claims, the usual moiety of the sum assured taken as the amount which has been at risk throughout the whole year. No apportionment must however be made with regard to the claims, for though the estimate is being made at the end of the year, it bears the date, as it were, of the beginning, and knowledge then impossible must not now be taken for granted; and if to deal with the surrenders, &c., and the new business, as suggested above, seems to imply the possession of unwarranted information, it must be remembered that the method is but an attempt to eliminate a

mode of termination, or irregular commencement, which cannot but be considered anomalous. The reserve value of the death claims in their entirety should in the next place be computed; and in making similar calculations for the adjusted amounts of the policies surrendered, lapsed and forfeited, and of the new assurances, it will be sufficiently accurate for the present purpose, if the risk premiums be halved, like the sums assured, and the valuation made on the basis of the reduced amounts.

In compliance, then, with the above recommendations, we may be said to have for each age the sum assured and reserve value, first, of the gross number of the policies in force; secondly, of the death claims; thirdly, of such a proportion of the assurances terminated otherwise than by death as is left by the foregoing adjustment; and lastly, of a similar proportion of the new business: if, therefore, to the first set of figures, the second and third be added, and the last be deducted therefrom (for the first includes the total new assurances instead of one half, and the last is the actual moiety), the resulting quantities will be the proper equivalent of the amount of sum assured which has undergone a whole year's risk, and the reserve value at the end of the time. Each of these quantities being then multiplied by the probability of dying in a year at such an age as was attained at the commencement of the year in question, the sums of the two sets of products will give the amount of sum assured which, according to the mortality table, should be claimed, with the amount which is in hand as reserve value, the difference being the true measure of the strain on the funds. The actual experience will be shown by the summation of the sums assured and reserve values of the death claims, as given above, the balance between assurance and value being the actual strain; and from the comparison of the actual strain with the measure, the profit or loss from mortality may be ascertained—the less being the actual strain, the greater the profit or the less the loss. It will not be overlooked that the effects of the claims on the profit fund might be equally well ascertained by the multiplication by the probability of death of the difference only between sum assured and value, instead of that of each of them separately; but the additional insight gained by the more extended return is well worth the slight additional labour involved. For instance, if both for estimate and actual experience the average proportion of sum assured not supported by reserve value be obtained, the comparison of these proportions will show the *quality* of the claims, in the same way as the comparison of the sums

assured shows the amount or *quantity;* while by contrasting the estimated with the actual *strain,* the combined result of the two considerations is discovered.* The subject of this paper being financial, a comparison as to *number* need not be referred to.

But an element of inaccuracy remains in the above given process, in that claims are not in practice paid all together at the end of the year, as is theoretically assumed. One with another, all the deaths may be considered to take place half way through the year; and if one month is required for proof of the fact, and three more must elapse before the claim is matured, the moneys will eventually become payable two months before the end of it. It will probably, therefore, be considered a sufficient adjustment if two months' interest be added both to the sum assured estimated to fall in, and to the amount which has actually done so.

It may be that there are some who would be willing to approach less closely to an accurate measure of the death strain than as above, for the sake of being able to gauge the mortality being experienced month by month as a year rolls on; and if there be the means of making a fair approximation to the amount of the probable surrenders, &c., and new business, it would seem that an estimate of a coming year's mortality might be obtained at the beginning, by making a rough valuation as at the end of it, or by basing the computations on the valuation in possession, and making such adjustment as is possible for increase in the reserve value of the estimated claims. In such cases, however, the actual claims would have to be valued one by one, for advantage could not be taken of the Class Book grouping.

So far for ordinary whole life assurances, and as the same general principles apply to every other class of policies, it is unnecessary to deal separately with each. It must suffice to say, that the contribution of all to the estimate of mortality, both under sum assured and reserve value, must be obtained by the use of the

* For the sake of an example, taking the table for a single age given on p. 159, the proportion of strain, as estimated is 47·6 per-cent, and for the supposed actual results 36·7, 73·9, 37·0, and 76·5 per-cent respectively. To compare the four suppositions one with another, the respective percentages the actual figures are of the estimated should be found, and it will be seen that the relation is as follows:—

As to amount of sum assured, or *quantity.*	As to proportion of strain, or *quality.*	As to real cost to the profit fund, or *strain.*
100·0	77·0	77·0
100·0	155·3	155·3
128·4	77·8	100·0
62·2	160·6	100·0

the last column being obtained either directly from the strains, or from a multiplication of the other two.

appropriate probability of the claim becoming payable during the year; but it will be remembered that there is great variation in the balances really at risk under different sorts of policies—the strain, for instance, caused by the failure of an endowment assurance being very different from that due to a claim under a short period policy. The greater the reserve, the less will the mortality be felt; but, at the same time, a diminution of the possible loss is also a lessening of the possible profit.

Hitherto an annual valuation has been taken for granted; for although the majority of offices declare bonus only at quinquennial or other intervals, it is not unusual for the Valuation Class Book to be balanced at the end of every year, and a test investigation to be thereupon made. In such cases, the foregoing remarks do not require revision, but the profit or loss of any year must be accumulated at interest until the end of the period, the total or net result being the aggregate or balance of the different accumulations. But it is not easy to see how any satisfactory estimate can be obtained for those companies which have valuations of liability only at intervals of more than one year. As the age increases during the period, the probability of death also increases, while the difference between sum assured and value diminishes, so that there is a certain amount of counteraction in the influences at work; but even if it were practicable to obtain a formula for an estimate applicable to values at the middle of the period, an actuarial investigation would have to be made as at that date, which is out of the question. Moreover, the time within the period at which the claims fall in must not be left out of account, so that altogether it would seem that no satisfactory test can be applied to the mortality, unless annual valuations are the rule.

With regard to the labour involved in the use of the suggested measure, seeing that for each age the amount of assurances and consequent liability are obtained without reference to it, the principal additional trouble is the valuation of the policies written off and on, as referred to above, for the adjusting and multiplying by the probability of death is no lengthy matter. The four-figure logarithm card is an invaluable aid, and it will probably be found in most cases that the calculations will be sufficiently accurate if made in quinquennial groups of ages, with such a value for q as best meets the case; but after all, no question as to additional work should have weight, if it be acknowledged that the proposed is the true method of measuring mortality.

In conclusion, although it has been assumed throughout that

some value is retained for every assurance, it may be well to call attention to the fact that if reduction of reserve increases the strain of the claims, the pressure must be greater when there is no reserve at all, and greater still, if not ruinous, when it is not limited to the full amount of the sum assured, but a further addition has to be made for anomalies in the valuation.

Since writing these pages, I have learned that a method of measuring mortality, almost identical with that advocated in them, has been already adopted by Mr. Hardy, who had himself the intention of submitting a paper on the subject. It is not the first time that the same line of thought has been independently followed by two different persons, but it is to be hoped that such a coincidence will not deprive the Institute of the abler dissertation.

DISCUSSION.

The PRESIDENT (Mr. J. Hill Williams) said—This paper treats of a subject which is of practical interest to us all. There may not be much novelty in it, but it is always interesting to see rules that have been more or less applied by somewhat of a rule-of-thumb method, correctly analyzed, as they have been here, and reduced to a very ingenious form, showing how they can be applied scientifically. The paper shows also the great value of the labours of this Institute, especially in the instruction of its younger members. I do not think we could have a better instance of that, than the production of a paper like this.

Mr. C. WALFORD thought the paper an acknowledgment of the principle of treating the life fund as if it were made up, as indeed it is, of the reserves of the policies which are in existence in the company. That theory has been developed to a very large extent by Professor Elizur Wright, of Boston, United States; but he has carried it to an extreme which appears dangerous. At all events, if it should have a tendency to interfere with the theory of the reserve fund being the property of the collective body of the policyholders, then the danger would begin. He referred particularly to Professor Wright's book, published two or three years ago, the "*Politics and Mysteries of Life Insurance.*" He concluded by introducing the Hon. Mr. Washburn, of Boston, who has been President of one of the largest insurance companies in the State of Massachusetts.

Mr. MACFADYEN—The central idea in this paper is that the true measure of the death-strain is the difference between the total sum you may be called on to pay, and the reserve you have in hand to meet it. This idea is not at all new. It has occurred probably to most actuaries, and has been applied by, at all events one, Mr. Hardy, in his office calculations. A German writer discussing German insurance in the *Journal*

of the Institute recently, also strongly insisted on this as the true measure of the death-strain. Mr. Higham's analysis seems to consider that an examination of mortality experience is yearly made; and in the course of the paper reference is even made to some companies taking monthly estimates. So far from considering monthly mortality estimates useful, I am prepared to go to the length of saying that in most offices even yearly estimates are valueless. In any company with less than 5000 risks, the mortality experience of a year is no basis to found anything practical upon. When thinking of true mortality tables, we are too apt to forget that it is only in infinity that our theory and the facts will always coincide. It is all very well to talk of an H^M reserve, an H^{M5} reserve, a Carlisle reserve, or any other. If there are few risks on the books, one basis is practically as near the truth as another; and the one that holds the most in hand, is only better than the others, not because it is more correct, but because it leaves less at the sport of chance. With few risks the future is utterly unknown from anything that has happened in the past. I think, then, that in most offices, yearly, to say nothing of monthly, estimates of mortality are valueless, and that a quinquennium is as short a time as ought to elapse before publishing the results of the mortality experience. One other point Mr. Higham has not adverted to. Since the amount at risk is the difference between the nominal amount at risk and the reserve, the present method of fixing the maximum insurance an office will itself retain, stands condemned. Instead of the present hard and fast lines of £10,000, £5,000, and so on, acted on each case, whether there be any existing reserve or not; the truer method of arriving at such maximum insurance would be to consider the £10,000, £5,000, &c., as the utmost amount that must be at risk after the reserve is deducted from the possible amount payable. As this method of regarding maximum enables the companies to increase, without undue risk, the amount of their holdings in individual cases, there is reason to hope that it may be generally adopted.

Mr. H. Ambrose Smith wished to call attention to the fact that the paper applies only to the practice of a limited number of offices. The great majority of companies which make statements in their annual reports of their mortality as experienced, say that it came within their estimates,—they will never admit that it exceeds that estimate; but they only deal with the mortality, pecuniary results not being hinted at. There are, however, a few offices which empirically condescend upon their monetary results as well, and this paper shows that the subject can be otherwise than superficially handled. The discussion has brought to my recollection an opinion offered many years ago by the late Mr. Spens, who used to lay down that, granted an office has formed a resolution to abide by a certain maximum, when the value of a given policy has attained some magnitude, that maximum may be ignored (supposing the office has a fund in hand), and a new policy issued for its amount of the reserve without disturbing the equilibrium of the company.

Mr. Justican thought there was a greater display of symbols in Mr. Higham's demonstrations than was really necessary. For instance, the relation that exists between the value of a policy at the beginning

of the year, accumulated for one year, and the value at the end of the year, is very simply established.

Since $$ {}_{n}V_x + P_x = vp_{x+n}\ {}_{n+1}V_x + vq_{x+n}, $$

which is self-evident, therefore,

$$ ({}_{n}V_x + P_x)(1+i) = p_{x+n} \cdot {}_{n+1}V_x + q_{x+n}, $$

as given in the paper. This remark applies equally to the reversions.

Mr. R. P. HARDY mentioned that views like Mr. Higham's had been put forward by Mr. Sprague in his paper on the limitation of risk* in which the function $1-V$ plays a prominent part. But this paper goes further, and enables you exactly to earmark your profit, so as to see what financial profit has been derived from mortality, what from interest, and what from margins. As Mr. Walford had remarked, Mr. Elizur Wright has carried to an extreme the doctrine of the appropriation of reserve to individual policyholders; but in this country too we are a little bitten by the fallacy, when we consent to place special deposits in the hands of foreign and Colonial governments for the sole benefit of their policyholders: thus taking the money out of the common fund, and placing it beyond reach. Professor Wright describes two quantities—one which he calls "self-insurance" value, which we call "reserve", and the other a very curious function which he calls "insurance value". This latter will be found described in the preface of the large volume of 3 and 4 per-cent tables, 17 Offices' experience, published by Mr. Wright. He was very glad Mr. Higham had written the paper, and that the members of the Institute had not to wait till he might himself have leisure to take up the subject.

Mr. W. SUTTON objected to the title of the paper. The measure of the death-strain is not the correct term to use, but it should be the measure of the strain *by claims*. Bearing in mind what Mr. Ambrose Smith has mentioned as to the practice of many offices, of simply quoting the number of lives insured who have died in the year, in contradistinction to the amount of claims which have fallen in in the year, the distinction is rather important. He agreed with Mr. Justican as to the algebraical formulas, and was quite at a loss to see why the self-evident proposition

$$ {}_{n}V_x + P_x = p_{x+n}v \cdot {}_{n+1}V_x + q_{x+n}v $$

had not been made use of. The true measure recommended in the paper is the sum which the office would have in hand over and above the amount required to meet its liabilities if no claims had fallen in in the year. In other words, the true measure proposed is

$$ ({}_{n}V_x + P_x)(1+i) - {}_{n+1}V_x, $$

that is, the amount left in hand after providing for the office's liabilities under all its policies, none of its policies having in the course of the year become claims. There is another formula which is identical with that used by Mr. Higham, namely:—

$$ (P_x + d)(1 + a_{x+n}) + {}_{n}V_x = 1, $$

* See *Journal* xiii, 20.

which states that the amount which the office has in hand, plus what it will lose under the contract in the event of the policy becoming a claim, are together equal to the sum assured. The office clearly loses the capitalized value of the premiums, and it clearly loses the capitalized value of the interest on the sum assured; and those two together

$$=(P_x+d)(1+a_{x+n})=1-{}_nV_x.$$

With regard to the question of priority of publication of the method under discussion, he would refer the members to the 13th volume of the *Journal* containing the examination papers for 1863, and they would find that the twenty-second question for the second year's examination brings out very distinctly the method of treatment used in Mr. Higham's paper. The formula used by Mr. Higham for estimating the true sum at risk, had been for some years past associated in his (Mr. Sutton's) mind with Mr. Hardy's name, and for the following reason. In the 16th volume of the *Journal* (p. 234) there is a remarkable footnote in a paper by Mr. Sprague, in reference to the application of formula $1-{}_nV_x$, to denote the true sum at risk, in which he states that an application of this formula suggested by Mr. Hardy appears to him to place in a new and more favourable light, the reinsurance method advocated by the late Mr. Tucker.

The Hon. Mr. Washburn—This whole question of life insurance is one which deeply interests the American public. The amount of insurance there carried on is enormously large and is increasing every year. It calls for constant legislation on the part of the several commonwealths; and there is hardly a State in the Union but will, during these present winter sessions, be considering in some form or other the questions which this Institute is considering. All of us who know anything about life insurance, know that many theories have been propounded; but, in America, we endeavour to reduce the matter to as simple and direct a question as possible, so that the masses of people who need insurance, can have it at the least cost, and yet be satisfied that if their money is placed in our offices its return will be certain. Reference has been made here to Mr. Elizur Wright, a gentleman whose name is pretty well known on this side of the water, as having given great attention to the theory of life insurance. He is a man of large heart and generous impulses, somewhat theoretical, perhaps, and somewhat in advance of most of us, who throws out from day to day theories and ideas, which may or may not be practical, but which deserve to be carefully considered and discussed, as they are, both here and in America. There are probably in them a great many things from which you will dissent, but he is in the main doing a good work. We Americans regard English life insurance institutions as founded upon a rock. We believe that your funds are invested so that there is no question as to their security, and that whatever you have done has been safely and thoroughly done. We, as younger people, are more venturesome. Perhaps it is our nature and our habit; we do not stay so long as you to consider a question, nor are we as critical or careful; but I think we may be of service each to the other. You will find in England the excessive caution peculiar to old age. On the other hand, our young, go-ahead ideas may possibly be

of some service to you. We conduct life insurance in a way which, perhaps, would hardly be allowed here; the system of canvassing and solicitation is carried, perhaps, to an extent which would hardly be suffered here, and ought not, I think, to be suffered with us to the extent it is. The course that we have pursued of extensive pamphleteering, circulars, &c., has had its day. I think there is less of that today than there was a few years ago, and there will be less, probably, in the future. We feel that there is committed to our care a great and sacred trust—than which none more sacred could be committed to our hands—a trust for the widow and the fatherless.

Mr. C. D. HIGHAM, in reply, said, he did not claim for the paper the merit of entire novelty; but in the effort to do something for the Institute, which had done a great deal for him, he did not find that anything had been written in the *Journal* on this subject, and thought there could be no harm in laying down in black and white what had hitherto been more or less a matter of general theory. He agreed with Mr. Macfadyen that it would be absurd to base any conclusions on a month's experience, or even a year's, and he had intended to make it clear that the surplus or deficiency spoken of is for the actuary's information, and not for the public's. He had not mentioned the maximum of risk, because it had been given by Mr. Sprague in the 13th volume of the *Journal*. He differed with Mr. Justican and Mr. Sutton, and thought it desirable that every formula should be demonstrated mathematically in spite of its apparently evident truth.

A Comparison of various Methods of Graduation of a Mortality Table considered in reference to the Valuation of the Liability of an Average Life Office under its Assurance Contracts. By WILLIAM SUTTON, *Actuary to the Registry of Friendly Societies.*

[Read before the Institute, 27 February 1877.]

WHAT is the best method to use in the graduation of a mortality table, although not a new question, is one which always must have a practical interest for the profession; and as the various methods of graduation which have been devised from time to time can all boast of their respective admirers, the subject is one which cannot easily be disposed of at present. In the following paper it is not proposed to open up any discussion on the general principles involved, nor is it proposed to go into any detail as to how far these general principles have been carried out in the

various well known methods of graduation. The purpose of the paper is of a somewhat different nature. Mortality tables have little if any value except for their application to those financial matters into which the duration of life enters as an important element; and it would therefore appear to be a not unreasonable notion to institute a comparison between various methods of graduation; not to ascertain to what extent they respectively conform to established mathematical principles, but to ascertain to what extent they respectively affect that most important question to all concerned in the business of life assurance, what amount an office should retain at its periodic valuation to meet the liability under its policies. This amount, as we all know, will vary considerably with the data employed, the said data being the rate of interest and graduated mortality table. It is not however proposed in this paper to touch at all upon the comparative effect of employing various rates of interest, nor is it intended to deal with the comparative effect of using various mortality experiences. The rate of interest is assumed to be the same throughout, as also the mortality experience on which the mortality table is based, the only variable element introduced being the method of adjustment of the mortality experience used. It may perhaps be asked, why confine such an investigation to the single matter above stated? Why not carry it at least another stage, and extend it to an investigation of the effect of graduation upon the premiums? but to this there is the one sufficient answer, that except for purposes of limitation, premiums are not based on this or that mortality experience, but are practically fixed by that most efficacious instrument, competition.

Part I.—*Calculation and Comparison of Elementary Values.*

The mortality experience of twenty assurance offices, the publication of which is a lasting record of the great services this Institute has rendered to the business of life assurance, seemed to afford the best possible material for the object in view, and I take this the most fitting opportunity of stating that, in selecting this mortality experience, I am not in any way to be considered as implying that there appear to me to exist any grounds for being dissatisfied with Mr. Woolhouse's method of graduation. On the contrary, it will, I think, be found in the figures hereafter given that if smoothness of progression combined with close adherence to

the facts be the points to which to have regard in the graduation of a mortality table, then Mr. Woolhouse's method of graduation will be found to comply with both these requirements in a very remarkable manner. Apart from other self-evident reasons, I may add that a great inducement to select the Institute experience was the fact that I conceived, as will be seen hereafter, that the said experience as published would itself afford without difficulty the one thing needed in making my comparisons, namely the means of constructing an office which should fairly represent the position of an average office.

Having therefore fixed upon a suitable mortality experience, the next point to consider was which, among the many known methods of graduation—I use the word known advisedly—would do best for the purpose, and it will perhaps be sufficient to state without further comment that I selected the following four, namely:—

1. The second method laid down by the late Mr. J. Finlaison in his Report on the Law of Mortality of the Government Life Annuitants, March, 1829.

2. The method adopted by Mr. G. W. Berridge in the graduation of the peerage mortality experience, published by Messrs. A. H. Bailey and A. Day.

3. A graduation by the aid of the formula given by Mr. W. M. Makeham, as an improvement on that of the late Mr. Gompertz.

4. The method devised by Mr. W. S. B. Woolhouse for the graduation of the mortality experience of 20 offices, collected and published by the Institute of Actuaries.

Before giving any figures in illustration of the comparative effects of these various methods of graduation, it will be desirable to give a short description of each method.

The first mentioned was stated by Mr. John Finlaison to have been used by him in the graduation of the mortality experiences on which the Government Annuity Tables were based; but as pointed out by Mr. H. Ambrose Smith, in vol. xiii, p. 58, of the *Journal of the Institute of Actuaries*, whatever method was used for the graduation of the tables mentioned by Mr. J. Finlaison, this as described by him certainly was not.

Let P_{x-2}, P_{x-1}, P_x, P_{x+1}, P_{x+2}, each be found from the formula

$$P_x = \frac{p'_{x-2} + p'_{x-1} + p'_x + p'_{x+1} + p'_{x+2}}{5},$$

where p'_x denotes the probability, according to the given mortality experience, of living a year at age x, then

$$p_x = \text{adjusted probability of living a year at age } x$$

$$= \frac{P_{x-2}+P_{x-1}+P_x+P_{x+1}+P_{x+2}}{5}$$

$$= \frac{5p'_x+4(p'_{x-1}+p'_{x+1})+3(p'_{x-2}+p'_{x+2})+2(p'_{x-3}+p'_{x+3})+(p'_{x-4}+p'_{x+4})}{25}$$

$$= \cdot 20p'_x+\cdot 16(p'_{x-1}+p'_{x+1})+\cdot 12(p'_{x-2}+p'_{x+2})+\cdot 08(p'_{x-3}+p'_{x+3})$$
$$+\cdot 04(p'_{x-4}+p'_{x+4}).$$

This method, it will be seen, brings in the unadjusted probabilities of living for the age x, and for the four immediately preceding and succeeding ages, giving to each a weight inversely proportionate to its distance from the age under consideration. In practically applying this formula the method adopted is to sum every five unadjusted probabilities of living (or dying), and place the total opposite the middle age. This being done throughout, every five of these totals are again summed as before, and the total divided by 25, *i.e.* multiplied by ·04, is the adjusted probability of living (or dying) at the age opposite to which it is placed. Thus—

Age (x)	p'_x			p_x
25	·99486			
26	·99308			
27	·99353	4·96628		
28	·99217	4·96316		
29	·99264	4·96272	24·81342	·99254
30	·99174	4·96087		
31	·99264	4·96039		
32	·99168			
33	·99169			

The values of p_x for age 93 and upwards were not obtained by Mr. Finlaison's method, but are the same as those given by Mr. Woolhouse's method.

Mr. Berridge's method of graduation is fully described in a short paper by him, published in vol. xii of the *Journal*, page 220.

The logarithms of the unadjusted probabilities of living a year are added together for every 10 years, beginning with age 20, up to age 79, and then these totals are redistributed by means of a formula in finite differences, the result being that the sum of every ten adjusted logarithms agrees with the sum of every ten unadjusted logarithms. From ages 80 to 95, the sum of every 8

adjusted logarithms was made to agree with the unadjusted, and, as far as possible, the method described by Mr. Berridge for joining this part of the table on to the other part was followed. The last part of the graduated table still shows a break between 79 and 80; but I have no reason to think I have failed to apply correctly the process sketched out by Mr. Berridge.

The following are the sums of every set of unadjusted and adjusted logarithmic probabilities :—

Ages.	Unadjusted.	Adjusted.
20–29	997066	997065
30–39	996140	996140
40–49	994676	994675
50–59	990890	990891
60–69	980825	980833
70–79	956468	956478
80–87	727064	727063
88–95	560353	560351

In applying Mr. Makeham's formula, $l_x = dg^{q^x}s^x$, I have proceeded as follows. The unadjusted numbers living at ages 20, 40, 60, 80, were taken, and a set of constants deduced therefrom, from which a series of adjusted values of $\log l_x$ was obtained. This process was repeated with the unadjusted numbers living at ages 30, 50, 70, 90, thus furnishing a second series of adjusted values of $\log l_x$. The two sets of values were then added together, and the sum halved gave the final values of $\log l_x$. The effect of this combination of the two series is as follows :—Let ${}^1\mathrm{L}$ and ${}^2\mathrm{L}$ denote the adjusted numbers living at any given age deduced from the use of the first and second set of constants respectively, then

$$\log \text{(adjusted number living)} = \tfrac{1}{2}(\log {}^1\mathrm{L} + \log {}^2\mathrm{L})$$
$$= \tfrac{1}{2}\log ({}^1\mathrm{L} \times {}^2\mathrm{L})$$

that is, adjusted number living $= \sqrt{{}^1\mathrm{L} \times {}^2\mathrm{L}}$.

This application of the method does not differ materially from Mr. Woolhouse's use of it, the only important distinction being that whereas in the present method the unadjusted numbers living are made use of to furnish the constants, Mr. Woolhouse has employed constants derived from the numbers living as already adjusted by his own method. It is worth noting that in giving the adjusted values for the $\mathrm{H^{MF}}$ Table, obtained by constants deduced from his own adjusted table, "for the purpose of exhibiting

how far the formula may be relied upon as a near approximation", Mr. Woolhouse does not compare them with his own adjusted values, which in this case would be the unadjusted values, but with the unadjusted values from which his own table was deduced. My meaning will be clearer perhaps from the following table:—

Table H^{MF}.

Age.	Number Living. Unadjusted. (1)	Differences between (2) and (1).	Woolhouse. (2)	Differences between (3) and (2).	Makeham. (3)	Differences between (3) and (1).
20	9554	+ 7	9561	+18	9579	+25
25	9230	+10	9240	− 4	9236	+ 6
30	8904	...	8904	−17	8887	−17
35	8535	+ 2	8537	−13	8524	−11
40	8128	+ 5	8133	...	8133	+ 5
45	7696	− 2	7694	+ 3	7697	+ 1
50	7183	− 2	7181	+10	7191	+ 8
55	6586	−11	6575	+ 8	6583	− 3
60	5847	−10	5837	− 4	5833	−14
65	4914	...	4914	...	4909	− 5
70	3805	+19	3824	−16	3808	+ 3
75	2571	+23	2594	+ 2	2596	+25
80	1411	+ 9	1420	+22	1442	+31

The degree of approximation to the unadjusted facts obtained by the use of Makeham's formula for the purpose of the present paper is shown in the following table:—

Table H^{M}.

Age.	Number Living. Unadjusted	Difference.	Makeham's method. as explained above.
20	10,000	...	10.000
25	9,668	− 7	9,661
30	9,347	−35	9,312
35	8,970	−25	8.945
40	8 552	− 8	8,544
45	8 104	−14	8,090
50	7,565	− 9	7,556
55	6,929	−22	6,907
60	6,134	−29	6,105
65	5,128	−12	5,116
70	3,944	+ 1	3,945
75	2,649	+19	2,668
80	1,448	+21	1,469

Mr. Woolhouse's method of graduation has been so ably explained by himself that no reference to it is required on the present occasion beyond a very few words. For a most clear and

comprehensive account of the rationale of his process, I have only to refer to vol. xv, pp. 390 and 391. A full account will also be found in the Institute of Actuaries' Life Tables. Put in a form somewhat similar to that given above for Mr. Finlaison's formula, the resulting formula is

$$\text{adjusted number living at age } x = \cdot 20 l'_x + \cdot 192(l'_{x-1}+l'_{x+1}) + \cdot 168(l'_{x-2}+l'_{x+2}) \\ + \cdot 056(l'_{x-3}+l'_{x+3}) + \cdot 024(l'_{x-4}+l'_{x+4}) \\ - \cdot 016(l'_{x-6}+l'_{x+6}) - \cdot 024(l'_{x-7}+l'_{x+7}).$$

It was originally proposed to take as another method of graduation for the purpose of this paper that employed by the late Mr. Griffith Davies in the graduation of the Equitable experience, and which is explained at some length in his Treatise on Annuities. It was found, however, on a close examination of the method in question, that in its practical application it requires to be dealt with more or less arbitrarily; and this was in itself considered sufficient to render it unsuitable for the present purpose. From the facts he had to deal with being in a peculiar form, Mr. Davies was obliged to have recourse to a very ingenious method of deducing the number living at certain ages, and then by a method of interpolation to obtain the numbers living at the intervening ages. It may be of interest to state that the same method of deducing the numbers living at certain ages, when applied to the Institute experience, Healthy Males, brought out the following results, showing clearly that in this respect, at any rate, Mr. Davies's process is a very good approximation :—

Ages.	Unadjusted.	G. Davies's Approximation.
20	9616	9615
30	8987	8985
40	8223	8223
50	7274	7276
60	5898	5890
70	3793	3779
80	1392	1401
85	541	543
90	150	145

The following table gives the numbers living at ages from 20 upwards, for the unadjusted results of the Institute Healthy Males, and the corresponding numbers living, as adjusted by means of the various methods above described, and for the purpose of comparison, the corresponding numbers living are also given for the Carlisle and 17 Offices tables.

TABLE I.—*Number Living at Age x.*

Age x.	1 20 Offices' Experience, H^M unadjusted.	2 Col. 1, adjusted by Mr. Woolhouse's method.	3 Col. 1, adjusted by Mr. Berridge's method.	4 Col. 1, adjusted by Mr. Finlaison's method.	5 Col. 1, adjusted by Mr. Makeham's method.	6 17 Offices' Experience.	7 Carlisle.	Age x.
20	100000	100000	100000	100000	100000	100000	100000	20
1	99424	99367	99417	99389	99325	99270	99294	1
2	98728	98699	98864	98744	98651	98537	98604	2
3	98112	98023	98256	98091	97974	97803	97915	3
4	97353	97360	97623	97438	97293	97062	97225	4
5	96685	96714	96971	96792	96610	96319	96535	5
6	96188	96073	96299	96150	95922	95570	95829	6
7	95521	95430	95611	95497	95231	94816	95123	7
8	94903	94988	95002	94833	94515	94057	94384	8
9	94161	94092	94193	94150	93832	93291	93563	9
30	93467	93392	93465	93448	93124	92521	92644	30
1	92696	92671	92726	92724	92408	91740	91708	1
2	92013	91937	91975	91990	91683	90954	90772	2
3	91247	91192	91212	91243	90949	90157	89852	3
4	90490	90437	90438	90486	90205	89353	88951	4
5	89704	89668	89652	89714	89448	88542	88046	5
6	88965	88881	88853	88924	88677	87718	87143	6
7	88178	88071	88040	88111	87892	86886	86223	7
8	87335	87238	87215	87277	87094	86046	85287	8
9	86437	86385	86373	86421	86276	85192	84335	9
40	85518	85514	85517	85550	85438	84349	83333	40
1	84674	84633	84643	84668	84582	83456	82250	1
2	83788	83745	83749	83772	83701	82570	81100	2
3	82890	82846	82834	82863	82796	81671	79951	3
4	82013	81924	81897	81931	81863	80751	78785	4
5	81044	80978	80934	80969	80902	79807	77619	5
6	80042	79990	79943	79970	79908	78833	76470	6
7	79041	78955	78922	78931	78881	77820	75337	7
8	77922	77873	77868	77843	77816	76768	74236	8
9	76833	76749	76779	76711	76711	75675	73202	9
50	75652	75591	75649	75539	75563	74535	72200	50
1	74403	74383	74477	74329	74370	73347	71232	1
2	73107	73135	73257	73075	73131	72107	70213	2
3	71862	71852	71988	71779	71834	70813	69146	3
4	70627	70515	70663	70432	70484	69461	68029	4
5	69288	69124	69279	69024	69075	68050	66880	5
6	67697	67670	67833	67552	67605	66575	65631	6
7	66133	66150	66319	66016	66069	65035	64434	7
8	64554	64564	64734	64414	64466	63431	63087	8
9	62932	62909	63074	62739	62793	61758	61560	9
60	61337	61177	61335	60990	61045	60012	59819	60
1	59426	59361	59516	59156	59224	58192	57816	1
2	57494	57459	57612	57235	57324	56295	55747	2
3	55504	55469	55624	55228	55349	54318	53662	3
4	53429	53390	53550	53141	53294	52262	51597	4
5	51283	51232	51391	50977	51162	50128	49557	5
6	49048	49007	49150	48737	48954	47918	47520	6
7	46756	46725	46828	46441	46675	45637	45396	7
8	44465	44394	44432	44102	44326	43288	43481	8
9	42006	42030	41970	41710	41915	40880	41461	9
70	39443	39620	39449	39275	39448	38424	39425	70
1	37233	37156	36883	36784	36937	35929	37389	1
2	34918	34628	34283	34227	34391	33408	35189	2
3	32134	32033	31666	31600	31823	30875	32791	3
4	29604	29379	29049	28948	29248	28347	30230	4
5	26486	26699	26451	26287	26684	25839	27504	5
6	23987	24073	23898	23674	24148	23370	24877	6

TABLE I.—*continued.*

Age x	1 20 Offices' Experience, HM unadjusted.	2 Col. 1, adjusted by Mr. Woolhouse's method.	3 Col. 1, adjusted by Mr. Berridge's method.	4 Col. 1, adjusted by Mr. Finlaison's method.	5 Col. 1, adjusted by Mr. Makeham's method.	6 17 Offices' Experience.	7 Carlisle.	Age x
77	21432	21512	21396	21137	21661	20959	22315	77
8	19101	19045	18981	18700	19242	18623	19918	8
9	16756	16699	16670	16366	16912	16379	17750	9
80	14476	14477	14482	14141	14693	14249	15649	80
1	12439	12383	12460	12053	12604	12248	13744	1
2	10451	10426	10614	10120	10662	10394	11905	2
3	8650	8639	8937	8365	8883	8697	10230	3
4	6861	7034	7414	6812	7279	7168	8686	4
5	5626	5635	6032	5466	5856	5805	7307	5
6	4410	4452	4784	4327	4618	4616	6026	6
7	3452	3474	3671	3390	3563	3590	4860	7
8	2700	2671	2701	2623	2684	2728	3810	8
9	1932	2032	1886	1984	1970	1999	2972	9
90	1563	1517	1235	1443	1404	1414	2332	90
1	1204	1093	748	1018	970	956	1724	1
2	827	751	413	696	648	611	1232	2
3	455	487	205	443	417	363	887	3
4	152	285	90	259	257	197	657	4
5	152	140	34	127	152	95	493	5
6	108	51	11	46	85	40	378	6
7	...	9	...	8	...	14	296	7
						4	230	8

The next step was to compute the values of a_x, A_x, and P_x, relating to each table, and these are given in Tables II, III, and IV, at 3 per-cent interest.

TABLE II.—*Annuity Values at* 3 *per-cent Interest* (a_x).

Age x.	1 20 Offices' HM, unadjusted.	2 20 Offices' HM, Mr. Woolhouse's adjustment.	3 20 Offices' HM, Mr. Berridge's adjustment.	4 20 Offices' HM, Mr. Finlaison's adjustment.	5 20 Offices' HM, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
20	22·056	22·043	22·061	22·029	22·013	21·797	21·694	20
1	21·851	21·848	21·850	21·830	21·827	21·616	21·504	1
2	21·665	21·656	21·638	21·631	21·636	21·430	21·304	2
3	21·456	21·460	21·425	21·429	21·439	21·239	21·098	3
4	21·269	21·254	21·211	21·219	21·236	21·043	20·885	4
5	21·060	21·038	20·995	21·002	21·028	20·842	20·665	5
6	20·804	20·814	20·775	20·776	20·814	20·635	20·442	6
7	20·578	20·582	20·552	20·546	20·593	20·423	20·212	7
8	20·333	20·347	20·32[illegible]	20·310	20·372	20·205	19·981	8
9	20·108	20·109	20·094	20·072	20·136	19·982	19·761	9
30	19·865	19·867	19·858	19·829	19·898	19·754	19·556	30
1	19·631	19·623	19·617	19·584	19·654	19·519	19·348	1
2	19·370	19·373	19·370	19·332	19·404	19·279	19·134	2
3	19·119	19·117	19·119	19·075	19·147	19·032	18·910	3
4	18·857	18·855	18·860	18·812	18·884	18·780	18·675	4
5	18·593	18·587	18·596	18·542	18·615	18·521	18·433	5
6	18·310	18·314	18·327	18·269	18·340	18·255	18·183	6
7	18·028	18·037	18·051	17·990	18·059	17·983	17·928	7
8	17·747	17·756	17·768	17·707	17·771	17·703	17·669	8
9	17·470	17·469	17·479	17·419	17·478	17·417	17·405	9
40	17·187	17·176	17·184	17·124	17·179	17·123	17·143	40
1	16·880	16·876	16·883	16·821	16·873	16·821	16·890	1

TABLE II.—*continued.*

Age x.	1 20 Offices' HM, unadjusted.	2 20 Offices' HM, Mr. Woolhouse's adjustment.	3 20 Offices' HM, Mr. Berridge's adjustment.	4 20 Offices' HM, Mr. Finlaison's adjustment.	5 20 Offices' HM Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
42	16·570	16·566	16·574	16·511	16·563	16·512	16·610	42
3	16·252	16·248	16·260	16·193	16·246	16·195	16·389	3
4	15·918	15·924	15·910	15·869	15·924	15·870	16·130	4
5	15·592	15·594	15·613	15·539	15·597	15·540	15·863	5
6	15·261	15·260	15·281	15·205	15·264	15·204	15·585	6
7	14·918	14·923	14·943	14·868	14·927	14·864	15·294	7
8	14·586	14·585	14·600	14·528	14·585	14·519	14·986	8
9	14·238	14·242	14·251	14·184	14·239	14·171	14·654	9
50	13·892	13·896	13·898	13·836	13·889	13·820	14·303	50
1	13·550	13·545	13·540	13·483	13·535	13·465	13·932	1
2	13·204	13·188	13·178	13·126	13·178	13·107	13·558	2
3	12·835	12·826	12·813	12·764	12·818	12·747	13·180	3
4	12·451	12·462	12·445	12·399	12·456	12·385	12·798	4
5	12·072	12·094	12·074	12·031	12·091	12·021	12·408	5
6	11·727	11·724	11·701	11·662	11·724	11·656	12·014	6
7	11·365	11·353	11·328	11·291	11·356	11·290	11·614	7
8	10·992	10·981	10·953	10·919	10·989	10·923	11·218	8
9	10·614	10·608	10·579	10·547	10·620	10·555	10·841	9
60	10·216	10·236	10·205	10·175	10·252	10·188	10·491	60
1	9·861	9·866	9·832	9·805	9·883	9·822	10·180	1
2	9·498	9·498	9·462	9·438	9·517	9·457	9·875	2
3	9·134	9·134	9·094	9·075	9·153	9·096	9·567	3
4	8·773	8·774	8·730	8·714	8·791	8·737	9·246	4
5	8·415	8·418	8·369	8·357	8·432	8·382	8·917	5
6	8·062	8·064	8·014	8·003	8·076	8·032	8·578	6
7	7·711	7·712	7·663	7·650	7·725	7·686	8·228	7
8	7·351	7·360	7·319	7·298	7·378	7·347	7·869	8
9	7·015	7·007	6·981	6·948	7·037	7·013	7·499	9
70	6·695	6·657	6·649	6·600	6·701	6·685	7·123	70
1	6·305	6·311	6·325	6·258	6·371	6·364	6·737	1
2	5·925	5·975	6·009	5·928	6·048	6·049	6·373	2
3	5·632	5·653	5·701	5·613	5·733	5·742	6·044	3
4	5·296	5·348	5·401	5·311	5·424	5·441	5·752	4
5	5·097	5·061	5·109	5·024	5·124	5·148	5·512	5
6	4·797	4·782	4·826	4·746	4·832	4·863	5·277	6
7	4·530	4·512	4·551	4·475	4·548	4·585	5·059	7
8	4·235	4·249	4·284	4·210	4·274	4·315	4·838	8
9	3·973	3·992	4·024	3·955	4·008	4·053	4·592	9
80	3·737	3·742	3·771	3·714	3·752	3·799	4·365	80
1	3·479	3·507	3·515	3·488	3·505	3·553	4·119	1
2	3·265	3·290	3·250	3·279	3·268	3·312	3·898	2
3	3·063	3·089	2·975	3·086	3·040	3·077	3·672	3
4	2·978	2·908	2·694	2·903	2·822	2·846	3·454	4
5	2·740	2·739	2·411	2·727	2·613	2·617	3·229	5
6	2·601	2·570	2·131	2·548	2·412	2·391	3·033	6
7	2·423	2·393	1·861	2·350	2·220	2·167	2·873	7
8	2·191	2·206	1·605	2·128	2·036	1·946	2·776	8
9	2·152	1·987	1·368	1·897	1·858	1·728	2·665	9
90	1·741	1·740	1·151	1·687	1·684	1·516	2·499	90
1	1·329	1·487	·957	1·462	1·510	1·309	2·481	1
2	·991	1·229	·784	1·205	1·329	1·109	2·577	2
3	·856	·951	·628	·951	1·129	·921	2·687	3
4	1·644	·677	·481	·677	·884	·748	2·736	5
5	·693	·415	·311	·415	·545	·592	2·757	5
6	...	·178	...	·178	...	·468	2·704	6
						·371	2·559	7
						·243	...	8

TABLE III.—*Value of* 1 *payable at Death*, 3 *per-cent Interest* (A_x).

Age x.	1 20 Offices' H^M, unadjusted.	2 20 Offices' H^M, Mr. Woolhouse's adjustment.	3 20 Offices' H^M, Mr. Berridge's adjustment.	4 20 Offices' H^M, Mr. Finlaison's adjustment.	5 20 Offices' H^M, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
20	·32847	·32886	·32832	·32925	·32972	·33600	·33901	20
1	·33444	·33451	·33448	·33506	·33514	·34128	·34455	1
2	·33985	·34011	·34066	·34083	·34070	·34669	·35037	2
3	·34595	·34584	·34685	·34674	·34644	·35226	·35637	3
4	·35139	·35183	·35308	·35284	·35235	·35797	·36252	4
5	·35748	·35812	·35938	·35917	·35841	·36383	·36898	5
6	·36493	·36465	·36578	·36574	·31464	·36985	·37548	6
7	·37152	·37139	·37229	·37245	·37108	·37603	·38218	7
8	·37865	·37824	·37887	·37931	·37751	·38236	·38890	8
9	·38521	·38518	·38562	·38626	·38439	·38586	·39531	9
30	·39228	·39221	·39249	·39333	·39132	.39552	·40129	30
1	·39910	·39934	·39953	·40048	·39842	·40235	·40734	1
2	·40670	·40662	·40671	·40781	·40571	·40936	·41357	2
3	·41401	·41407	·41402	·41530	·41320	·41654	·42010	3
4	·42164	·42170	·42154	·42297	·42 85	·42389	·42694	4
5	·42933	·42950	·42924	·43080	·42868	·43144	·43399	5
6	·43757	·43745	·43710	·43878	·43670	·43917	·44117	6
7	·44579	·44553	·44512	·44689	·44489	·44710	:44870	7
8	·45397	·45372	·45336	·45514	·45327	·45524	·45624	8
9	·46204	·46207	·46178	·46353	·46181	·46358	·46393	9
40	·47029	·47060	·47037	·47212	·47052	·47214	·47156	40
1	·47922	·47935	·47915	·48093	·47942	·48093	·47893	1
2	·48825	·48836	·48814	·48996	·48845	·48995	·48621	2
3	·49751	·49762	·49729	·49922	·49769	·49919	·49352	3
4	·50725	·50707	·50661	·50863	·50706	·50863	·50108	4
5	·51674	·51669	·51612	·51828	·51660	·51826	·50885	5
6	·52638	·52642	·52580	·52801	·52629	·52804	·51694	6
7	·53637	·53621	·53566	·53784	·53611	·53795	·52542	7
8	·54604	·54608	·54565	·54774	·54606	·54798	·53439	8
9	·55618	·55605	·55581	·55774	·55615	·55812	·54406	9
50	·56625	·56613	·56610	·56787	·56634	·56836	·55429	50
1	·57621	·57635	·57650	·57815	·57665	·57870	·56509	1
2	·58629	·58676	·58705	·58856	·58705	·58912	·57598	2
3	·59703	·59729	·59769	·59910	·59754	·59960	·58699	3
4	·60822	·60792	·60840	·60975	·60808	·61015	·59812	4
5	·61926	·61863	·61920	·62046	·61871	·62075	·60948	5
6	·62931	·62939	·63006	·63121	·62939	·63139	·62096	6
7	·63985	·64020	·64095	·64200	·64012	·64205	·63260	7
8	·65072	·65103	·65186	·65284	·65081	·65274	·64413	8
9	·66172	·66190	·66275	·66368	·66155	·66344	·65512	9
60	·67333	·67274	·67365	·67452	·67227	·67414	·66531	60
1	·68366	·68353	·68149	·68529	·68302	·68480	·67436	1
2	·69424	·69424	·69528	·69597	·69368	·69541	·68325	2
3	·70483	·70484	·70599	·70656	·70428	·70595	·69222	3
4	·71535	·71532	·71661	·71707	·71482	·71610	·70157	4
5	·72577	·72569	·72711	·72748	·72528	·72673	·71112	5
6	·73606	·73600	·73747	·73778	·73566	·73694	·72103	6
7	·74628	·74626	·74768	·74805	·74597	·74700	·73122	7
8	·75677	·75650	·75771	·75832	·75598	·75689	·74168	8
9	·76655	·76678	·76756	·76851	·76592	·76662	·75216	9
70	·77587	·77700	·77720	·77865	·77570	·77617	·76340	70
1	·78723	·78706	·78664	·78860	·78531	·78553	·77465	1
2	·79830	·79685	·79585	·79823	·79472	·79469	·78525	2
3	·80683	·80623	·80482	·80739	·80389	·80364	·79483	3
4	·81663	·81510	·81357	·81619	·81289	·81239	·80334	4
5	·82242	·82345	·82206	·82455	·82163	·82092	·81083	5

TABLE III.—*continued.*

Age x.	1 20 Offices' H^M, unadjusted.	2 20 Offices' H^M, Mr. Woolhouse's adjustment.	3 20 Offices' H^M, Mr. Berridge's adjustment.	4 20 Offices' H^M, Mr. Finlaison's adjustment.	5 20 Offices' H^M, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
76	·83116	·83160	·83031	·83265	·83013	·82923	·81717	76
7	·83893	·83916	·83832	·84054	·83841	·83732	·82352	7
8	·84752	·84711	·84610	·84826	·84638	·84518	·82996	8
9	·85515	·85461	·85366	·85569	·85414	·85281	·83713	9
80	·86203	·86187	·86103	·86269	·86159	·86021	·84374	80
1	·86955	·86874	·86850	·86928	·86878	·86740	·85090	1
2	·87577	·87506	·87622	·87537	·87569	·87440	·85734	2
3	·88166	·88090	·88421	·88099	·88233	·88126	·86392	3
4	·88414	·88617	·89240	·88632	·88868	·88799	·87027	4
5	·89107	·89110	·90065	·89145	·89476	·89465	·87682	5
6	·89512	·89601	·90830	·89667	·90062	·90123	·88253	6
7	·90030	·90118	·91668	·90244	·90621	·90774	·88719	7
8	·90706	·90663	·92413	·90890	·91158	·91419	·89002	8
9	·90819	·91301	·93104	·91562	·91676	·92053	·89325	9
90	·92016	·92020	·93734	·92173	·92182	·92673	·89809	90
1	...	·92756	·94300	·92829	·92689	·93276	·89861	1
2	...	·93508	·94805	·93577	·93217	·93857	·89582	2
3	...	·94317	·95257	·94317	·93799	·94405	·89261	3
4	...	·95116	·95686	·95116	·94512	·94910	·89118	4
5	...	·95878	·96182	·95878	·95500	·95362	·89057	5
6	...	·96568	·97087	·96568	...	·95726	·89212	6
7	...	·97087	...	·97087	...	·96005	·89633	7
						·96382	...	8

TABLE IV.—*Annual Premiums throughout Life to assure 1. Interest at 3 per-cent* (P_x).

Age x.	1 20 Offices' H^M, unadjusted.	2 20 Offices' H^M, Mr. Woolhouse's adjustment.	3 20 Offices' H^M, Mr. Berridge's adjustment.	4 20 Offices' H^M, Mr. Finlaison's adjustment.	5 20 Offices' H^M, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
20	·01425	·01427	·01423	·01430	·01432	·01473	·01494	20
1	·01464	·01464	·01463	·01468	·01469	·01508	·01531	1
2	·01500	·01501	·01504	·01506	·01505	·01545	·01571	2
3	·01541	·01540	·01546	·01546	·01544	·01583	·01613	3
4	·01578	·01581	·01589	·01588	·01585	·01624	·01657	4
5	·01621	·01625	·01634	·01633	·01627	·01665	·01703	5
6	·01674	·01672	·01679	·01680	·01672	·01709	·01751	6
7	·01721	·01721	·01727	·01729	·01718	·01755	·01802	7
8	·01775	·01772	·01776	·01780	·01767	·01803	·01853	8
9	·01825	·01825	·01828	·01833	·01819	·01853	·01904	9
30	·01880	·01880	·01881	·01888	·01872	·01905	·01952	30
1	·01935	·01936	·01938	·01946	·01929	·01960	·02002	1
2	·01997	·01996	·01996	·02006	·01988	·02018	·02054	2
3	·02058	·02058	·02058	·02069	·02052	·02079	·02110	3
4	·02123	·02124	·02122	·02135	·02117	·02143	·02170	4
5	·02191	·02193	·02190	·02205	·02185	·02210	·02233	5
6	·02266	·02265	·02261	·02277	·02258	·02280	·02300	6
7	·02343	·02340	·02336	·02353	·02334	·02355	·02371	7
8	·02422	·02419	·02415	·02433	·02415	·02434	·02444	8
9	·02502	·02502	·02499	·02517	·02500	·02517	·02521	9
40	·02586	·02589	·02586	·02605	·02588	·02605	·02599	40
1	·02680	·02682	·02679	·02699	·02682	·02698	·02677	1
2	·02779	·02780	·02777	·02798	·02781	·02797	·02757	2

TABLE IV.—*continued.*

Age x.	1 20 Offices' H^M, unadjusted.	2 20 Offices' H^M, Mr. Woolhouse's adjustment.	3 20 Offices' H^M, Mr. Berridge's adjustment.	4 20 Offices' H^M, Mr. Finlaison's adjustment.	5 20 Offices' H^M, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
43	·02881	·02885	·02881	·02904	·02886	·02903	·02838	43
4	·02999	·02996	·02990	·03016	·02997	·03014	·02925	4
5	·03114	·03114	·03106	·03134	·03112	·03133	·03018	5
6	·03238	·03238	·03229	·03258	·03236	·03258	·03117	6
7	·03370	·03367	·03360	·03390	·03366	·03391	·03225	7
8	·03504	·03504	·03488	·03528	·03504	·03530	·03343	8
9	·03650	·03648	·03644	·03673	·03649	·03678	·03476	9
50	·03802	·03801	·03800	·03828	·03803	·03835	·03622	50
1	·03960	·03963	·03965	·03992	·03967	·04000	·03785	1
2	·04128	·04136	·04140	·04166	·04141	·04176	·03957	2
3	·04316	·04320	·04327	·04353	·04325	·04361	·04140	3
4	·04521	·04516	·04525	·04551	·04518	·04558	·04335	4
5	·04738	·04725	·04736	·04761	·04726	·04767	·04545	5
6	·04945	·04946	·04960	·04985	·04947	·04988	·04771	6
7	·05175	·05182	·05199	·05223	·05181	·05224	·05015	7
8	·05427	·05434	·05453	·05477	·05429	·05474	·05272	8
9	·05698	·05702	·05723	·05748	·05693	·05741	·05532	9
60	·06003	·05987	·06012	·06036	·05974	·06025	·05790	60
1	·06295	·06291	·06319	·06342	·06276	·06328	·06032	1
2	·06613	·06613	·06645	·06668	·06596	·06649	·06283	2
3	·06955	·06956	·06994	·07013	·06937	·06992	·06551	3
4	·07320	·07319	·07365	·07382	·07301	·07357	·06847	4
5	·07709	·07705	·07760	·07775	·07690	·07745	·07171	5
6	·08123	·08120	·08181	·08195	·08106	·08159	·07528	6
7	·08568	·08566	·08630	·08648	·08548	·08599	·07924	7
8	·09062	·09049	·09108	·09139	·09024	·09068	·08363	8
9	·09565	·09576	·09618	·09669	·09530	·09567	·08853	9
70	·10083	·10148	·10160	·10246	·10072	·10100	·09398	70
1	·10777	·10766	·10738	·10865	·10654	·10668	·10013	1
2	·11528	·11425	·11354	·11523	·11276	·11274	·10651	2
3	·12165	·12119	·12010	·12210	·11939	·11921	·11285	3
4	·12970	·12840	·12710	·12933	·12654	·12612	·11897	4
5	·13489	·13585	·13456	·13688	·13417	·13352	·12444	5
6	·14338	·14382	·14251	·14492	·14234	·14143	·13019	6
7	·15171	·15230	·15102	·15353	·15112	·14991	·13591	7
8	·16190	·16138	·16013	·16282	·16049	·15900	·14216	8
9	·17196	·17121	·16991	·17270	·17056	·16875	·14971	9
80	·18198	·18174	·18046	·18300	·18131	·17924	·15728	80
1	·19413	·19277	·19237	·19369	·19286	·19053	·16624	1
2	·20534	·20399	·20618	·20457	·20518	·20278	·17505	2
3	·21700	·21543	·22242	·21561	·21840	·21616	·18492	3
4	·22225	·22676	·24155	·22707	·23252	·23091	·19539	4
5	·23825	·23834	·26404	·23920	·24765	·24734	·20732	5
6	·24858	·25096	·29024	·25274	·26396	·26577	·21883	6
7	·26302	·26563	·32042	·26942	·28143	·28658	·22905	7
8	·28425	·28282	·35470	·29058	·30026	·31029	·23571	8
9	·28813	·30570	·39325	·31605	·32077	·33740	·24373	9
90	·33571	·33585	·43572	·34300	·34344	·36837	·25669	90
1	...	·37294	·48189	·37704	·36928	·40404	·24818	1
2	...	·41954	·53148	·42434	·40028	·44498	·25093	2
3	...	·48338	·58197	·48338	·44063	·49143	·24209	3
4	...	·56719	·64608	·56719	·50162	·54302	·23857	4
5	...	·67748	·73372	·67748	·61814	·59885	·23705	5
6	...	·81954	·97087	·81954	...	·65219	·25085	6
7	...	·97087	...	·97087	...	·70014	...	7
						·77558	...	8

The first thing to be noted in Table II is the remarkable closeness in the annuity values as given by the unadjusted results, and Mr. Woolhouse's adjustment (Inst. H^M 3 per-cent), which in itself is another proof, if proof were necessary, of the satisfactory results that method of adjustment has yielded.

Mr. Finlaison's method gives annuity values throughout smaller than the unadjusted, and bearing in mind the purpose to which Mr. Finlaison's method is stated by him to have been applied, this is a point in itself of considerable practical importance.

As a further verification of the effect of an adjustment by this method, it should be stated that it has been applied to the experience for the graduation of which it was devised, and has been found to yield the like results.

The following table gives the means for a general comparison of the effect of the various methods of graduation upon annuity values.

Table of Annuity Values (a_x—3 *per-cent*) H^M.

Age (x)	Unadjusted.	Woolhouse.	Finlaison.	Berridge.	Makeham.
20	22·056	22·012	22·029	22·061	22·013
30	19·865	19·867	19·829	19·858	19·898
40	17·187	17·176	17·124	17·184	17·179
50	13·892	13·896	13·836	13·898	13·889
60	10·216	10·236	10·175	10·205	10·252
70	6·695	6·656	6·600	6·649	6·701
80	3·737	3·742	3·714	3·771	3·752

For the purpose of comparing the policy reserves brought out by the employment of the various tables, the following table has been constructed, which gives the value of 100 $\{A_{x+n}-P_x(\frac{1}{2}+a_{x+n})\}$ for each table, for various ages of entry and for various durations of policies; and there is placed alongside each a column showing the percentages which the policy reserves by other tables, bear to those based on Mr. Woolhouse's adjustment. In any application of this table, it should always be carefully borne in mind that it does not give all the information required as regards with profit policies having vested reversionary bonus additions, because there is no doubt, as will be seen further on, that the value of such bonus additions will in general have an appreciable effect in any comparison of the actual reserves required by the various tables for the liabilities of an office, where such reversionary bonus additions exist.

TABLE I.—*continued.*

Age x	1 20 Offices' Experience, HM unadjusted.	2 Col. 1, adjusted by Mr. Woolhouse's method.	3 Col. 1, adjusted by Mr. Berridge's method.	4 Col. 1, adjusted by Mr. Finlaison's method.	5 Col. 1, adjusted by Mr. Makeham's method.	6 17 Offices' Experience.	7 Carlisle.	Age x
77	21432	21512	21396	21137	21661	20959	22315	77
8	19101	19045	18981	18700	19242	18623	19918	8
9	16756	16699	16670	16366	16912	16379	17750	9
80	14476	14477	14482	14141	14693	14249	15649	80
1	12439	12383	12460	12053	12604	12248	13744	1
2	10451	10426	10614	10120	10662	10394	11905	2
3	8650	8639	8937	8365	8883	8697	10230	3
4	6861	7034	7414	6812	7279	7168	8686	4
5	5626	5635	6032	5466	5856	5805	7307	5
6	4410	4452	4784	4327	4618	4616	6026	6
7	3452	3474	3671	3390	3563	3590	4860	7
8	2700	2671	2701	2623	2684	2728	3810	8
9	1932	2032	1886	1984	1970	1999	2972	9
90	1563	1517	1235	1443	1404	1414	2332	90
1	1204	1093	748	1018	970	956	1724	1
2	827	751	413	696	648	611	1232	2
3	455	487	205	443	417	363	887	3
4	152	285	90	259	257	197	657	4
5	152	140	34	127	152	95	493	5
6	108	51	11	46	85	40	378	6
7	...	9	...	8	...	14	296	7
						4	230	8

The next step was to compute the values of a_x, A_x, and P_x, relating to each table, and these are given in Tables II, III, and IV, at 3 per-cent interest.

TABLE II.—*Annuity Values at 3 per-cent Interest* (a_x).

Age x.	1 20 Offices' HM, unadjusted.	2 20 Offices' HM, Mr. Woolhouse's adjustment.	3 20 Offices' HM, Mr. Berridge's adjustment.	4 20 Offices' HM, Mr. Finlaison's adjustment.	5 20 Offices' HM, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
20	22·056	22·043	22·061	22·029	22·013	21·797	21·694	20
1	21·851	21·848	21·850	21·830	21·827	21·616	21·504	1
2	21·665	21·656	21·638	21·631	21·636	21·430	21·304	2
3	21·456	21·460	21·425	21·429	21·439	21·239	21·098	3
4	21·269	21·254	21·211	21·219	21·236	21·043	20·885	4
5	21·060	21·038	20·995	21·002	21·028	20·842	20·665	5
6	20·804	20·814	20·775	20·776	20·814	20·635	20·442	6
7	20·578	20·582	20·552	20·546	20·593	20·423	20·212	7
8	20·333	20·347	20·323	20·310	20·372	20·205	19·981	8
9	20·108	20·109	20·094	20·072	20·136	19·982	19·761	9
30	19·865	19·867	19·858	19·829	19·898	19·754	19·556	30
1	19·631	19·623	19·617	19·584	19·654	19·519	19·348	1
2	19·370	19·373	19·370	19·332	19·404	19·279	19·134	2
3	19·119	19·117	19·119	19·075	19·147	19·032	18·910	3
4	18·857	18·855	18·860	18·812	18·884	18·780	18·675	4
5	18·593	18·587	18·596	18·542	18·615	18·521	18·433	5
6	18·310	18·314	18·327	18·269	18·340	18·256	18·183	6
7	18·028	18·037	18·051	17·990	18·059	17·983	17·928	7
8	17·747	17·756	17·768	17·707	17·771	17·703	17·669	8
9	17·470	17·469	17·479	17·419	17·478	17·417	17·405	9
40	17·187	17·176	17·184	17·124	17·179	17·123	17·143	40
1	16·880	16·876	16·883	16·821	16·873	16·821	16·890	1

TABLE V.—*continued.*

Age at Entry.	Duration of Policy.	Twenty Offices—Healthy Males. Mr. Woolhouse's adjustment.	Unadjusted.		Mr. Berridge's method.		Mr. Finlaison's method.		Mr. Makeham's method.		Institute H^M.		17 Offices.		Carlisle.	
43	20	39·478	39·621	100·3	39·682	100·5	39·643	100·4	39·401	99·8	39·975	101·3	39·571	100·2	37·965	96·2
	30	59·171	58·981	99·7	59·232	100·1	59·369	100·3	58·934	99·6	59·381	100·3	58·900	99·5	56·527	95·5
	40	75·204	75·246	100·1	75·058	99·8	75·292	100·1	75·155	99·9	75·303	100·1	74·822	99·5	71·730	95·4
	50	86·221	86·221	100·0	89·464	103·7	86·476	100·3	86·530	100·4	86·453	100·3	87·421	101·4	82·014	95·1
45	5	11·785	11·808	100·2	11·890	100·8	11·858	100·6	11·855	100·6	12·826	108·8	11·972	101·5	10·753	91·2
	10	22·648	22·777	100·6	22·866	101·0	22·773	100·5	22·688	100·2	23·387	103·3	22·847	100·9	21·991	97·1
	15	33·844	33·963	100·4	34·115	100·8	33·996	100·5	33·767	99·8	34·390	101·6	33·928	100·3	33·360	98·6
	20	44·800	44·816	100·1	45·164	100·8	44·990	100·4	44·732	99·8	45·191	100·9	44·846	100·1	42·692	95·3
	30	65·028	64·813	99·7	64·784	99·6	65·143	100·2	64·661	99·4	65·241	100·3	64·397	99·0	62·889	96·7
	40	79·025	79·017	99·9	81·023	102·5	79·032	100·0	79·788	100·9	79·176	100·2	79·699	100·8	76·428	96·7
50	5	14·001	14·128	100·9	14·140	101·0	14·077	100·6	13·987	99·9	14·824	105·9	14·056	100·4	14·195	101·4
	10	26·474	26·591	100·4	26·686	100·8	26·588	100·4	26·337	99·5	27·080	102·3	26·425	99·8	26·721	100·9
	15	38·678	38·682	100·0	39·009	100·8	38·843	100·4	38·559	99·7	39·112	101·1	38·610	99·8	37·004	95·7
	20	50·502	50·232	99·5	50·554	100·1	50·686	100·4	50·185	99·4	50·758	100·5	50·062	99·1	48·729	96·5
	30	70·065	70·094	100·1	69·873	99·7	70·138	100·1	69·989	99·9	70·186	100·2	69·534	99·2	66·753	95·3
	40	83·508	83·496	100·0	87·460	104·7	83·801	100·3	83·876	100·5	83·791	100·3	84·942	101·7	78·946	94·6
55	5	16·552	16·560	100·1	16·666	100·7	16·628	100·5	16·413	99·2	17·242	104·2	16·463	99·4	16·577	100·2
	10	30·437	30·337	99·7	30·708	100·9	30·579	100·5	30·316	99·6	30·930	101·6	30·332	99·6	28·311	93·0
	15	43·889	43·497	99·1	43·862	99·9	44·062	100·4	43·539	99·2	44·180	100·7	43·365	98·8	41·693	95·0
	20	56·071	55·723	99·4	55·642	99·3	56·155	100·2	55·584	99·1	56·339	100·5	55·168	98·4	53·709	95·8
	30	73·809	73·756	99·9	76·279	103·3	73·781	99·9	74·764	101·3	73·999	100·2	74·606	101·1	70·734	95·8
60	5	19·174	19·060	99·4	19·391	101·1	19·287	100·6	19·167	100·0	19·750	103·0	19·159	99·9	16·588	86·5
	10	34·851	34·395	98·7	34·740	99·7	35·009	100·5	34·551	99·1	35·191	101·0	34·327	98·5	32·202	92·4
	15	49·047	48·643	99·2	48·484	98·8	49·112	100·1	48·564	99·0	49·360	100·7	48·063	98·0	46·223	94·2
	20	60·786	60·768	99·9	60·426	99·4	60·834	100·1	60·757	99·9	60·947	100·3	60·119	98·9	56·206	92·5
	30	78·609	78·563	99·9	83·808	106·6	78·972	100·5	79·135	100·7	78·984	100·5	80·527	102·4	72·445	92·2
65	5	22·556	22·121	98·1	22·243	98·6	22·662	100·5	22·194	98·4	22·961	101·8	21·969	97·4	21·675	96·1
	10	39·493	39·094	99·0	38·680	97·9	39·506	100·1	38·914	98·5	39·866	100·9	38·349	97·1	37·921	96·0
	15	53·498	53·540	100·1	52·960	99·0	53·506	100·0	53·461	99·9	53·690	100·3	52·725	98·6	49·487	92·5
	20	64·154	64·129	99·9	67·476	105·2	64·055	99·9	65·537	102·2	64·419	100·4	65·324	101·8	60·941	95·0
70	5	25·907	25·807	99·6	25·218	97·3	25·857	99·8	25·517	98·5	26·368	101·8	25·048	96·7	24·531	94·7
	10	43·134	43·481	100·8	42·710	99·0	43·093	99·9	43·333	100·5	43·372	100·5	42·601	98·8	38·653	89·6
	15	56·242	56·438	100·4	60·490	107·6	56·081	99·7	58·121	103·4	56·569	100·6	57·983	103·1	52·637	93·6
	20	69·289	69·420	100·2	76·960	111·1	69·765	100·7	70·185	101·3	69·841	100·8	72·311	104·3	61·624	88·9

TABLE IV.—*continued.*

Age x.	1 20 Offices' H^M, unadjusted.	2 20 Offices' H^M, Mr. Woolhouse's adjustment.	3 20 Offices' H^M, Mr. Berridge's adjustment.	4 20 Offices' H^M, Mr. Finlaison's adjustment.	5 20 Offices' H^M, Mr. Makeham's adjustment.	6 17 Offices.	7 Carlisle.	Age x.
43	·02884	·02885	·02881	·02904	·02886	·02903	·02838	43
4	·02999	·02996	·02990	·03016	·02997	·03014	·02925	4
5	·03114	·03114	·03106	·03134	·03112	·03133	·03018	5
6	·03238	·03238	·03229	·03258	·03236	·03258	·03117	6
7	·03370	·03367	·03360	·03390	·03366	·03391	·03225	7
8	·03504	·03504	·03488	·03528	·03504	·03530	·03343	8
9	·03650	·03648	·03644	·03673	·03649	·03678	·03476	9
50	·03802	·03801	·03800	·03828	·03803	·03835	·03622	50
1	·03960	·03963	·03965	·03992	·03967	·04000	·03785	1
2	·04128	·04136	·04140	·04166	·04141	·04176	·03957	2
3	·04316	·04320	·04327	·04353	·04325	·04361	·04140	3
4	·04521	·04516	·04525	·04551	·04518	·04558	·04335	4
5	·04738	·04725	·04736	·04761	·04726	·04767	·04545	5
6	·04945	·04946	·04960	·04985	·04947	·04988	·04771	6
7	·05175	·05182	·05199	·05223	·05181	·05224	·05015	7
8	·05427	·05434	·05453	·05477	·05429	·05474	·05272	8
9	·05698	·05702	·05723	·05748	·05693	·05741	·05532	9
60	·06003	·05987	·06012	·06036	·05974	·06025	·05790	60
1	·06295	·06291	·06319	·06342	·06276	·06328	·06032	1
2	·06613	·06613	·06645	·06668	·06596	·06649	·06283	2
3	·06955	·06956	·06994	·07013	·06937	·06992	·06551	3
4	·07320	·07319	·07365	·07382	·07301	·07357	·06847	4
5	·07709	·07705	·07760	·07775	·07690	·07745	·07171	5
6	·08123	·08120	·08181	·08195	·08106	·08159	·07528	6
7	·08568	·08566	·08630	·08648	·08548	·08599	·07924	7
8	·09062	·09049	·09108	·09139	·09024	·09068	·08363	8
9	·09565	·09576	·09618	·09669	·09530	·09567	·08853	9
70	·10083	·10148	·10160	·10246	·10072	·10100	·09398	70
1	·10777	·10766	·10738	·10865	·10654	·10668	·10013	1
2	·11528	·11425	·11354	·11523	·11276	·11274	·10651	2
3	·12165	·12119	·12010	·12210	·11939	·11921	·11285	3
4	·12970	·12840	·12710	·12933	·12654	·12612	·11897	4
5	·13489	·13585	·13456	·13688	·13417	·13352	·12444	5
6	·14338	·14382	·14251	·14492	·14234	·14143	·13019	6
7	·15171	·15230	·15102	·15353	·15112	·14991	·13591	7
8	·16190	·16138	·16013	·16282	·16049	·15900	·14216	8
9	·17196	·17121	·16991	·17270	·17056	·16875	·14971	9
80	·18198	·18174	·18046	·18300	·18131	·17924	·15728	80
1	·19413	·19277	·19237	·19369	·19286	·19053	·16624	1
2	·20534	·20399	·20618	·20457	·20518	·20278	·17505	2
3	·21700	·21543	·22242	·21561	·21840	·21616	·18492	3
4	·22225	·22676	·24155	·22707	·23252	·23091	·19539	4
5	·23825	·23834	·26104	·23920	·24765	·24734	·20732	5
6	·24858	·25096	·29024	·25274	·26396	·26577	·21883	6
7	·26302	·26563	·32042	·26942	·28143	·28658	·22905	7
8	·28425	·28282	·35470	·29058	·30026	·31029	·23571	8
9	·28813	·30570	·39325	·316[illegible]5	·32077	·33740	·24373	9
90	·33571	·33585	·43572	·34300	·34344	·36837	·25669	90
1	...	·37294	·48189	·37704	·36928	·40404	·24818	1
2	...	·41954	·53148	·42434	·40028	·44498	·25093	2
3	...	·48338	·58497	·48338	·44063	·49143	·24209	3
4	...	·56719	·64608	·56719	·50162	·54302	·23857	4
5	...	·67748	·73372	·67748	·61814	·59885	·23705	5
6	...	·81954	·97087	·81954	...	·65219	·25085	6
7	...	·97087	...	·97087	...	·70014	...	7
						·77558	...	8

The first thing to be noted in Table II is the remarkable closeness in the annuity values as given by the unadjusted results, and Mr. Woolhouse's adjustment (Inst. H^M 3 per-cent), which in itself is another proof, if proof were necessary, of the satisfactory results that method of adjustment has yielded.

Mr. Finlaison's method gives annuity values throughout smaller than the unadjusted, and bearing in mind the purpose to which Mr. Finlaison's method is stated by him to have been applied, this is a point in itself of considerable practical importance.

As a further verification of the effect of an adjustment by this method, it should be stated that it has been applied to the experience for the graduation of which it was devised, and has been found to yield the like results.

The following table gives the means for a general comparison of the effect of the various methods of graduation upon annuity values.

Table of Annuity Values (a_x—3 per-cent) H^M.

Age (x)	Unadjusted.	Woolhouse.	Finlaison.	Berridge.	Makeham.
20	22·056	22·012	22·029	22·061	22·013
30	19·865	19·867	19·829	19·858	19·898
40	17·187	17·176	17·124	17·184	17·179
50	13·892	13·896	13·836	13·898	13·889
60	10·216	10·236	10·175	10·205	10·252
70	6·695	6·656	6·600	6·649	6·701
80	3·737	3·742	3·714	3·771	3·752

For the purpose of comparing the policy reserves brought out by the employment of the various tables, the following table has been constructed, which gives the value of 100 $\{A_{x+n}-P_x(\frac{1}{2}+a_{x+n})\}$ for each table, for various ages of entry and for various durations of policies; and there is placed alongside each a column showing the percentages which the policy reserves by other tables, bear to those based on Mr. Woolhouse's adjustment. In any application of this table, it should always be carefully borne in mind that it does not give all the information required as regards with profit policies having vested reversionary bonus additions, because there is no doubt, as will be seen further on, that the value of such bonus additions will in general have an appreciable effect in any comparison of the actual reserves required by the various tables for the liabilities of an office, where such reversionary bonus additions exist.

to be adjusted. In this case, also, the arithmetical work is best performed by a continuous scheme; and if the differences are taken as before, it will be found that the series of

$$\Delta^3 l_x = [3(l'_{x-1} - l'_{x+4}) + (l'_{x+9} - l'_{x-6})] \times \cdot 008.$$

It is, however, preferable to work on the number dying at each age; for the method is equally applicable to p_x and q_x, or l_x and d_x. Sometimes Mr. Woolhouse's method fails to bring out a regular series in cases where only a small number of lives are involved; but when actuaries have to perform actual work they are obliged to have a regular series. In most instances, the method he had described brings out a series which progresses regularly, and of which the third difference comes to a very small quantity; and the results are practically always very close to the original values.

Mr. P. GRAY said that, many years ago, when entering on the study of life contingencies, along with his friend, Mr. Ambrose Smith, they sought to verify Mr. Finlaison's method of adjustment. But all their attempts were futile, and their results had but little resemblance to those in the report. They were not a little discouraged by their failure, and there was no doubt other beginners had been similarly discouraged. He had been informed very recently by Mr. A. J. Finlaison, that there was reason to believe that Mr. J. Finlaison had subjected his data to some modification before applying his method of adjustment. If so, this certainly ought to have been stated in the report, since the omission of it could hardly fail to have had a prejudicial effect on Mr. Finlaison's reputation as a computer. The form in which he presented his adjusted value of p_x was (using modern notation, Report p. 10):—

$$p_x = \frac{5p'_x + 4(p'_{x+1} + p'_{x-1}) + 3(p'_{x+2} + p'_{x-2}) + 2(p'_{x+3} + p'_{x-3}) + (p'_{x+4} + p'_{x-4})}{25},$$

When otherwise arranged, the significance of this expression becomes more apparent, thus:—

$$p_x = \frac{p'_{x-4} + 2p'_{x-3} + 3p'_{x-2} + 4p'_{x-1} + 5p'_x + 4p'_{x+1} + 3p'_{x+2} + 2p'_{x+3} + p'_{x+4}}{25},$$

it being understood that the probabilities in the numerator are those given by the observation. Each adjusted probability is here made to depend upon nine of the observed probabilities, that for the age under treatment being the central term, while the others are ranged in order on both sides of it. The central term is affected with the coefficient 5, and those on both sides of it are symmetrically affected with the coefficients 4, 3, 2, 1, respectively, as they recede from the central term. The sum of the coefficients is 25, which, therefore, forms the denominator of the expression. The expression seems to promise well for the purpose in view, being a sort of average of nine adjoining terms, in which each is weighted in accordance with its proximity to the central term. But it is subject to two drawbacks, the first of which is the labour involved in its application. It was stated by Mr. John Finlaison (Report, p. 10) and by Mr. A. G. Finlaison (Report, p. 25, August 1860) to be laborious, and he (Mr. Gray) in working upon it with Mr. Smith found it, in the way in which they applied it, to have been truly so described. But a most effectual remedy had

since been found for this inconvenience. Dr. Wittstein, in a paper in vol. 17 of the *Journal of the Institute* (translated by Mr. Sprague) had shown how Mr. Finlaison's function might be formed in almost the easiest manner conceivable. The other drawback is one that Dr. Wittstein seems to have overlooked. Whatever may be said in favour of Mr. Finlaison's expression, as affording a highly probable approximation to the value sought, is, on consideration, seen to be applicable only when the terms entering it are each the result of observations directed to the determination of the same value; while here the terms involved are those of successive values, forming generally an increasing or a decreasing series, according as the probability of dying or that of living was the function employed. In consequence, if, as here, the series treated partake at all of a geometrical character, the influence of the terms on one side of the central term will always be greater than the influence of those on the other side of it; and the result in a series of probabilities thus formed will be a constant tendency to deviation in the one direction or the other, according to circumstances. Mr. A. G. Finlaison informs us in his report (p. 38) that he noticed this result in the excess of the mortality given by his adjusted probabilities over that given by the unadjusted; and he (Mr. Gray) had also found it while working on the H^M table, by comparison with Mr. Woolhouse's results, whose method of adjustment was certainly not subject to any such source of error. Mr. Finlaison equalized the deaths according to the two series by multiplying his probabilities by a constant factor. Whether this was the best way, or a legitimate way, of correcting the error of his process, might be matter for debate; but the method of adjustment under consideration, when applied in the manner shown by Dr. Wittstein, is so delightfully simple that it is most desirable that some legitimate method of rendering it more generally available may be found.

Mr. Walford said that when early mortality tables were made, there was no accurate record of the ages of the lives on which they were based, the ages being simply those given by friends; and, of course, graduation became a necessity. And so any table of mortality based upon insured lives in small numbers would present variations at certain ages, both young and old, which would require the same process of graduation. When large masses of lives are got, with regard to whose ages there can be no question, as they will be eventually by reason of registration, actuaries will be able to obtain real results from raw materials, and the less such tables are submitted to graduation, the better for all concerned. Such tables would afford the means of actually testing whether the climacteric periods which are believed in, are actually true or otherwise. He himself believed that there is something in them. But in England, as the Registration Act prolongs its operation year by year, and as the certainty regarding age increases, we shall have the means once for all of settling the question. But when it is settled, there certainly will be a little difficulty in following strictly the ups and downs. It would not do, for instance, to charge a man a smaller premium at 56 than at 55 or 50. Actuaries will have to consider those questions when they arise, and decide how they shall be dealt with. Probably the climacteric periods will be very few, and be capable of being softened down in such a way as not to present any great difficulties in calculating annuities from them.

Mr. Berridge said that since he had made his graduation, he had come to the conclusion that an average between two graduations was better than any one graduation which could be devised. That is very strongly brought out in Mr. Woolhouse's method, which is really an average of five series. Mr. Sutton has taken the average of two series; and even much more simple methods of graduation would give better results, if there was an average of two series, than would be obtained from any one process, provided that there was no assumed law involved in making the averages. If an arithmetical mean is introduced in any part of the calculation, there is a slight error. Mr. Woolhouse, instead of introducing an arithmetical mean, introduces a mean which is the result of the second or third difference of the curve, as the case might be, and he therefore avoids that difficulty. Mr. Woolhouse's method follows the curve exactly; in fact, the late Mr. Samuel Brown had complained to him (Mr. Berridge) that it was no graduation at all, because it was too near the original.

Mr. Hardy said that he noticed with very great satisfaction the new title under which Mr. Sutton appeared before them that evening. Mr. Sutton held the certificate of the Institute, and had been through its curriculum; and therefore it was very gratifying to see him appointed to a very important office under Government. He (Mr. Hardy) had always looked upon the method of graduation employed as almost as important as the facts; and he might say that the facts took the shape of the graduation rather than preserved their original features. Probably the climacteric periods mentioned by Mr. Walford are brought about by human institutions. For instance, we regard a man as of age at 21, which is the third period of seven years; and perhaps the periods are affected by social laws rather than by anything in nature itself which makes any particular period dangerous to life. He thought that graduation was an endeavour to apply science to discover the real law that is supposed to underlie the facts. The facts are not in sufficient quantity at all ages to enable us to get the true law, and methods of graduation are attempts to bring out such a law from the indications that the facts present.

Mr. Bailey—Graduation assumes a law, and bends the facts to it.

Mr. Sutton, in reply, said that the present paper dealt only with what some persons might consider to be the unimportant portion of the subject. On taking the lives existing at the close of the observation over which the Institute experience extended, and treating them as his average office, he found that the results, on the assumption of the lives entering and existing at the various ages all assuring for the same amount, did not agree with a summary which he had made of the sixth-schedule returns of nineteen out of the twenty offices which had contributed their experience, the percentage of risk at the various ages of the total sum assured not tallying at all with those brought out on the assumption of the lives assuring for the same amount. That necessitated, to a great extent, the rewriting and reconsideration of that portion of the paper; and might be taken as an explanation of the fact that there was only one part of the paper before them. He was strongly of Mr. Bailey's opinion that, in all cases where they had large masses of figures to deal with, as in the Institute experience, the facts should be taken as they are. He

thought that it was the greatest compliment that could be paid to Woolhouse's method of graduation to compare it with the facts, not comparing, however, the numbers living, or probabilities of living, or probabilities of dying, or mean duration of life, but taking actual annuity values, which are the starting point for all practical purposes. With regard to Mr. John Finlaison's graduated Government life annuities, he might say that he had been himself in the same unfortunate position as Mr. Gray had described. He had tried for many hours by twisting and turning about the rough figures given by Mr. Finlaison, to ascertain how he had got his graduated results, and he had not succeeded. Among other attempts which he made, one was the very method of construction of the table by differencing, which Mr. A. J. Finlaison had called attention to that evening; but up to the present he had been unsuccessful. He had used Mr. John Finlaison's method many times, and found it extremely easy, and was very much puzzled to find him speaking of it as laborious, as compared with the other method. Mr. Gray had mentioned that the method had appeared in the *Journal* in a translation of the German paper by Dr. Wittstein. It seemed rather remarkable that, although Dr. Wittstein's paper was written some forty years, at least, after Mr. John Finlaison's method was known, no mention was made in it of the latter. Dr. Wittstein was probably under the impression that the method recommended by him was a new one. Whether the financial effect pointed out on p. 183 would obtain, if instead of the probabilities of living were taken the numbers living, he was not sure, but he was rather inclined to think that it would not.

Mr. A. J. Finlaison said that it would make a difference at the end of the table where the numbers were larger. Therefore, it would make the annuity value larger.

Mr. Sutton said that if they could always rely on the values on each side of the central value being some smaller and some larger, he was not sure that it would not be so; but, in its present form, with regard to the p_x, it was not so. In the paper now before the Institute, he had carefully endeavoured to abstain from raising any discussion as to the comparative merits of various methods of graduation. The sole point under consideration was to compare their financial effects, and it was from no other point that he had endeavoured to deal with them.

The President said that as they had the advantage of having present that evening Herr Wilhelm Lazarus, of Germany, one of their foreign members, they would be glad to hear a few remarks from him.

Herr Wilhelm Lazarus said that he had no doubt that English actuaries took an interest in what actuaries were doing in Germany in the way of scientific work, and they might have wondered that they did not see in Germany an Institute of a somewhat similar kind to that which had existed in Great Britain for so long a time. In Germany, however, they laboured under such great difficulties that it was not possible to establish an Institute there. They tried to do so in the year 1865, but the attempt was unsuccessful. The actuaries are not united together in a particular town, as is the case in France and in England, but are spread over the country in such a way that they cannot hold meetings. In 1865, a society, whose name might be translated as "The Association for the Science of Life Insurance",

was established, but soon faded away. But before it died, or fell asleep—for he did not know whether it was really dead—it gave birth to a work which was now going forward, and a work in which they had been guided by an example which the Institute of Actuaries had already twice given.* The mortality experience of the German insurance companies had been worked out, and perhaps it would interest the meeting to hear some of the particulars of the work. It was in the year 1868 that the plan was formed by the Association, and it was adopted in the following year, and sent to the companies, who, after some hesitation, agreed to give all their experience in nearly the same manner in which it had been given in England; so that they would have the experience of lives and not policies, together with the strict dates of birth and death up to the end of 1870. In 1875 the cards were sent back by the companies; but then another delay took place. It was proposed to bring these cards up to a later date by means of the experience which had been collected in the meantime; and last summer the cards came in with the experience up to the end of 1875. There are four different sorts of cards, relating to healthy lives, males and females; and diseased lives, males and females. Then there are other cards, which refer to a kind of industrial assurance, where the medical examination is not made so accurately; but these have been separated from the others, because it is expected that they will show a different mortality experience. And then there are cards relating to cases in which no medical examination has been made at all. The whole number of cards amounts to 720,511. The numbers are:—Healthy lives—males, 282,000; females, 101,000. Diseased lives—males, 76,000; females, 25,000. For the industrial business there were about as many males, and 76,000 females; and in endowment and annuity insurances there were about 20,000 of each sex. It is expected that 100,000 additional cards will be received from societies which have not yet sent them, but have promised to do so. The work is now being carried on in Berlin, and it is hoped that when it is completed, there will be a mortality experience, so far as Germany was concerned, which may be relied upon as sufficient.

A Comparison of various Methods of Graduation of a Mortality Table, considered in reference to the Valuation of the Liability of an Average Life Office under its Assurance Contracts. By WILLIAM SUTTON, *Actuary to the Registry of Friendly Societies.*

PART II.—*On the Determination of an Average Life Office.*

[Read before the Institute, 30 April 1877.]

IN the first part of this paper, it has been shown to what extent the reserve under a single policy is affected by various methods of graduating the mortality experience on which such

* It seems desirable to point out that this is a mistake. The 17 Offices' Experience was taken out before the Institute was founded, and several actuaries took part in it who never joined the Institute.—ED. *J.I.A.*

reserve is based. We will now endeavour to obtain some notion of the financial effect upon the total reserve made by an office at a periodic valuation of its liabilities.

This can only be done, either by taking for our data those of individual offices, and comparing the results brought out, or by constructing an office which shall represent, with more or less accuracy, the case of actually existing offices. The first course, apart from its impracticability, would have the objection of only dealing with particular cases, in a matter where the particular cases may differ materially from one another, and would certainly differ materially from others which might have been chosen for the purpose; and to the second course the objection fairly applies that, in all probability, the average office as constructed does not exactly represent any existing individual office. As, however, an average office properly constituted would be a sort of average of all the varying circumstances of existing offices, it would on the whole show less deviation from any particular case, and consequently the results derived from such a supposition would require smaller adjustments to meet the case of any existing individual office.

When first considering this subject, one great inducement to write this paper arose from thinking that in the Institute experience the facts were given which would enable us at once to obtain what it is proposed throughout to call an average office. The total number of lives on the books of the 20 contributing offices at a given date is there given, together with their ages at entry and their ages at that date, and if the amounts for which they were respectively assured had been also given, the information would, it was considered, be quite sufficient for the purpose in view. In coming to this conclusion, the fact was not overlooked, that all the existing lives were not assured on the ordinary whole life scale, but included term assurances, contingent assurances, endowment assurances, &c. As we know, however, that these classes of business are only on the average a small percentage of the whole, it did not appear necessary to attach much importance to this point as regards the object in view. Of course no average office would be complete which did not take into account the vested reversionary bonuses attaching to the with-profit assurances, but an assumption to supply this could easily have been made which would, there is reason to think, have been sufficient for the purpose. As we all know, however, the Institute experience only gives particulars of the lives, not of the sums assured; and it was necessary, therefore, in the first instance to assume either that all persons who entered became assured for the same amount, or that the amount

for which a man effected his policy would depend upon his age at the time, which, speaking generally, would be tantamount to assuming that the older he was at the time of effecting his assurance, the larger would be the sum for which he would assure.

In the absence of any standard, it was decided to assume that the lives all entered for the same sum, and taking this sum to be 1, and omitting all lives entering under 20, the following table was constructed, which is merely the total lives existing at the various ages, Institute Experience, Healthy Males, at the close of the observations.

Table 1.—*Sum at risk at various Ages, on the assumption that all the Lives existing at close of Observations (Healthy Males—Institute Experience) were each assured for* 1.

Age.	Sum Assured.	Percentage of Total.	Age.	Sum Assured.	Percentage of Total.
20	65	·09	60	1,071	1·47
1	172	·24	1	993	1·36
2	295	·40	2	849	1·16
3	473	·65	3	807	1·11
4	643	·87	4	802	1·09
5	780	1·07	5	693	·95
6	992	1·36	6	605	·83
7	1,114	1·53	7	559	·77
8	1,341	1·84	8	496	·68
9	1,401	1·91	9	454	·62
30	1,617	2·23	70	383	·52
1	1,717	2·35	1	364	·50
2	1,767	2·43	2	266	·36
3	1,955	2·68	3	254	·35
4	1,996	2·72	4	245	·33
5	2,045	2·81	5	184	·25
6	1,995	2·73	6	174	·24
7	2,299	3·15	7	155	·21
8	2,171	2·97	8	110	·15
9	2,139	2·91	9	82	·11
40	2,192	3·00	80	87	·12
1	2,263	3·10	1	66	·09
2	2,247	3·08	2	45	·06
3	2,244	3·07	3	37	·05
4	2,289	3·11	4	26	·04
5	2,256	3·09	5	23	·03
6	2,133	2·92	6	15	·02
7	2,060	2·82	7	15	·02
8	2,053	2·81	8	8	·01
9	1,921	2·61	9	8	·01
50	1,798	2·46	90	2	·01
1	1,776	2·43	1	1	
2	1,638	2·24	2	2	
3	1,656	2·27	3	1	
4	1,499	2·04			
5	1,329	1·82			
6	1,352	1·85		73,101	100·
7	1,270	1·74			
8	1,147	1·57			
9	1,119	1·52			

Ages.	Sum Assured.	Percentage of Total.
20–24	1,648	2·25
25–29	5,628	7·71
30–34	9,052	12·41
35–39	10,649	14·57
40–44	11,235	15·36
45–49	10,423	14·25
50–54	8,367	11·44
55–59	6,217	8·50
60–64	4,522	6·19
65–69	2,807	3·85
70–74	1,512	2·06
75–79	705	·96
80–84	261	·36
85 & upwards	75	·09
	73,101	100·

A first glance at this table is quite sufficient to show that the sums assured at the younger ages are too large in proportion to those in the middle part of life, and would seem to point unmistakeably to the inference that, on the average, the sum assured by a new entrant does to some extent depend upon his age. The only information available to test this was the returns to the Board of Trade; but on referring to these, it was found that besides not stating the number of policies existing at each age, as well as the sums assured, the latter varied so considerably that it was a matter of great difficulty to make any use of individual returns. It was therefore decided to take the returns of nineteen out of the twenty offices,* and group the sums at the various ages together. It should be added that in the case of each office, the first return under the Life Assurance Companies Act was used, so that the time that had elapsed since the close of the Institute observations should be as short as possible. The results are given *in extenso* in the following table.

* The remaining office having amalgamated with another office, there was no return available.

Table 2.—*Summary of Board of Trade Returns of* 19 *Offices under Schedule* 6—*Question* 2.

Age.	With Profits.	Percentage of Total.	Without Profits.	Percentage of Total.	Total.	Percentage of Total.	Bonuses.	Per-cent of Bonus on Total Sum Assured.	Percentage of Total Sum Assured at Risk. With.	Without.	With & Without.
	£		£		£		£				
20	93,650	·11	16,884	·12	110,534	·11	162	·15			
21	158,600	·19	34,280	·25	192,880	·20	290	·15			
22	264,930	·32	47,465	·34	312,395	·32	1,808	·58	1·65	1·54	1·6
23	355,376	·43	49,036	·35	404,412	·42	731	·18			
24	491,443	·60	66,157	·48	557,600	·58	1,249	·22			
25	539,440	·66	84,542	·61	623,982	·65	2,072	·33			
26	780,450	·95	105,793	·76	886,243	·92	1,660	·19			
27	954,884	1·16	130,036	·94	1,084,920	1·13	3,447	·32	5·46	4·07	5·2
28	1,021,670	1·24	139,313	1·00	1,160,983	1·21	5,018	·43			
29	1,189,680	1·45	105,662	·76	1,295,342	1·35	7,473	·58			
30	1,264,944	1·54	126,747	·91	1,391,691	1·45	8,103	·58			
31	1,589,707	1·93	201,555	1·45	1,791,262	1·86	14,557	·81			
32	1,540,415	1·87	197,578	1·42	1,737,993	1·81	16,565	·95	9·76	6·73	9·3
33	1,688,589	2·05	180,035	1·29	1,868,624	1·94	22,194	1·19			
34	1,948,544	2·37	230,343	1·66	2,178,887	2·27	24,933	1·14			
35	2,017,805	2·45	246,450	1·77	2,264,255	2·35	29,843	1·32			
36	1,978,497	2·41	297,862	2·14	2,276,359	2·37	34,978	1·54			
37	1,965,905	2·39	264,182	1·90	2,230,087	2·32	43,599	1·96	12·30	9·29	11·8
38	2,065,843	2·51	247,157	1·78	2,313,000	2·41	50,381	2·18			
39	2,092,710	2·54	235,704	1·70	2,328,414	2·42	50,892	2·19			
40	2,259,972	2·75	190,858	1·37	2,450,830	2·55	61,826	2·52			
41	2,169,404	2·64	398,735	2·87	2,568,139	2·67	66,426	2·59			
42	2,194,983	2·67	356,994	2·57	2,551,977	2·65	67,756	2·66	13·54	11·97	13·3
43	2,236,307	2·72	327,709	2·36	2,564,016	2·67	85,325	3·33			
44	2,271,034	2·76	389,658	2·80	2,660,692	2·77	86,169	3·24			
45	2,330,464	2·83	289,165	2·08	2,619,629	2·72	97,635	3·73			
46	2,415,875	2·94	387,938	2·79	2,803,813	2·92	105,999	3·78			
47	2,284,587	2·78	354,354	2·55	2,638,941	2·74	113,090	4·29	14·05	12·88	13·8
48	2,213,429	2·69	380,514	2·74	2,593,943	2·70	121,337	4·6[illegible]			
49	2,309,890	2·81	378,420	2·72	2,688,310	2·80	126,904	4·72			
50	2,211,536	2·69	471,349	3·39	2,682,885	2·79	136,911	5·10			
51	2,210,094	2·69	413,999	2·98	2,624,093	2·73	160,783	6·13			
52	2,220,004	2·70	299,385	2·15	2,519,389	2·62	162,179	6·44	13·12	13·92	13·2
53	2,119,770	2·58	458,537	3·30	2,578,307	2·68	163,537	6·34			
54	2,021,382	2·46	292,415	2·10	2,313,797	2·41	177,000	7·65			
55	1,960,225	2·38	387,008	2·78	2,347,233	2·44	188,257	8·02			
56	1,846,378	2·24	404,559	2·91	2,250,937	2·34	178,610	7·93			
57	1,693,186	2·06	317,888	2·29	2,011,074	2·09	171,781	8·54	10·72	12·75	11·0
58	1,612,887	1·96	404,646	2·91	2,017,533	2·10	188,540	9·35			
59	1,707,656	2·08	260,024	1·86	1,967,680	2·05	187,643	9·54			
60	1,512,469	1·84	244,312	1·76	1,756,781	1·83	176,114	10·02			
61	1,358,128	1·65	179,058	1·29	1,537,186	1·60	179,799	11·70			
62	1,342,619	1·63	264,243	1·90	1,606,862	1·67	192,821	12·00	7·86	9·27	8·07
63	1,179,491	1·43	412,734	2·97	1,592,225	1·66	184,765	11·60			
64	1,075,315	1·31	188,156	1·35	1,263,471	1·31	178,995	14·17			

TABLE 2—(*continued*).

Age.	With Profits.	Per-centage of Total.	Without Profits.	Per-centage of Total.	Total.	Percentage of Total.	Bonuses.	Per-cent of Bonus on Total Sum Assured.	Percentage of Total Sum Assured at Risk. With.	Without.	With and Without.
	£		£		£		£				
65	1,039,905	1·26	209,902	1·51	1,249,807	1·30	178,042	14·25			
66	1,064,081	1·29	236,683	1·70	1,300,764	1·35	190,962	14·68			
67	903,862	1·10	165,199	1·19	1,069,061	1·11	163,532	15·30	5·52	6·64	5·68
68	769,526	·91	157,001	1·13	926,527	·96	151,146	16·31			
69	765,411	·93	154,624	1·11	920,035	·96	161,013	17·50			
70	730,788	·89	235,348	1·69	966,136	1·00	165,113	17·09			
71	617,096	·75	182,208	1·31	799,304	·83	124,737	15·61			
72	630.095	·77	124,739	·90	754,834	·79	138,217	18·31	3·53	6·02	3·88
73	474,828	·58	168,476	1·21	643,304	·67	117,861	18·32			
74	442,800	·54	127,121	·91	569,921	·59	118,039	20·71			
75	403,730	·49	146,279	1·05	550,009	·57	108,658	19·76			
76	265,909	·32	131,933	·95	397,842	·41	65,090	16·36			
77	295,571	·36	69,062	·50	364,633	·38	89,977	24·68	1·63	3·26	1·86
78	224,729	·27	48,802	·35	273,531	·28	61,910	22·63			
79	154,408	·19	57,044	·41	211,452	·22	48,655	20·65			
80	181,968	·22	53,179	·38	235,147	·24	49,395	21·01			
81	123,019	·15	45,253	·33	168,272	·18	35,279	20·97			
82	94,298	·11	45,483	·33	139,781	·15	29,787	21·31	·66	1·32	·78
83	93,180	·11	29,349	·21	122,529	·14	208,31	17·00			
84	57,999	·07	9,925	·07	67,924	·07	19,524	28·74			
85	48,914	·06	13,631	·10	62,545	·07	16,618	26·57			
86	24,300	·03	7,889	·06	32,189	·03	8,923	27·72			
87	22,849	·03	8,000	·06	30,849	·03	6,399	20·74			
88	22,498	·03	9,390	·07	31,888	·03	9,805	30·75			
89	8,300	·01	3,175	·02	11,475	·01	2,231	19·44			
90	12,750	·02	100	...	12,850	·01	2,989	22·87	·20	·34	·20
91	8,100	·01	4,700	·03	12,800	·01	4,061	31·73			
92	1,300	...	300	...	1,600	...	318	19·88			
93	6,000	·01	...	...	6,000	·01	2,851	47·52			
94	1,000	...	...	...	1,000	...	412	41·20			
95	...	...	100	...	100	...	...	...			
97	250	...	...	...	250	...	119	47·60			
101	..	...	300	...	300	...	...				
	82,249,631	...	13,904,634	...	96,154,265	...	5,967,601	6·21	100·	100·	100·

Before making any comparison between these results and those of Table 1, it will be well to call attention to two or three points in reference to them. In the first place, the sums stated are in general not less reassurances; and in the second place, a difference of practice also will be found in dealing with lives not accepted at the ordinary rate, some offices giving the lives at their true age,

and treating the addition to the premium as an extra premium, while other offices have given the lives at their rated-up ages, whereas the figures of Table 1 are deduced from lives accepted at the tabular rate of premium.

The following table shows the percentage of the total sum assured at risk for every group of 5 ages, according to Table 1 and according to Table 2.

Table 3.—*Showing percentage of Total Sum Assured at risk at various Present Ages.*

Ages.	According to Institute experience (Lives).	According to Board of Trade Returns of 19 Offices.
20–24	2·25	1·63
25–29	7·71	5·26
30–34	12·41	9·34
35–39	14·57	11·87
40–44	15·36	13·31
45–49	14·25	13·88
50–54	11·44	13·23
55–59	8·50	11·02
60–64	6·19	8·07
65–69	3·85	5·68
70–74	2·06	3·88
75–79	·96	1·86
80–84	·36	·77
85 & upwards	·09	·20
	100·	100·

A comparison of these results certainly tends to show that, assuming the lives to enter at the various ages in the proportions indicated by the Institute experience, the further assumption that they all assure for the same amount on the average, irrespective of age, would not be borne out by the figures. Thinking it possible that the reassurances may have some effect on the results, it was thought desirable to have recourse to the Board of Trade returns again, and take out the amounts of the reassurances at the various ages, and these are given in the following table:—

TABLE 4.—*Whole-Life Reassurances—Board of Trade Returns of* 19 *Offices*.

Present Ages.	Reversionary Bonus.	Sums Assured (With and Without Profits).
	£	£
20–24	814	127,650
25–29	1,070	212,775
30–34	10,241	389,664
35–39	11,877	561,925
40–44	19,816	860,252
45–49	32,592	903,230
50–54	50,584	799,885
55–59	56,989	848,150
60–64	34,062	458,600
65–69	22,747	192,225
70–74	29,475	172,525
75–79	3,808	74,250
80 & uprds.	518	13,950
	274,593	5,615,081

After deducting these from the amounts given at the corresponding ages in Table 2, we get the following as the net total sums at risk for groups of five ages.

TABLE 5.—*Board of Trade Returns of* 19 *Offices—Whole Life—With and Without Profits*.

Present Ages.	Total Sum Assured.	Reassurances.	Net Sum Assured.	Percentage of Total.	Reversionary Bonus.	Reassurances.	Net Reversionary Bonus.
	£	£	£		£	£	£
20–24	1,577,821	127,650	1,450,171	1·60	4,240	814	3,426
25–29	5,051,470	212,775	4,838,695	5·35	19,670	1,070	18,600
30–34	8,968,457	389,664	8,578,793	9·48	86,352	10,241	76,111
35–39	11,412,115	561,925	10,850,190	11·98	209,693	11,877	197,816
40–44	12,795,654	860,252	11,935,402	13·18	367,502	19,816	347,686
45–49	13,344,636	903,230	12,441,406	13·75	564,965	32,592	532,373
50–54	12,718,471	799,885	11,918,586	13·16	800,410	50,584	749,836
55–59	10,594,457	848,150	9,746,307	10·76	914,831	56,989	857,842
60–64	7,756,525	458,600	7,297,925	8·07	912,494	34,062	878,432
65–69	5,466,194	192,225	5,273,969	5·82	844,695	22,747	821,948
70–74	3,733,499	172,525	3,560,974	3·93	663,967	29,475	634,492
75–79	1,797,467	74,250	1,723,217	1·90	369,290	3,808	365,482
80–84	733,653	9,950	723,703	·80	154,816	518	154,298
85–89	168,946	1,000	167,946	·19	43,976	...	43,976
90–94	34,250	3,000	31,250	·03	10,581	...	10,581
95–99	650	...	650	...	119	...	119
	96,154,265	5,615,081	90,539,184	...	5,967,601	274,593	5,693,008

If now we proceed to endeavour to make the figures given by the Institute experience, agree with these, it will be seen that it is

necessary to make some assumption as to the amount of the policy depending upon the age at the time of its being effected. For this purpose it will be sufficient to take the various groups of ages at entry, 20–29, 30–39, &c.

Taking, then, the 73,101 lives of Table 1, we get the following results.

TABLE 6.—*Institute Experience, Healthy Males—Total Sum Assured at risk at various Present Ages, classified according to Ages at Entry.*

Ages at Entry.	Present Ages.						
	20-29	30-39	40-49	50-59	60-69	70-79	Upwards.
20–29	7,276	9,260	3,907	1,210	376	35	6
30–39	...	10,441	12,038	5,299	1,857	437	20
40–49	...	...	5,713	6,079	2,631	699	86
50–59	...	...	...	1,996	1,996	685	129
60–69	...	...	...	...	469	327	71
70–	...	...	...	...	...	34	24
	7,276	19,701	21,658	14,584	7,329	2,217	336

whereas the corresponding totals at present ages given by Table 5 would be

5,084	15,688	19,679	17,487	10,155	4,262	746

If we assume, taking 1 as our standard policy, that those who enter between 20–29 assure for ·70

30–39 „ ·883

40–49 and upwards „ 1·486

then Table 6 would take the following form.

TABLE 7.—*Existing Sums assured at risk—Institute Experience, Healthy Males—on assumption of Sum assured varying with Age at Entry.*

Ages at Entry.	Present Ages.						
	20-29	30-39	40-49	50-59	60-69	70-79	Upwards.
20–29	5,093	6,482	2,734	847	263	24	4
30–39	...	9,219	10,630	4,680	1,640	386	18
40–49	...	...	8,489	9,032	3,909	1,039	128
50–59	...	...	...	2,966	2,966	1,018	192
60–69	...	...	...	...	697	486	105
70–	...	...	...	...	...	50	36
	5,093	15,701	21,853	17,525	9,475	3,003	438

This would give us the following proportions of sums assured at risk at the various ages :—

Present Ages.	Percentage of Total Sum at risk, on assumption of Sum Assured varying with Age at Entry.	Percentage according to Table 1.
20–29	6·97	6·95
30–39	21·47	21·46
40–49	29·88	26·92
50–59	23·96	23·93
60–69	12·95	13·89
70–79	4·11	5·83
80 & upwards	·66	1·02
	100·	100·

It will be seen that although these percentages agree fairly well with those obtained from the Board of Trade returns, yet at ages 40–49, and ages beyond 60, there is a sensible difference, and on the whole the results cannot be considered quite satisfactory. When we come to consider the reasons why the proportion of the sums at risk at the various ages, as given by the Institute experience, do not agree with the results given by the Board of Trade returns, several disturbing causes will at once occur to us. One cause is undoubtedly the existence of duplicate policies, that is, that whereas in the Institute experience only one policy is considered as attaching to each life, we know, as a matter of fact, in practice this is not so, as in many cases large assurances on a life are effected simultaneously by policies in a number of offices, and in other cases a person on whose life a policy has been effected with an office years ago, will effect a further assurance, either in the office in which he is already assured or in another office. There are many persons, moreover, who on the principle of not having all their eggs in one basket will simultaneously take out policies in more than one office. All these cases would combine in making the results of the Institute experience differ from those given by the Board of Trade returns. Another cause is that already mentioned, namely, that whereas the Board of Trade returns relate only to whole life assurances, the Institute experience includes not only whole life risks, but term assurances, endowments, endowment assurances, contingent assurances, &c. A further cause is the fact previously stated, that rated-up or diseased lives are included in the Board of Trade returns, and this would have the

result of increasing the percentage at risk at the older ages, as compared with that at the younger ages. The assumption that lives entering at the older ages assure on the average for larger amounts than those entering at the younger ages, although not in itself an unreasonable one, does not, as has been shown, enable us to establish an agreement between the Institute experience and the Board of Trade returns without a somewhat exaggerated use of the assumption, and in the face of the disturbing causes just mentioned, we can readily understand the reason for this.

As in taking out the necessary facts from the Board of Trade returns each office had to be separately dealt with, and as some of the results might prove to be of sufficient interest to preserve, the offices were grouped into two classes:—

A. Offices contributing their experience both to the Institute of Actuaries, and to the Joint Committee of the Scottish Managers' Association and the Faculty of Actuaries.

B. Offices contributing their experience only to the Institute of Actuaries.

The following tables afford opportunity for some interesting comparisons, which, if they serve no other purpose, illustrate the difficulties surrounding the determination of an average office.

Of course it is to be distinctly borne in mind how various are the constitutions of the offices included under both classes. In each group there are offices doing a special class of business, and having distinguishing features as regards the distribution of profits; but as between Class A and Class B many of these special features will pretty well balance one another, and it may be very safely accepted that on the average the various differences here brought out are not far, practically, from the truth. One point is well brought out, at any rate, and that is that the distribution of the risks in Classes A and B differs materially, the latter showing unmistakeably a larger proportion at risk at the older ages than the former, which, as we should be led to suppose, goes much closer to the proportions of the combined risks than the former. Any discussion bearing on this point would be quite out of place in a paper like the present, and it will be sufficient on this occasion merely to point out the fact. As regards the amount of the reversionary bonus additions, it is quite certain that it affords in itself no criterion whatever as to the profits made by the various companies. The true test of this is of course the actual surplus appropriated for division *coupled with the data and method used*

in the determination of the said surplus. The figures before us must only be taken as some indication of the proportions in which policyholders exercise the option of a reversionary bonus addition.

TABLE 8.—*Showing the sums assured at risk, as per Table 2, divided into Classes* A *and* B.

Present Ages.	CLASS A (10 OFFICES). Whole Life Assurances.—Sums Assured. With Profits.	CLASS A. Without Profits.	CLASS A. Reversionary Bonus.	CLASS B (9 OFFICES). Whole Life Assurances.—Sums Assured. With Profits.	CLASS B. Without Profits.	CLASS B. Reversionary Bonus.
	£	£	£	£	£	£
20–24	1,039,054	148,728	3,160	324,945	65,094	1,080
25–29	3,407,314	340,753	14,878	1,078,810	224,593	4,792
30–34	5,900,854	568,125	64,117	2,131,345	368,138	22,235
35–39	7,293,248	702,811	150,990	2,827,512	588,544	58,703
40–44	7,941,625	972,939	272,984	3,190,075	691,015	94,518
45–49	8,099,118	1,037,936	407,004	3,455,127	752,455	157,961
50–54	7,452,509	1,077,747	612,157	3,330,277	857,938	188,253
55–59	6,226,644	1,059,431	737,240	2,593,688	714,694	177,591
60–64	4,301,308	759,845	745,255	2,166,714	528,658	167,239
65–69	2,866,348	466,412	683,234	1,676,437	456,997	161,461
70–74	1,684,753	475,681	531,970	1,210,854	362,211	131,997
75–79	733,673	242,107	283,863	610,674	211,013	85,427
80–84	238,671	70,267	104,754	311,798	112,922	50,062
85–89	53,839	13,261	29,624	73,022	28,824	14,352
90 & upwds.	9,000	700	4,460	20,400	4,800	6,240
	57,247,958	7,936,743	4,645,690	25,001,673	5,967,891	1,321,911
Deduct	*2,425,549	793,588	167,270	1,731,379	1,073,796	83,133
	54,822,409	7,143,155	4,478,420	23,270,294	4,894,095	1,238,778
No. of Policies	102,446	8,277	...	31,783	5,548	...
Average amount of Policy	£534	£863	£560 (with and without.)	£732	£882	£754 (with and without.)

The first thing to be noticed in this table is the remarkable difference in the proportion of the without-profit business to the with-profit. In Class A the with-profit business is about 7¾ times the without-profit as regards the amount assured, whereas in Class B it is about 4¾ times. The number of with-profit policies in Class A is to the number of without-profit policies as 102 to 8, or about 12½ to 1, whereas in Class B it is as 5¾ to 1. The average in amount of the with-profit policies in Class A is £534, while in

* The sums here deducted are those relating to offices which did not furnish particulars as to the number of policies in force.

Class B it is £732, whereas the average amount of the without-profit policies is nearly the same in both classes; and whereas the average policy (with or without profits) is £560 in Class A, it is £754 in Class B, or about 35 per-cent greater.

Table 9.—*Showing the Premiums payable, corresponding to the Sums Assured, given in Table* 8.

Present Ages.	Class A (10 Offices).				Class B (9 Offices).			
	Whole Life Assurances.				Whole Life Assurances.			
	Premiums With Profits.	Average Premium per cent.	Premiums Without Profits.	Average Premium per-cent.	Premiums With Profits.	Average Premium per-cent.	Premiums Without Profits.	Average Premium per-cent.
20–24	20,943	2·015	2,545	1·711	6,384	1·965	1,165	1·790
25–29	74,720	2·193	6,591	1·934	23,324	2·161	4,423	1·969
30–34	139,992	2·372	12,030	2·117	50,354	2·362	7,852	2·133
35–39	187,260	2·568	16,585	2·360	72,214	2·555	13,881	2·359
40–44	220,508	2·777	25,586	2·630	89,272	2·798	18,884	2·732
45–49	243,425	3·005	30,548	2·943	105,612	3·057	22,264	2·959
50–54	240,451	3·226	35,074	3·254	109,798	3·297	27,932	3·256
55–59	216,050	3·470	40,445	3·818	91,256	3·517	25,050	3·505
60–64	158,590	3·687	35,032	4·610	83,962	3·874	20,138	3·811
65–69	112,669	3·931	21,274	4·561	64,731	3·862	18,602	4·070
70–74	68,113	4·043	22,642	4·760	50,178	4·144	16,059	4·434
75–79	32,038	4·367	13,218	5·447	25,229	4·131	9,323	4·419
8084	11,807	4·936	3,966	5·644	13,177	4·227	6,842	6·061
8589	2,478	4·603	832	6·274	3,336	4·569	1,493	5·181
90 & uprds.	500	5·555	46	...	1,395	6·923	488	9·936
	1,729,544	3·021	266,414	3·357	790,222	3·161	194,396	3·257

The above table gives a summary of the premiums payable on the sums assured, particulars of which are given in Table 8. It will be noticed that whereas the average with-profit premium in Class A is 3·021 per-cent, that in Class B is 3·161, and that the average without-profit premiums are 3·357 and 5·257 per-cent respectively. Of course, in dealing with these figures, it must not be forgotten that there are many circumstances which affect any comparisons that may be made. Among others, may be mentioned the application of bonus to the reduction of premium, and the assurances by way of single premium, so that in any deductions drawn from the above table these points must be borne in mind. Assuming that the application of bonus to the reduction of premium does not largely affect these figures, it will be seen that the common practice of considering the average premium payable as about 3 per-cent is not very wide of the mark. The assurances affected on

the without-profit scale would not probably be subject to so many disturbing influences as regards the premium payable, and it is noteworthy how closely the without-profit premium agrees in the list class so far as about present ages 50–54. For older ages the without-profit premiums in Class A give a larger average per-cent than those in Class B, and it is not easy to say how this arises. It may be that the without-profit business in Class B is of older standing than that of Class A, and this probably is, to a great extent, true, but it may be that the business of Class A contains a great deal more of assurances on rated-up lives. Whatever the cause may be, however, the fact itself is none the less worthy of notice.

The following table gives particulars of the reassurances of the two classes :—

TABLE 10.—*Giving particulars of the Reassurances of Classes* A *and* B. (*See Table* 8.)

Present Ages.	Class A. Reassurances. Sums Assured With & Without Profits.	Class A. Reassurances. Percentage of Total at Risk at same Ages.	Class A. Reassurances. Premiums Payable.	Class B. Reassurances. Sums Assured With & Without Profits.	Class B. Reassurances. Percentage of Total at Risk at same Ages.	Class B. Reassurances. Premiums Payable.
	£		£	£		£
20–24	94,650	7·97	1,841	33,000	8·46	611
25–29	162,900	4·35	3,246	49,875	3·83	996
30–34	227,014	3·51	5,209	162,650	6·51	3,714
35–39	333,625	4·17	7,675	228,300	6·68	5,355
40–44	572,252	6·42	16,254	288,000	7·42	8,343
45–49	590,780	6·47	18,751	312,450	7·43	10,250
50–54	556,784	6·53	19,098	243,101	5·80	8,631
55–59	631,453	8·67	24,029	216,697	6·55	8,793
60–64	379,400	7·50	18,188	79,200	2·91	3,723
65–69	142,725	4·28	7,053	49,500	2·32	2,591
70–74	166,125	7·69	8,242	6,400	·41	455
75–79	68,350	7·00	4,070	5,900	·72	457
80–84	7,950	·26	521	2,000	·47	304
85–89	...	...		1,000	·99	58
90 & uprds.	...	...		3,000	1·22	405
	3,934,008	6·04	136,177	1,681,073	5·43	54,686

This table would seem to indicate that in Class A the system of reassurance is more extensively carried on than in Class B. The average premiums per-cent on the reassurances of Class A and Class B are 3·462 and 3·253 respectively.

Grouping now, the with and without profits together in each class, the following table shows the proportions at risk at the various present ages.

TABLE 11.—*Showing the proportion of the Total Sum Assured at risk at the various ages.* (*See Table* 8.)

Present Ages.	Class A.		Class B.		Classes A and B.	
	Without deducting Reassurances.	After deduction of Reassurances.	Without deducting Reassurances.	After deduction of Reassurances.	Without deducting Reassurances.	After deduction of Reassurances.
20–24	1·82	1·78	1·26	1·22	1·63	1·60
25–29	5·75	5·85	4·21	4·28	5·26	5·35
30–34	9·92	10·19	8·08	7·98	9·34	9·48
35–39	12·27	12·51	11·03	10·88	11·87	11·98
40–44	13·68	13·62	12·53	12·27	13·31	13·18
45–49	14·02	13·95	13·59	13·31	13·88	13·74
50–54	13·03	13·02	13·52	13·47	13·23	13·16
55–59	11·18	10·86	10·68	10·55	11·02	10·76
60–64	7·76	7·64	8·70	8·95	8·07	8·07
65–69	5·11	5·21	6·89	7·12	5·68	5·82
70–74	3·33	3·25	5·08	5·34	3·88	3·93
75–79	1·50	1·48	2·65	2·78	1·86	1·91
80–84	·47	·51	1·37	1·44	·77	·80
85–89	·10	·11	·33	·34	·20	·19
90 & uprds.	...	·02	·08	·07		·03
	100·	100·	100·	100·	100·	100·

This investigation had been carried thus far, when, acting on a suggestion, for which I have to thank my friend Mr. R. P. Hardy, one of the Honorary Secretaries of this Institute, it was found that the experience of the ten contributing Scotch offices had been taken out in such a form that it was possible to ascertain therefrom the actual sums assured at risk on 31 Dec. 1863, for healthy males and females, whole life, with and without profits, together with the respective present ages, and ages at entry. Accordingly, to test the accuracy of the above conclusions, application was made to Mr. James Meikle, of the Scottish Provident Institution, by whom the experience of the ten Scotch offices was taken out. A large portion of this experience has been published in a volume, entitled "Report by the Joint Committee, Appointed "by the Managers' Association of the Life Assurance Offices in "Scotland, and the Faculty of Actuaries in Scotland, to collect "the Statistics of the Mortality Experience of the Scottish Life "Assurance Offices to 31 December 1863", and dated May 1869, and Mr. Meikle, with extreme kindness and courtesy, took the trouble to furnish me with particulars of the sums assured, with the ages at which the assurances were taken out, and the ages at 31 Dec. 1863. The following is a summary of the particulars:—

TABLE 12.—*Scottish Life Assurance Offices' Mortality Experience to* 31 *December* 1863.

Ages at Entry.	Sums Assured—Healthy Males and Females—With and Without Profits.				Present Ages.
	Assurances effected.	No. of Lives entering.	Assurances existing, 31 Dec. 1863.	Percentage of Total.	
	£		£		
20–24	2,964,550	5,415	649,040	1·52	20–24
25–29	7,195,290	12,134	2,382,280	5·59	25–29
30–34	8,842,590	13,381	4,411,100	10·35	30–34
35–39	8,261,360	10,704	5,839,590	13·70	35–39
40–44	6,622,450	7,714	6,385,240	14·99	40–44
45–49	4,141,100	4,904	6,630,820	15·56	45–49
50–54	2,590,770	2,808	5,478,580	12·36	50–54
55–59	1,310,070	1,363	4,151,970	9·74	55–59
60–64	552,300	572	3,092,310	7·25	60–64
65–69	131,710	159	1,978,280	4·64	65–69
70–	7,280	...	1,029,600	2·42	70–74
			415,710	·98	75–79
			174,950	·40	80–92
	42,619,470	59,154	42,619,470	100·	

It should be stated that where a policy was for a larger amount than £10,000, it has been reckoned as for £10,000. The effect of this would be to eliminate to a great extent the reassurances. Of the total assurances here given, those in their first year of duration are given in the following table :—

TABLE 13.—*Scottish Life Assurance Offices' Mortality Experience to* 31*st December* 1863.

Ages at Entry.	Assurances effected in Year ending 31 Dec. 1863, and existing at same date.	Percentage of Total.	Assurances existing 31 Dec. 1863.	Percentage of Total.
	£		£	
20–24	288,560	7·47	2,964,550	6·96
25–29	659,6[illegible]0	17·05	7,195,290	16·88
30–34	747,080	19·33	8,842,590	20·75
35–39	747,550	19·34	8,261,360	19·38
40–44	567,270	14·68	6,622,450	15·54
45–49	416,290	10·77	4,141,100	9·72
50–54	227,930	5·89	2,590,770	6·08
55–59	123,580	3·20	1,310,070	3·07
60–64	67,110	1·74	552,300	1·30
65–69	20,260	·53	131,710	·32
	3,865,310	100·	42,619,470	100·

From this table it will be seen that when the aggregate sum assured at 31 Dec. 1863, is apportioned according to ages at entry, the amounts corresponding to the various ages at entry differ but

little in their proportions to one another from the proportions in which they are found to enter in the year 1863. This would seem to indicate that although the rate of mortality increases with the age, the rate at which policies go off the books by death, lapses, or surrenders, would appear to be pretty constant at all ages. In other words, as the rate of mortality increases, the rate of lapsing (by surrenders or otherwise) decreases, and it is by no means self-evident that this conclusion is altogether erroneous.

Referring again to the particulars of the sums assured (Table 12) we find that the average sum assured *per life* is as follows:—

Ages at Entry.	Average Sum Assured per Life.	Percentage of Average Sum Assured per life for all Ages at Entry.
	£	
20–29	579	80·42
30–39	709	98·47
40–49	854	118·61
50–59	935	130·00
60–69	936	130·00
All Ages at Entry	720	...

This result would seem to indicate that the amount for which a person assures does, to some extent, depend upon the age at entry, in other words, that policies effected at the younger ages are, on the average, of smaller amount than policies effected in the middle and latter portion of life. It is to be noticed, however, that the results just brought out do not relate to policies but *lives*, and in order more completely to investigate this point, particulars of number of policies relating to the assurances effected in year 1863, and existing at 31 Dec. 1863, were kindly furnished by Mr. Meikle, from which the following results have been obtained:—

Ages at Entry.	Average Amount of Policy.		Percentage of Average Policy.
	£	£	
20–24	411	463	82·7
25–29	491		
30–34	514	553	98·7
35–39	598		
40–44	644	663	118·4
45–49	691		
50 and upwards		662	118·4
All Ages at Entry		560	...

The first point to be noticed in connection with these results is that throughout, the sum assured per policy for various ages at entry, bears almost exactly the same ratio to the average policy

for all ages at entry, as the sum assured per life for the same ages at entry bears to the average sum assured per life for all ages at entry; and would appear almost to show beyond a doubt that the amount of a policy does in some degree vary with the age at entry. It is not a little remarkable that the average policy for all ages at entry, agrees exactly with the average policy deduced from the Board of Trade returns of the Offices under Class A (see Table 7). If, now, we endeavour to make a rough approximation to the average policy from the average sum assured per life, we shall find that the figures still agree pretty closely with those just given.

Referring to the before-quoted Report of the Joint Committee, we find that the total number of policies for all classes of risks was 115,254, and that there were 20,505 duplicates, giving 94,749 lives, and reducing the average sums assured per life in the ratio of 115,254 to 94,749, that is 1 to ·8224, we get as our average policies:—

£475 for Ages at Entry 20–29	£702 for Ages at Entry 40–49
583 „ 30–39	592 for all Ages.

These, it will be seen, are in excess of the amounts derived from the actual figures for the single year 1863, and it may be that this result arises from the fact that in obtaining the average sum assured per life, the entire experience has been made use of. A far more probable cause, however, is that the duplicate policies for ordinary whole life assurances bear a greater proportion to the lives than in other classes of risks.

In order to give at one glance a comparative view of the distribution of the risks according to present ages for each group of ages at entry, the following table has been drawn up. To make it as complete as circumstances would permit, the figures have been given not only for Mr. Manly's Table 27 (see vol. xiv p. 292), but also for the "model office", proposed by Mr. King in a paper read at the last meeting of this Institute, and for an average office founded on the lives existing according to the Institute experience (healthy males), on the assumption that the sums assured per life are the same as those given by the Scotch Life Assurance Offices' mortality experience to 31 Dec. 1863. It should be borne in mind in examining Mr. King's figures, that instead of having entrants at all ages 20, 21, &c., he only uses entrants for ages 20, 25, and so on. This difference is, however, more apparent than real, as it will be seen on reference to his paper that his ages 20, 25, and so on, are practically the central ages of quinquennial groups of entrants. In the following table, these ages at entry are called 20–24, 25–29, &c.

TABLE 14.—*Showing, according to various data, and according to various assumptions, the proportion of the Total Sum Assured at risk at various Present Ages, classified according to Ages at Entry, the Total Existing Sum Assured at risk for all Present Ages being* 100.

	Present Ages.															
	20-24.				25-29.				30-34.				35-39.			
Ages at Entry.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.	Lives existing 31 Dec. 1863, Inst. H^M.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.	Lives existing 31 Dec. 1863, Inst. H^M.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.*	Lives existing 31 Dec. 1863, Inst. H^M.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.	Lives existing 31 Dec. 1863, Inst. H^M.
20–24	·19	1·52	1·39	1·82	·13	2·02	·95	2·29	·10	1·35	·76	1·42	·08	·89	·66	·82
25–29	†	...	...	*	·88	3·57	3·74	3·92	·66	4·52	2·86	4·54	·44	3·34	2·44	3·44
30–34	...	...	...	...	...	...	...	...	3·28	4·48	4·53	4·94	2·19	5·08	3·60	5·48
35–39	...	...	...	...	...	...	...	...	...	...	...	...	5·48	4·40	3·98	3·7(
40–44	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45–49	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	...
50–54	...	...	...	...	...	...	...	..	...	...	...	...	...	...	...	...
	·19	1·52	1·39	1·82	1·01	5·59	4·69	6·21	4·04	10·35	8·15	10·90	8·19	13·71	10·68	13·44

	Present Ages.											
	40-44.				45-49.				50-54.			
Ages at Entry.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.	Lives existing 31 Dec. 1863, Inst. H^M.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.	Lives existing 31 Dec. 1863, Inst. H^M.	Manly's Table 27.	Scotch Offices' Experience.	Mr. King's Model Office.	Lives Existing 31 Dec. 1863, Inst. H^M.
20–24	·06	·54	·58	·46	·05	·36	·51	·28	·04	·14	·46	·14
25–29	·32	2·44	2·15	2·29	·30	1·41	1·90	1·29	·23	·92	1·69	·75
30–34	1·75	3·96	3·09	4·72	1·31	3·31	2·72	3·24	1·10	1·89	2·38	1·88
35–39	3·72	4·52	3·20	4·39	3·07	4·14	2·75	3·92	2·41	2·76	2·38	2·61
40–44	6·57	3·52	2·99	3·28	4·38	3·87	2·40	3·87	3·50	3·08	2·02	3·18
45–49	...	...	...	...	5·48	2·48	2·05	2·14	3·72	2·55	1·62	2·46
50–54	...	...	...	...	...	...	...	...	3·94	1·51	1·35	1·38
	12·42	14·98	12·01	15·14	14·59	15·57	12·33	14·74	14·94	12·85	11·90	12·40

* The figures under this head are based on the assumption that the lives existing 31 Dec. 1863, Inst. H^M, entering between Ages 20-29, 30-39, 40-49, 50 and upwards, assure for sums in the ratio of 80·42, 98·47, 118·61, 130·00, and 130·00 respectively.

† Read throughout, in connection with the figures under this heading, 20 for 20-24, 25 for 25-29, &c., &c.

TABLE 14—*continued.*

PRESENT AGES.

Ages at Entry.	55-59. Manly's Table 27.	55-59. Scotch Offices' Experience.	55-59. Mr. King's Model Office.	55-59. Lives Existing 31 Dec. 1863, Inst. H^M.	60-64. Manly's Table 27.	60-64. Scotch Offices' Experience.	60-64. Mr. King's Model Office.	60-64. Lives Existing 31 Dec. 1863, Inst. H^M.	65-69. Manly's Table 27.	65-69. Scotch Offices' Experience.	65-69. Mr. King's Model Office.	65-69. Lives Existing 31 Dec. 1863, Inst. H^M.
20–24	·03	·10	·42	·08	·02	·03	·37	·05	·02	·01	·31	·01
25–29	·19	·38	1·46	·37	·15	·22	1·23	·24	·10	·08	1·01	·12
30–34	·88	1·09	2·04	1·17	·66	·52	1·70	·63	·55	·28	1·34	·33
35–39	1·97	1·63	2·05	1·50	1·53	1·22	1·72	1·03	1·10	·41	1·36	·53
40–44	2·85	2·12	1·70	2·21	2·19	1·48	1·41	1·31	1·97	1·01	1·12	·78
45–49	3·07	1·94	1·33	2·04	2·41	1·28	1·08	1·43	1·75	·86	·84	·75
50–54	2·63	1·64	1·04	2·54	2·19	1·26	·83	1·33	1·53	·78	·64	·78
55–59	1·97	·85	·75	·64	1·31	·86	·56	·76	1·10	·59	·42	·70
60–64	...	...	...	...	·55	·39	·42	·33	·33	·52	·29	·39
65–69	...	...	...	...	...	...	...	...	·11	·11	·18	·11
70–74	...	...	...	...	...	...	...	...	...	...	...	...
	13·59	9·75	10·79	9·55	11·01	7·26	9·32	7·11	8·56	4·65	7·51	4·50

Ages at Entry.	70-74. Manly's Table 27.	70-74. Scotch Offices' Experience.	70-74. Mr. King's Model Office.	70-74. Lives Existing 31 Dec. 1863, Inst. H^M.	75-79. Manly's Table 27.	75-79. Scotch Offices' Experience.	75-79. Mr. King's Model Office.	75-79. Lives Existing 31 Dec. 1863, Inst. H^M.	80-84. Manly's Table 27.	80-84. Scotch Offices' Experience.	80-84. Mr. King's Model Office.	80-84. Lives Existing 31 Dec. 1863, Inst. H^M.	85 and upwards. Manly's Table 27.	85 and upwards. Scotch Offices' Experience.	85 and upwards. Mr. King's Model Office.	85 and upwards. Lives Existing 31 Dec. 1863, Inst. H^M.
20–24	·01	...	·23	·00	·01	...	·15	·00	·00	...	·07	...	...	...	·03	...
25–29	·06	·00	·76	·03	·04	...	·49	·00	·02	...	·24	...	·01	...	·11	...
30–34	·33	·13	1·01	·19	·22	·01	·65	·03	·11	·01	·32	·01	·04	...	·13	·01
35–39	·66	·20	·99	·25	·44	·09	·61	·12	·26	·01	·28	·01	·14	...	·11	·00
40–44	1·31	·31	·82	·39	·77	·12	·52	·16	·39	·04	·24	·06	·19	·00	·09	·00
45–49	1·31	·44	·60	·41	·66	·14	·36	·18	·33	·03	·16	·06	·19	·01	·07	·02
50–54	1·10	·53	·46	·47	·66	·30	·28	·26	·33	·04	·13	·09	·17	·02	·05	·02
55–59	·66	·47	·30	·33	·44	·16	·19	·16	·20	·13	·09	·09	·11	·02	·04	·03
60–64	·22	·20	·19	·24	·15	·11	·12	·15	·09	·07	·06	·07	·04	·01	·03	·02
65–69	·07	·12	·12	·12	·04	·05	·07	·07	·02	·02	·04	·03	·01	·00	·02	·02
70–74	·01	·01	...	·04	·00	·00	...	·02	·00	·00	...	·02	·00	·00	...	·01
	5·74	2·41	5·48	2·47	3·43	·98	3·44	1·15	1·75	·35	1·68	·44	·90	·06	·68	·13

Table 15.—*Showing, according to various data, and according to various assumptions, the proportion at risk at various Present Ages on the assumption that the Total Sum assured at risk for all ages is* 100.

Present Ages.	Board of Trade Returns of 19 Offices.	Board of Trade Returns of 19 Offices. Class A.	Board of Trade Returns of 19 Offices. Class B.	Scotch Offices' Mortality Experience.	Mr. King's Model Office.	Institute Experience HM.*	Present Age.	Manly's Table 27.
20–24	1·60	1·78	1·22	1·52	1·39	1·82	20	·20
25–29	5·35	5·85	4·28	5·59	4·69	6·21	25	1·01
30–34	9·48	10·19	7·98	10·35	8·15	10·90	30	4·03
35–39	11·98	12·51	10·88	13·71	10·68	13·44	35	8·15
40–44	13·18	13·62	12·27	14·98	12·01	15·14	40	12·37
45–49	13·74	13·95	13·31	15·57	12·33	14·74	45	14·52
50–54	13·16	13·02	13·47	12·85	11·90	12·40	50	14·88
55–59	10·76	10·86	10·55	9·75	10·79	9·55	55	13·52
60–64	8·08	7·64	8·95	7·26	9·32	7·11	60	10·95
65–69	5·82	5·21	7·12	4·65	7·51	4·50	65	8·52
70–74	3·93	3·25	5·34	2·41	5·48	2·47	70	5·71
75–79	1.90	1·48	2·78	·98	3·44	1·15	75	3·41
80–84	·80	·51	1·44	·35	1·63	·44	80	1·75
85 & uprds.	·22	·13	·41	·06	·68	·13	85 & upds.	·98

The foregoing table gives a comparative view of the distribution of the risks of an office according to the various data and other material mentioned in the course of this paper. It will at once be seen, that while no two can be said to be identical, yet some of them agree pretty well. For instance, Mr. King's "model office" and the Board of Trade returns of Class B run closely together at many ages, while the Scotch Offices' mortality experience agrees on the whole very well with the healthy males (lives existing 31 Dec. 1863) of the Institute experience on the assumption as regards the latter that the sum assured increases with the age at entry. On the other hand, Mr. King's "model office" and the Board of Trade returns of Class B vary considerably from the two just mentioned. On a future occasion, when dealing with the question of valuations, it is proposed to consider these figures more closely with a view to examining in detail the causes of these variations.

Mr. Manly's Table 27, it will be seen, differs very materially from the other results, and the cause of this it is not difficult to trace. It arises, there would appear to be no doubt, in the assumption he has made as to the proportions of sums assured effected at various ages. These are given in the following table,

* On the assumption that the lives existing 31 Dec. 1863, and entering between ages 20–29, 30–39, 40–49, 50 and upwards, assure for sums in the ratio of 80·42, 98·47, 118·61, 130·00, and 130·00 respectively.

together with the corresponding proportions as deduced from the Scotch Life Offices' Mortality Experience for the year 1863, which have been already given on page 207.

TABLE 16.—*Showing the proportions of Sums Assured effected at various ages.*

Ages at Entry.	Scotch Life Offices' Mortality Experience for Year ending 31 Dec. 1863.	Age at Entry.	Manly's Table 27.
20–24	6·96	20	·68
25–29	16·88	25	3·08
30–34	20·75	30	11·44
35–39	19·38	35	19·24
40–44	15·54	40	23·08
45–49	9·72	45	19·23
50–54	6·08	50	13·95
55–59	3·07	55	6·97
60–64	1·30	60	1·93
65–69	·32	65 & uprds.	·40

As another proof of the fact just pointed out, it may be stated that the average net premium Inst. H^M 3 per-cent, according to Mr. Manly's Table 27, is 2·81 per-cent, corresponding to about age 42, whereas, as we have previously seen, the average premium should correspond to about age 37, or an age 5 years younger.

The writer cannot refrain from making a few remarks on Mr. Manly's essay above referred to. At the time that essay was written, nothing in any way resembling Mr. Manly's Table 27, on which the results established in the essay are founded, had ever been published; and it is no exaggeration to state that, to the facts there so carefully brought out, as well as to the methods of investigation so ably laid down or indicated, the actuarial profession is at least as much indebted as to any other of the valuable communications, published from time to time, through the medium of this Institute.

In conclusion, I have to state that I am greatly indebted to Mr. C. J. Harvey, of the Prudential Assurance Society, for making the extracts from the Board of Trade returns, also to Messrs. J. Whitcher, of the London Life Association, and T. H. Cooke, of the Northern Assurance Company, for their valuable assistance in the course of the numerous investigations connected with the subject of the paper, and to Mr. Charles Windett, of the London and Provincial Law Assurance Society, for many of the necessary calculations.

Discussion.

The President (Mr. J. Hill Williams) thought that Mr. Sutton had shown conclusively, that any attempt to constitute a model office is very difficult indeed, not to say impossible. One part of the paper he fully endorsed—namely, the allusion to Mr. Manly's very useful and interesting paper. Considering the materials available at the time it was written, it is a remarkable instance of foresight combined with great industry. That paper has given more subject for thought than almost any of those which have appeared in the *Journal of the Institute;* it has also furnished the actuary for several years past with the most forcible arguments, when it has been necessary to change the basis of valuation from an erroneous to a more nearly true mortality experience table.

Mr. Manly said that when he wrote his essay he certainly had little or no materials to base his assumed office upon; and the difference between his assumptions and the actual facts, arose from his wishing to err, if at all, upon the right side, making the assumptions as regards the proportions of the sum assured greater rather than less at the older ages, so that the results should not really show the excessive differences that might follow from other assumptions. For instance, if he had assumed the sums assured larger at the younger ages than at the older, the differences brought out by the various tables would have been much larger than were represented in the essay. It seemed probable, judging from Mr. King's tables, that Mr. Sutton's results would take an intermediate position between Mr. King's and his own. It is true that if the statistics collected by the Institute for the mortality experience had been extended to the sums assured, we should have had more materials to work upon in applying the scientific results to practical examples; but at the same time he did not think that the mortality experience should have been based upon any other facts than those actually used. If, as suggested by Mr. Sutton in his previous paper, second and subsequent assurances on lives had been taken as new lives, the effect of multiplying the deaths would have been far more serious than any loss we might have sustained by omitting those particulars altogether.

Mr. Macfadyen was of opinion that it is not possible to arrive at the true average life office. The material is defective; and to build an average life office out of that material, is like making bricks without straw. Mr. Sutton has pointed out many of these deficiencies; and another, of minor importance, is that the application of an average sum assured carries with it a disturbing element, arising from the fact that in most offices the great majority of the policies will be very much under the average amount, a very few large ones redressing the deficiency,—the most probable amount to be assured being as different a thing from the average amount assured, as is the most probable age at entry from the average age at entry. But even were it possible to arrive at the average life office, since it is certain that this company would seldom, probably never, represent the real office, actuaries would be chary of transferring conclusions drawn from the hypothetical experience to their own company. In anything that was to be put on record, or was to go forth to the public in any way, though the labour would be greater, they would prefer to value out the main

classes by the various tables wanted, rather than make a leap in the dark by trusting the average office results. He had on two occasions tried the actual by the hypothetical results—in both cases from curiosity merely. On the first occasion the actual results were very different from the hypothetical. On the second they were much alike—a likeness, however, equally close whether Mr. Manly's, Mr. King's, or indeed any reasonable hypothesis as to a model office were taken—thus making it evident that the likeness was not from any virtue of the hypothetical method, but from the resemblance of the tables compared. It seemed, therefore, a dangerous thing to use the average office practically, unless indeed the conclusions drawn are of the broadest. No doubt this might, in a sense, be said of employing a mortality table. But in employing the latter, all that is meant is, that though actual mortality and theoretical may be different, yet in the long run they will on the whole closely coincide; while for the former purpose you are called on to state the actual definite cash results of applying certain mortality tables to a certain definite distribution of risks.

Mr. Bailey said that the object of the average office is to try experiments, and without some such plan we should not have had so clear a conception of how insufficient a reserve the Carlisle table gives for ordinary life offices. He continued:—The old proverb says, "It is exceedingly easy to be wise after the event"; and as I happened to be one of the honorary secretaries of the Institute at the time this information was collected, and know what labour and trouble was expended upon it, I regret now that further details were not obtained, such as the sum assured and the reversionary bonus. I think the prevailing idea in the minds of the committee at that time, was to obtain the mortality on assured lives, as distinguished from policies; and that idea was so deeply impressed in their minds that it did not occur to them to go into other matters of which great use could have been made. But it must be remembered that it was no easy matter to persuade the companies to contribute their experience at all, and it was therefore necessary to be careful that no information was asked for which was not absolutely necessary. I do not think it occurred to any gentleman to ask for the sum assured, which would have caused but little extra labour to furnish, and would probably have been freely given.

Mr. Sutton, in reply, said that his remark, quoted by Mr. Manly, as to the form in which the Twenty Offices' Experience was taken out, was intended to apply only to bringing out the effect of selection; for he did not intend to say that the mortality experience proper should be based on policies rather than lives. Mr. Macfadyen had objected to the use of the word "average" on this occasion, but the remarks he had made would seem to show that he objects to all averages. At all events, the objections advanced by him would apply to any average of whatever kind or description. Mr. Macfadyen had spoken of the "probable sum assured". This was a new phrase to him (Mr. Sutton), and he did not understand it. He had only to say further that in his paper he had carefully avoided the expression or even indication of any theory whatever, and had certainly not gone so far as to show any preference for one average office over another, that being a subject which remains for consideration.

Does Vaccination afford any Protection against Small-pox? By T. B. SPRAGUE, M.A., *a Vice-President of the Institute of Actuaries.*

A SATISFACTORY anser to the question I hav placed at the hed of this articl can only be got by means of an examination of statistics carefully compiled. If we coud obtain statistics shoing the rate of mortality from small-pox among two bodies of persons, similarly circumstanst as to ages, occupations, conditions of life, and other things; and differing only to this extent, that all the persons in the one body had been vaccinated, and none at all in the other body; then an examination of such statistics woud giv a conclusiv anser to the question. But such statistics ar clearly unattainabl. Those that ar forthcoming wil be more or less deficient in the accuracy we hav supposed; and our conclusions will therefore only hav a greater or less degree of probability, acording as the statistics ar more or less complete and accurat.

I had supposed until lately that the statistics compiled by different persons at various times and publisht in medical works, prove beyond all reasonabl dout that vaccination afords an almost certain and complete protection from smalpox. It apears, however, that in certain quarters the efficacy of vaccination is now altogether denied. Vaccination is denounst as in all cases useless, and in many positivly injurios, becaus it introduces an animal poison into the system, which offen produces very serios ilness. Such is the vew taken by the National Anti-Compulsory Vaccination Leag, som of whose publications hav recently been brot under my notis by Mr. H. Pitman of Manchester.

From the nature of the case statistics on the subject must be furnisht by members of the medical profession, and the actuary can hav litl or nothing to do with the preparation of them. He may, however, usefully examin such statistics when givn to the world, and state, without being considerd presumptuos, what conclusions may in his opinion be fairly drawn from them. Such is the task which I propose to undertake on the present ocasion.

It will be convenient in the first instance to state the opinions which apear to be generally held by competent medical men with regard to the bearing of vaccination on the mortality from smalpox. These seem to be fairly summarized by Dr. William Robertson, F.R.S.E., in the 7th Annual Report on the vaccination of children born in Scotland during 1870, as follos:

1. That, as a protection agenst smalpox, a recent succesful

vaccination is as complete a safegard as that aforded by a previos atak of smalpox, whether natural or inoculated.

2. That during an epidemic of smalpox a very small proportion of persons recently and succesfully vaccinated fall victims to the disease.

3. That during smalpox epidemics, persons who hav been succesfully vaccinated during infancy, but who hav ataind adult life without having been revaccinated, ar apt to contract smalpox in considerabl numbers (but not necessarily in large *proportion*); and that in such persons the disease is not nearly so fatal as in those unprotected by vaccination.

4. That during an epidemic persons who hav never been succesfully vaccinated contract the disease in large numbers and in large proportion, and that such persons die in a proportion somtimes amounting to 1 in 4.

5. That the phrase "succesfully vaccinated" is offen improperly uzed; the application of the vaccine virus, and the subsequent apearance of a "sore" on the arm, not being conclusiv evidence of "succesful" vaccination, unless the characteristic mark results.

6. That there ar many adults who suppose themselvs to hav been "succesfully" vaccinated, who hav never in fact had cow-pox.

7. That there is in general no risk incurd in either vaccinating or revaccinating a helthy subject.

8. That so far from vaccination or revaccination being dangerous or inexpedient while an atak of smalpox is prevailing or imminent, it is precisely in anticipation of, or during the course of, such an epidemic, that these operations ar most urgently required and their protectiv value most strikingly illustrated.

9. That of those succesfully revaccinated in anticipation of an epidemic of smalpox, hardly any contract the disease, altho they may be in close and daily atendance on the sik for months after their revaccination.

What sort of statistics then ar brot forward by medical men in support of these propositions? My information on the subject is principally derived from a very readabl litl work by Dr. J. Thorburn of Manchester, entitld *Vaccination: a condenst summary of the evidence in its favor, and of the objections urged agenst it* (Simpkin, Marshall, and Co., 1870, price 6*d.*), and it wil be sufficient for my purpose to make a few extracts from this. As I can only undertake to deal with the question from a purely statistical point of vew, it wil be understood that I quote only from the statistical parts of

Dr. Thorburn's work, but I may mention that it would be very erroneos to suppose that it is entirely statistical. On the contrary, it deals with a number of other questions in a most interesting and satisfactory manner. Without further preface, I wil now quote som of the statistics givn by Dr. Thorburn.

On p. 9 is givn a table taken from Seaton's *Handbook of Vaccination* (London, 1868), shoing the comparativ mortality in England from smalpox at varios periods.

Periods compared.	Annual Deths by Smal-pox in England and Wales.	Annual Rate per million of the Population.
1. Average of 30 years previos to introduction of vaccination, estimated by Dr. Lettsom, and Sir Gilbert Blane	...	3,000
2. Average of 3 years (1838–40)* when vaccination was to a great extent diffused, but before public provision was made for its gratuitos performance	11,944	770
3. Average of 9† of the years (1841–53) when vaccination was gratuitosly provided, but was not obligatory	5,221	304
4. Average of the 10 years (1854–63) during which vaccination has been to a certain extent obligatory	3,351	171

"* The present system of registering deths commenst only in 1837."
"† During the years 1843–46, causes of deth were not distinguisht in the "Reports of the Registrar-General."

Page 18. "Out of 100,000 persons ther died annually of smalpox, in the century before vaccination, and the half century since, compared—

	Before Vaccination.	After Vaccination.
In Rhenish Prussia	90·8	9·0
„ Sweden	205·0	15·8
„ Copenhagen	312·8	28·6
„ West Prussia . . .	227·2	35·6
„ East Prussia	332·1	55·6
„ Berlin	342·2	17·6."

These figurs ar stated to be quoted from Oesterlen's *Handbuch der Medicinischen Statistik*, Tübingen 1865; and Dr. Thorburn says that they are condenst chiefly from Simon's *Blue-book* of 1857, "Papers relating to the history and practis of vaccination."

The foregoing statistics giv no information as to the relativ mortality among vaccinated and unvaccinated persons who ar atakt with the disease, and it may be argued that the diminisht mortality is the result of an improovment in modern sanitary arangements or som other caus. In proof that this is not the case, Dr. Thorburn states that a great mortality stil takes place

among the unvaccinated compared with those who ar atakt in spite of vaccination, and he givs (p. 20) the folloing figurs in proof of this statement.

	Number ov cases treated.	Number who Died per-cent.	
		Ov Vaccinated Persons.	Ov Unvaccinated Persons.
France, 1816–41	16,397	1·0	16·2
Bohemia, 1835–55	15,640	5·1	29·8
Milan, 1830–51	10,240	7·6	38·3
Vienna, 1837–56	6,213	5·0	30·0
London Smalpox Hospital, 1836–56	9,000	7·0	35·0

"This table", he remarks, "shos the wonderful power which vaccination possesses to modify the danger of the disease, even in those cases where it has faild entirely to protect".

Page 22. "'By careful vaccination, carried out thoroly by the dispensary and poor-law physicians in Ireland, smalpox has been stampt out from the contry. In the ten years ending 1841, no fewer than 58,006 persons died in Ireland from smalpox, and many thousands sufferd disfigurment. During the next ten years, the number of fatal cases of the disease fell to 38,275, and from 1851 to 1861, the number decreast to 12,727. In 1866, 187 deths ocurd, and last year only 20 persons fell victims to this disease. During the current year (1868) I believ no cases of smalpox hav ocurd in our contry'. (Dr. Cameron *On the Preservation of Helth;* critique in Brit. and For. Med.-Chirur. Revew, October 1869.)"

Page 23. "In Marseilles there was a great outburst in 1828. The most careful serch was made, and it was found that there wer in the place 30,000 persons more or less perfectly vaccinated, and 8,000 unvaccinated. The folloing was the apalling result:—

	Ov the 8,000 Unvaccinated.	Ov the 30,000 Vaccinated.
Number atakt	4,000, or 1 in 2	2,000, or 1 in 15.
Of whom died	1,875, or 1 in 4¼	20, or 1 in 1,500.

Page 25.—"In Denmark, where upwards ov 9,000 pepl died ov smalpox in the 30 years previos to 1792, vaccination was partially introduced in 1802, and only 58 persons died between that date and 1810. It was then made most strictly compulsory, and for the next ten years not a deth ocurd. In the British cavalry, vaccination and revaccination hav been stringently enforst,

and out ov 44,600 men who past thro the ranks in 20 years, only 3 died ov smalpox."

Altho the mortality from smalpox has thus greatly diminisht, it miht be that the mortality from other causes shoud increase to such an extent that the total rate ov mortality woud be but litl influenst. Dr. Thorburn givs (p. 30) the folloing figurs in proof that this has not been the case:—

"*Annual Deth Rate per* 1,000 *Inhabitants.*

	1755-75.	1776-79.	1841-50.
"Sweden . . .	28·9	26·8	20·5
	1681-90.	1746-55.	1846-55.
"London . . .	42	35	25."

As a last quotation, I wil giv the folloing, contributed by Surgeon Thompson, of the 100th or Canadian Infantry:—

Page 58. "The regiment was statioud at Montreal, and between the end ov December 1867, and 28 February 1868, ther ocurd 13 cases ov smalpox, it being then prevalent in the town. At the latter date the whole regiment was revaccinated. Seven cases ocurd after this, but in every instance they wer new recruits or men who had been on furlo, not one ov those who had been revaccinated being atakt, altho smalpox stil continued prevalent in the city."

It apears to me that the previos statistics can leav no reasonabl dout as to the reality ov the protection aforded by vaccination. I wil supplement them by a few figurs taken from the Annual Reports of the Registrar-General for Scotland.

It is stated in the 20th Annual Report, p. 30, that the Scottish Vaccination Act aplies only to children born in Scotland since the 1st day of January 1864. The deths from smalpox in Scotland for the 20 years 1855–1874 inclusiv wer as follos:—

Deaths from Smalpox in Scotland, 1855–74.

Years.	Smalpox Deths.	Years.	Smalpox Deths.
1855	1,309	1865	383
1856	1,306	1866	200
1857	845	1867	100
1858	832	1868	15
1859	682	1869	64
1860	1,495	1870	114
1861	766	1871	1,442
1862	426	1872	2,446
1863	1,646	1873	1,126*
1864	1,741	1874	655 Estimated.

* In the Twelfth Annual Report on the vaccination of children born in Scotland, this number is estimated as 866: the correct number has been furnisht to me by Dr. Robertson.

The rapid diminution in the number of these deths from the year 1864 to 1868 is most remarkabl. It can scarcely be denied that this rapid and markt diminution in the number ov deths is in consequence ov the introduction ov compulsory vaccination. The results ov the year 1868 wer such that it woud probably not hav been unreasonabl to look forward to the total disapearance ov smalpox within a few years. But any such anticipation has been most signally disapointed by the experience of subsequent years. From the year 1869 the number ov deaths from smalpox has increast, at first sloly but afterwards more rapidly, til the year 1872, when they wer more in number than they had been at any previos time during the 20 years under consideration. In explanation ov the epidemic of smalpox ov the years 1871 and 1872, Dr. Robertson says, in his Report dated 5 April 1875 (p. 29), that "26,000 ov the children born in Scotland during the last ten years ar believd to hav escaped the provisions ov the Vaccination Act. * * * * * Besides, ther ar notoriosly living among us thousands of unvaccinated persons who wer born in Scotland before the Vaccination Act came into operation, or who hav emigrated from forein lands to this contry. * * * * * Our returns do not enabl us to giv any accurat information regarding revaccinations or the vaccinations ov adults." In illustration ov the benefits that vaccination has conferd, he givs the folloing tabl :—

Smalpox Deths in the Eight Principal Towns at different ages in 1874, *compared with those ov the ten years* 1855–1864, *in the same towns before the Vaccination Act came into operation.*

Ages.	Smalpox Deths, 1874.		Smalpox Deths, 1855-1864.	
	Number.	Percentage at each Age.	Number.	Percentage at each Age.
0– 5	89	25·00	4,062	75·03
5–20	109	30·62	660	12·19
20–60	156	43·82	687	12·67
60 and above	2	0·56	6	0·11

I confess that this explanation is not entirely satisfactory to myself. I see here nothing that at all explains why, notwithstanding the introduction of compulsory vaccination, the deths from smalpox shoud hav risen in the year 1872 so far beyond their number in any of the previos 17 years. I cannot however myself suggest any explanation, and must leav the question to be delt with by our officers of helth and other medical men.

In non ov the preceding statistics, with the exception of the last tabl, is any information givn as to the ages ov the persons who wer atakt with smalpox. I now propose to examin certain statistics, prepared by one ov the oponents ov vaccination, in which such information is givn.

These ar containd in a report by Dr. Leander Joseph Keller, Chief Medical Officer of the Imperial Austrian State Railways, of which a translation has been publisht by the abov mentiond Leag. It is difficult, however, to form an opinion from that translation as to the soundness of Dr. Keller's arguments, as his principal statistical tables ar omitted. Having, thro the kindness of the Honorary Secretary, Mrs. Mary C. Hume-Rothery, had the opportunity of perusing the report in the original German, it apears to me of sufficient interest to justify my laying a somwhat ful acount of it before the readers of the *Journal* of the Institute. Dr. Keller shal, as far as possibl, speak for himself, and I believ it wil be found that in the folloing translation nothing of any importance to his argument has been omitted.

Report on the sikness from Smalpox among the Employees of the Austrian State Railway Company in the year 1873.

This railway company has always in its employ about 37,000 oficials, porters, and workmen; forming, with the adition of wives and children, pensioners and others ("provisionisten"), a population of from 55,000 to 60,000 persons, who ar atacht to 195 railway stations and 11 factories. The company has engaged 80 medical men to atend the employees and their wives and children in sikness. These ar required to keep exact records of all the siknesses that ocur, and to send monthly and yearly statistical reports to the directors; and they ar all under the supervision of the chief medical officer. From the reports for the year 1873 it apears that there wer altogether 2,054 cases of smalpox. Of these 1,669 (or 81·26 per-cent) recoverd; 385 (or 18·74 per-cent) died. The persons who wer atackt may be clast thus:

Vaccinated	1337,	of whom	219	or 16·38	per-cent died.
Unvaccinated . . .	596,	"	148	" 24·83	"
Revaccinated . . .	46,	"	7	" 15·22	"
Had smalpox previosly .	11,	"	2	" 18·18	"
Doutful	64,	"	9	" 14·06	"

The ages of the persons who wer atackt ar givn in Tabl (A). When the mortality at different ages is examind, it is found

to follo almost exactly the same law as was shoen by the figurs for the year 1872. The mortality is heviest in the first two years of life, then gradually diminishes, reaches its minimum between 15 and 20, and from that point increases gradually, and with great rapidity at advanst ages. *This is just the law which human mortality in general follos.*

Considering, next, the mortality at different ages among the vaccinated and unvaccinated respectivly, we get som very important results. Taking all ages together, we find that the unvaccinated hav experienst a hiher mortality by 8·45 per-cent than the vaccinated. This result is not only almost the same as that for the year 1872, but it is the same as almost always ocurs, and must ocur, in tables of smalpox statistics; and it is just this result on which the frends and defenders of vaccination continually base their arguments. They say that, becaus the rate of mortality among the unvaccinated is always hiher than that among the vaccinated, the conclusion must follo that vaccination, even if it dos not protect from smalpox, yet causes the disease to take a much milder form.

A singl glance, however, at the tabl is enuf to sho that this conclusion is quite erroneos, and that vaccination has nothing to do with the loer mortality of the vaccinated; for we cannot help seeing at once, that the reason for the greater mortality among the unvaccinated is simply that the greater number of ataks ocur in the first two years of life, during which a much hiher mortality universally prevails. For, if we exclude from observation the first two years of life, we find that at the other ages the mortality among the vaccinated amounted to 13·76 per-cent (173 out of 1257), while the mortality of the unvaccinated was only 13·15 per-cent (48 out of 365); and that, consequently, at these ages the mortality of the vaccinated and the unvaccinated was almost identical,—in fact, rather lihter among the unvaccinated.

It will now ov course be supposed that the mortality among the unvaccinated during the first two years of life, must be much hevier than that among the vaccinated of the same age, since the mortality among all the unvaccinated is greater by 8·45 per-cent than that among the vaccinated. But this is not the case; on the contrary, we hav the mortality

among the unvaccinated in the 1st year of life, 45·24 per-cent,
„ „ 2nd „ 38·10 „

while it is

among the vaccinated in the 1st year of life, 60·46 per-cent,
„ „ 2nd „ 54·05 „

It is thus clear that, altho the mortality among the unvaccinated in the first two years of life is less than that among the vaccinated, yet, when all ages ar taken together, the rate of mortality among the unvaccinated becoms hevier in comparison, and must do so, becaus the number of the unvaccinated who wer atakt in the first two years of life is large; and the reason why it is large is, that there ar many more unvaccinated than vaccinated children of that age in the families of the men employd on the line and in the workshops. If the contrary had been the case, and all the children had been vaccinated before they wer three months old, then, as a matter of course, many more vaccinated children woud have been atakt; and since the mortality in these first two years of life is considerabl, the mortality among the vaccinated woud hav been much greater than among the unvaccinated, but we shoud not therefore hav been entitld to blame vaccination for the result.

The conclusions we hav here drawn from the figurs of the year 1873 apear just as clearly from Table (C), which givs the experience of the years 1872 and 1873 combined; and we may draw the conclusion that statistical tables of smalpox cases, in which the age and its normal deth-rate ar left out of acount, ar quite worthless for the decision of the question as to the value of vaccination, even if (as is seldom the case) they ar quite correctly and consientiosly drawn up.

Looking, now, at the mortality during 1873, among the revaccinated and among those who had previosly had smalpox, we see that among the former there is a mortality of 15·22 per-cent and among the latter 18·18 per-cent, from which the conclusion necessarily follos, that neither revaccination nor a previos atak of smalpox afords any protection agenst another atak, and that the only reason why the rate of mortality in both these classes of persons has been comparativly low, is that they hav containd no very yung children. The lihtest mortality, namely 14·06 per-cent, ocurs among those returnd as doutful with respect to vaccination.

The data givn in the tables ar collected from widely different localities,—Austria Proper, Bohemia, Moravia, Lower Austria, Hungary, and the Banat; and the facts that they include all ages, and that the observations hav been made by a variety of medical men who wer in constant asociation with the employees on the railway and in the works, wil probably giv them a greater value with the medical profession than ataches to reports from individual hospitals

into which only paupers ar admitted; since such reports ar not unfrequently colord to support the privat views of their authors, even when the facts ar faithfully givn.

Considering, now, from the same point of vew, the observations for the years 1872 and 1873 combined, we see the same regular progression which I hav alredy pointed out in each year separatly; but, the numbers being larger, the results sho remarkably litl fluctuation. In the two years taken together, there wer 2627 cases of smalpox. Of these, 2158 (or 82·15 per-cent) recoverd, and 469 (or 17·85 per-cent) died. Comparing the mortality among all the smalpox cases at different ages, we find the rate first diminishes and then increases, just as it always dos in observations upon human mortality. This same law holds good if we consider the mortality at different ages among the vaccinated and unvaccinated separatly. Thus we hav:—

Age.	Vaccinated.	Unvaccinated.
Under 1 year	57·14	43·78
1 to 2 years	52·05	38·96
2 „ 3 „	34·15	17·86
3 „ 4 „	21·88	16·88
4 „ 5 „	23·64	13·70
5 „ 10 „	19·23	7·76
10 „ 15 „	6·35	12·05
15 „ 20 „	6·15	7·14
20 „ 30 „	7·42	9·26
30 „ 40 „	15·05	15·62
40 „ 50 „	17·95	16·67
50 „ 60 „	35·00	33·33
60 „ 70 „	63·64	40·00

In these serieses we see that the mortality gradually diminishes from the first year of life, in which it is very large, till it reaches its minimum between the 15th and 20th years, both among the vaccinated and the unvaccinated. It then gradually increases with the age. The only breaks in this progression, and they ar but sliht, ar between the ages 4 and 5 for the vaccinated and 10 to 15 for the unvaccinated; and if the number of lives under observation had been larger, these irregularities woud certainly hav disapeard. If we now compare the rates of mortality among the vaccinated and unvaccinated at the different ages, we find that between 10 and 40, at which ages there ar comparativly few unvaccinated, these experienst a rather hiher mortality than the vaccinated; but a much loer rate at all other ages, and

especially at the very yungest. Notwithstanding that the mortality is thus so much lihter among the unvaccinated, yet when all ages ar taken together, the rate of mortality among them is 23·20 agenst 15·61 per-cent among the vaccinated, being a precisely similar result to the one we hav alredy seen in the observations for the year 1873. The hiher mortality among the unvaccinated is entirely due to the cases in the first two years of life; for, if we leav these out of acount, the mortality among the vaccinated was 13·37 per-cent (210 out of 1570), and that of the unvaccinated 12·82 per-cent (66 out of 515), so that the unvaccinated abov the age of 2 experienst a lihter mortality than the vaccinated of the same age. Those first two years of life, however, which turn the scale so much agenst the unvaccinated, sho nevertheless the striking phenomenon that the mortality is 55·06 per-cent among the vaccinated, and only 42·44 per-cent among the unvaccinated, from which it is clear that the hevier mortality among the unvaccinated as a whole, is in no way due to vaccination, but is a natural consequence of the greater number of the unvaccinated who wer atakt at the ages in question.

Among the revaccinated who wer atakt with smalpox (76 in number), the mortality amounts to 15·79 per-cent, which, considering that there wer among them no children under 4 years of age, must be regarded as a rather unfavorabl result, and at the same time shos how litl value is to be atacht to revaccination.

The number of those who had previosly had smalpox is very small, 13, and the mortality among them, 23·08 per-cent, is hih, the reason being that several of these cases ocur at advanst ages; but the fact that among these cases there wer 3 children between the ages of 5 and 10, certainly shos very clearly that even a previos atak of small-pox dos not long act as a protection agenst a second attack, and that any so-called immunity, even if it exists at all, can only be of very short duration.

The 86 patients who ar returnd as doutful in respect of vaccination sho a mortality of 12·79 per-cent, which, considering that there was no child among them under 3 years old, and no person older than 50, cannot be regarded as an especially favorabl result, since we hav alreddy shoen that the mortality among the unvaccinated, excluding children under two years of age, is only 12·82 per-cent. (See Table C.)

Collecting now our results, we conclude—

1. Not only the unvaccinated, but also the vaccinated, the

revaccinated, and those who had previosly had smalpox, wer atakt. Of these the vaccinated form the overwhelming majority, certainly becaus, with the exception of the first two years of life, there are many more vaccinated than unvaccinated persons.

2. In the first two years of life many more unvaccinated children than vaccinated wer atakt, becaus at this age the unvaccinated ar much more numerous than the vaccinated.

3. Disregarding extreme old age, the mortality is greatest in the first two years, but is always less among the unvaccinated children than among the vaccinated of the same age.

4. Putting these two years aside, the mortality is almost equal among the vaccinated and unvaccinated, but rather hevier among the vaccinated.

5. Altho the mortality among all the unvaccinated is usually greater than among the vaccinated, this is not to be ascribed to the absence of vaccination, but only to the greater mortality that ocurs in erly childhood.

6. The mortality at different ages, among both vaccinated, and unvaccinated, follos the general law of human mortality; and vaccination has no power to alter this law.

7. Looking at all these facts, vaccination apears to be utterly useless.

Any unprejudist person who observs even cursorily the facts here brot forward, cannot fail to admit the extreme importance of the element of age in all smalpox statistics. It is incomprehensibl that the frends of vaccination shoud stil apeal to statistics in which the age of the persons atakt is not mentiond, and argue from such statistics that vaccination afords a sure protection, becaus it is found that the mortality among the unvaccinated is usually much greater than among the vaccinated. It is true that almost all observations sho us that this is the fact, but there ar other causes which contribute to it besides the tender age of the unvaccinated, one only of which I wil now mention, namely, that non but helthy children ar vaccinated and that the sikly ar usually left unvaccinated.

What woud be thot of the argument that, becaus the mortality among children who go to scool is much less than among children who stay at home, therefore the non-atendance at scool is the cause of the greater mortality? Every clearheaded person at once sees the absurdity of such an argument, becaus it is just the very yung and the sikly children who stay at home; but

the frends of vaccination argue in just as absurd a manner about the vaccinated and unvaccinated.

Assuming Dr. Keller's statistics to be entirely trustworthy, which I see no reason to dout, it apears to me that he has certainly establisht two points.

First. That smalpox statistics in which no acount is taken of the age, ar of litl value, if any, and ar offen positivly misleading. This is clearly shoen by the folloing figurs taken (or deduced) from Dr. Keller's table (C) relating to the years 1872 and 1873 combined.

Age.	Vaccinated.			Unvaccinated.		
	Number of cases.	Deths.	Mortality per-cent.	Number of cases.	Deths.	Mortality per-cent.
Under 2	89	49	55·06	278	118	42·44
Over 2	1570	210	13·37	515	66	12·82
Total	1659	259	15·61	793	184	23·20

Here the rate of mortality among the vaccinated taken as a whole, is very much less than that among the unvaccinated; but when we divide the cases into those under and abov two years of age, we find that in each class the mortality of the vaccinated is greater than that of the unvaccinated. If, therefore, the age wer not taken into acount, the figurs relating to the vaccinated and unvaccinated of all ages woud lead us to an erroneos conclusion.

Secondly. That the mortality among persons atakt with smalpox depends almost entirely on the age, and is very litl (if at all) afected by the fact of their having been vaccinated or having had a previos atak of smalpox. In other words, the rate of mortality among the vaccinated persons who wer atakt, was quite as hevy as that among the unvaccinated.

It dos not however follo that vaccination is of no use. In order to determin as to this, we require further information beyond that furnisht by Dr. Keller's tables, namely, as to the comparativ numbers of the vaccinated and the unvaccinated among the population of 55,000 to 60,000, upon whom his observations wer made. It is not sufficient to say as he dos, that under the age of 2, there ar many more unvaccinated children than vaccinated and that therefore the number of unvaccinated persons

atakt with smalpox is much greater than that of the vaccinated; while abov the age of 2 the contrary is the case. In order to decide the question at issue, we require to kno, at all events approximatly, the respectiv numbers of the vaccinated and the unvaccinated. Thus, for instance, taking the age 5 to 10 in tabl (C), we find that 234 vaccinated children were atakt and 116 unvaccinated, the former number being almost exactly dubl the latter. If now vaccinated and unvaccinated children were atakt in the same proportion, it miht fairly be argued that vaccination is of no use; but if the proportion of vaccinated children who were atakt is greatly less than that of the unvaccinated, then it woud follo that vaccination is to a considerabl extent a protection against smalpox. To apply this to the abov instance; if the vaccinated children between 5 and 10 wer twice as numeros as the unvaccinated and no more, then the same proportion of them were atakt by the disease and we shoud hav som reason to believ that vaccination afords no protection. But if the vaccinated children wer much more numeros in proportion than the unvaccinated, say 10 times as many, then the proportion of them which were atakt by the disease was only one-fifth of the proportion of the unvaccinated that was so atakt, and vaccination aforded a very substantial tho' not a compleat protection.

On the whole I conclude that Dr. Keller's statistics certainly do not justify the conclusions he has drawn from them as to the uselessness of vaccination.

There is one other conclusion which the statistics apear to suggest. Thus we get from table (C), the folloing figurs.

Age.	Number of Persons atakt who wer respectivly		Ratio of the Vaccinated cases to the Unvaccinated.
	Vaccinated.	Unvaccinated.	
Under 3 months	1	33	·03
3 to 6 "	16	71	·23
6 " 9 "	16	57	·28
9 " 12 "	16	40	·40
1 " 2 years	40	77	·52
2 " 3 "	41	56	·73
3 " 4 "	64	77	·83
4 " 5 "	55	73	·75
5 " 10 "	234	116	2·02
10 " 15 "	189	48	3·94
15 " 20 "	275	42	6·55
20 " 30 "	364	54	6·74
30 " 40 "	219	32	6·84
40 " 70 "	129	17	7·59

We see here a remarkably steddy progression in the last colum which shos the ratio of the number of vaccinated persons atakt to that of the unvaccinated. If we adopt Dr. Keller's vews, we shal say that under the age of 5 the number of unvaccinated children atakt is greater than that of the vaccinated, becaus there ar more unvaccinated children of that age among the population; and that at higher ages the contrary is the case. This conclusion apears to me for various reasons improbabl. It is difficult to believ that, if any legal obligation or general practis as to vaccination exists, the unvaccinated can be in a great majority under the age of 5 and the vaccinated the more numeros abov that age. In order to decide this question, it woud be desirabl to hav information as to the age at which vaccination is generally performd among the population under consideration, and the relativ extent to which it has been practist in former as compared with recent years. In order for the abov explanation to be satisfactory, it woud be necessary that for a long series of years past—at least 20 and probably more—there shoud hav been a gradually groing neglect of vaccination, so that among the children between 1 and 2 years of age the unvaccinated ar twice as numeros as the vaccinated; between 5 and 10, on the contrary, the vaccinated ar twice as numeros as the unvaccinated; between 10 and 15, four times as numeros, and from 15 upwards, more than six times as numeros. Upon these points, no dout, Dr. Keller woud be abl to giv accurat information, and it is to be hoped he wil do so in one of his future yearly reports.

The more probabl conclusion, as it apears to me, is that we hav in the abov figurs, an indication of the gradual wearing out with laps of time of the protection aforded by vaccination. The rapid increas of the ratio between the ages of 5 and 20 apears to indicate that between those ages vaccinated persons becom much more susceptibl to the infection of smalpox, and that it is therefore hihly desirable that they shoud be revaccinated.

In conclusion, I trust I may be alowd to express a hope that this discussion of the statistics put forward by the oponents of vaccination, may be useful to its defenders, by indicating to them certain points on which further information is desirabl.

TABLE (A).—*Year* 1873.

Age.	Vaccinated. Atakt.	Vaccinated. Recoverd.	Vaccinated. Died.	Unvaccinated. Atakt.	Unvaccinated. Recoverd.	Unvaccinated. Died.	Revaccinated. Atakt.	Revaccinated. Recoverd.	Revaccinated. Died.	Second Ataks. Atakt.	Second Ataks. Recoverd.	Second Ataks. Died.	Doutful. Atakt.	Doutful. Recoverd.	Doutful. Died.	Total. Atakt.	Total. Recoverd.	Total. Died.	Mortality per-cent.
1st year of life. 1st quarter	1	...	1	32	10	22	...	...	...	...	...	...	...	...	...	33	10	23	69·70
2nd „	14	4	10	51	28	23	...	...	...	...	...	...	...	...	...	65	32	33	50·77
3rd „	14	5	9	49	31	18	...	...	...	...	...	...	...	...	...	63	36	27	42·86
4th „	14	8	6	36	23	13	...	...	...	...	...	...	...	...	...	50	31	19	38·00
Total of 1st year . . .	43	17	26	168	92	76	...	...	...	...	...	...	...	...	...	211	109	102	48·34
From 1 to 2 years . .	37	17	20	63	39	24	...	...	...	...	...	...	...	...	...	100	56	44	44·00
„ 2 „ 3 „ . .	31	20	11	35	29	6	...	...	...	...	...	...	...	...	...	66	49	17	25·76
„ 3 „ 4 „ . .	51	39	12	55	44	11	...	...	...	...	...	...	...	...	...	106	83	23	21·70
„ 4 „ 5 „ . .	48	36	12	54	46	8	3	3	...	...	...	...	1	1	...	106	86	20	18·87
„ 5 „ 10 „ . .	207	165	42	85	79	6	8	6	2	3	3	...	6	6	...	309	259	50	16·18
„ 10 „ 15 „ . .	164	154	10	35	30	5	1	1	...	1	...	1	6	5	1	207	190	17	8·21
„ 15 „ 20 „ . .	238	223	15	32	30	2	4	3	1	...	...	...	15	11	4	289	267	22	7·61
„ 20 „ 30 „ . .	277	254	23	39	35	4	12	11	1	1	1	...	17	16	1	346	317	29	8·38
„ 30 „ 40 „ . .	157	133	24	21	18	3	11	9	2	3	3	...	12	10	2	204	173	31	15·20
„ 40 „ 50 „ . .	55	44	11	4	3	1	7	6	1	2	2	...	7	6	1	75	61	14	18·67
„ 50 „ 60 „ . .	23	14	9	3	2	1	...	...	...	...	...	...	...	...	...	26	16	10	38·46
„ 60 „ 70 „ . .	6	2	4	2	1	1	...	...	...	1	...	1	...	...	...	9	3	6	66·67
Total	1337	1118	219	596	448	148	46	39	7	11	9	2	64	55	9	2054	1669	385	18·74
Per-cent . . .	...	83·62	16·38	...	75·17	24·83	...	84·78	15·22	...	81·82	18·18	...	85·94	14·06	...	81·26	18·74	
Excluding first two years	1257	1084	173	365	317	48										1743	1504	239	
Per-cent. . . .	...	86·24	13·76	...	86·85	13·15										...	86·29	13·71	

Table (C).—*Years* 1872 *and* 1873.

Age.	Vaccinated.			Unvaccinated.			Revaccinated.			Second Ataks.			Doutful.			Total.			Mortality per-cent.
	Atakt.	Recoverd.	Died.	Atakt.	Recoverd.	Died.	Atakt.	Recoverd.	Died.	Atakt.	Recoverd.	Died.	Atakt.	Recoverd.	Died.	Atakt.	Recoverd.	Died.	
1st year of life. 1st quarter	1	...	1	33	11	22	...	...	...	...	...	...	...	...	...	34	11	23	67·65
2nd „	16	5	11	71	41	30	...	...	...	...	...	...	...	...	...	87	46	41	47·13 } 46·40
3rd „	16	7	9	57	35	22	...	...	...	...	...	...	...	...	...	73	42	31	42·47
4th „	16	9	7	40	26	14	...	...	...	...	...	...	...	...	...	56	35	21	37·50
Total of 1st year . . .	49	21	28	201	113	88	...	...	...	...	...	...	...	...	...	250	134	116	46·40
From 1 to 2 years . .	40	19	21	77	47	30	...	...	...	...	...	...	...	...	...	117	66	51	43·59
„ 2 „ 3 „ . .	41	27	14	56	46	10	...	...	...	...	...	...	...	...	...	97	73	24	24·74
„ 3 „ 4 „ . .	64	50	14	77	64	13	...	...	...	...	...	...	1	1	...	142	115	27	19·01
„ 4 „ 5 „ . .	55	42	13	73	63	10	4	3	1	...	...	...	1	1	...	133	109	24	18·04
„ 5 „ 10 „ . .	234	189	45	116	107	9	9	7	2	3	3	...	7	7	...	369	313	56	15·18
„ 10 „ 15 „ . .	189	177	12	48	42	6	2	2	...	1	...	1	6	5	1	246	226	20	8·13
„ 15 „ 20 „ . .	275	258	17	42	39	3	5	4	1	...	...	...	16	12	4	338	313	25	7·40 } 13·36
„ 20 „ 30 „ . .	364	337	27	54	49	5	23	21	2	1	1	...	26	23	3	468	431	37	7·91
„ 30 „ 40 „ . .	219	186	33	32	27	5	19	16	3	4	4	...	19	17	2	293	250	43	14·68
„ 40 „ 50 „ . .	78	64	14	6	5	1	14	11	3	2	2	...	10	9	1	110	91	19	17·27
„ 50 „ 60 „ . .	40	26	14	6	4	2	...	...	...	...	...	...	...	...	...	46	30	16	34·78
„ 60 „ 70 „ . .	11	4	7	5	3	2	...	...	...	2	...	2	...	...	...	18	7	11	61·11
Total	1659	1400	259	793	609	184	76	64	12	13	10	3	86	75	11	2627	2158	469	17·85
Per-cent. . . .	...	84·39	15·61	...	76·80	23·20	...	84·21	15·79	...	76·92	23·08	...	87·21	12·79	...	82·15	17·85	
Excluding first two years	1570	1360	210	515	449	66										2260	1958	302	
Per-cent. . . .	...	86·63	13·37	...	87·18	12·82										...	86·64	13·36	

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On the Mortality amongst Assured Lives, and the requisite Reserves of Life Offices. *By* George King, *of the Alliance Assurance Company, Fellow of the Institute of Actuaries, Member of the Physical Society, &c.*

[Read before the Institute, 26 Mar. 1877.]

Part II.—*Financial.*

PERHAPS I owe to the Institute an apology for bringing forward so late in the day this second part of my paper. When first I resolved to enter on the investigation of which I have now the honor to submit a second series of results, the ground, at least in so far as the experience of the 20 Offices was concerned, was practically unoccupied; for although Mr. Sprague had written on the mortality prevailing among assured lives, yet, as he had not touched on financial matters, I conceived that there remained ample room for another worker. It was only when I was on the point of sending in the part of my paper which has already been printed in the *Journal*, that I heard of Mr. Berridge's labours; and not till the other day, when the calculations for this second part were all but finished, did I become aware that Mr. Sprague intended to deal with the question of the premiums for the insurance of recently selected lives. Had I known that these gentlemen were engaged very similarly to myself, I should probably have been deterred from my task: but seeing that in ignorance of their movements it has been completed, perhaps the results will not be considered valueless.

My work has been done quite independently, and different thinkers who proceed altogether without concert are not likely to follow exactly the same lines. It will be found, I think, that I enlarge on topics barely referred to by my fellow-authors, and that even where we approach each other, we still view the subject in different lights.

Taking up the threads where they were dropped in April last (*Journal*, vol. 19, p. 381), it is now my purpose to present tables of annuities on assured lives based upon the "Analyzed-Mortality tables", and to accompany them with such explanations and remarks as are necessary: to touch upon the measure of the risk premium, with which Mr. Sprague occupied himself in his late paper above mentioned: and to give illustrative examples of policy values derived from what I conceive to be the nearest approach to a "true table of mortality" which, in the present state of statistics, we are able to achieve. I shall then explain the construction of a "Model Office" —to adopt a ready coined, but not very felicitous phrase—from the "Mortality Experience", and give a comparison of its valuations by the principal data in use in this country: and, finally, I shall try to illustrate the methods by which,—with the aid of auxiliary tables, and formulas of convenient shape—the valuations of the model office may be practically turned to account. It is my hope that it will prove an implement by the judicious and cautious use of which may be solved with sufficient accuracy some of the most important problems that come before the actuary of a life office.

In the preparation of the "Analyzed-Mortality tables" I was much struck with the great vitality which prevailed at about ages 65 to 80 among the entrants at 25 and 30 years of age. When the comparative statement given in Table E (Vol. 19, p. 389) was completed, the peculiarity became very apparent, as the following extract from that table will show:—

Extract from Table E.

Present Age.	Annual Mortality per-cent for Age at Entry,		
	25	30	35
65 to 69	4·76	4·66	5·08
70 „ 74	5·53	5·57	7·56
75 „ 79	...	10·22	11·90

The mortality tables for all the other ages at entry nearly agreed with that for age at entry 35, and it was therefore evident that the disturbance lay in the tables for ages at entry 25 and 30. In the discussion which followed the reading of the paper, Mr.

Manly commented on the irregularity, and in my reply I stated that the figures were derived from the original data by the process explained, and that they had not in any way been arbitrarily altered. Both before that time and since, the matter has had my serious consideration, and had the calculations for the first part of the paper not been already too far advanced before the difficulty was observed, some slight modification might have been made in order to avoid it. There cannot be any reason why, among persons now aged 70 to 74, and who assured their lives at 25 and 30, the mortality per-cent should be only 5·53 and 5·57 respectively, while among persons of the same present age, but who assured five years later, at 35, it is 7·56, or 2 per-cent in excess. The fluctuation is manifestly accidental. Before proceeding to calculate the annuity values which form the foundation for the present part of the paper, I again looked into the matter, and came to the conclusion that it was desirable to make a slight correction. A process of grouping had already been resorted to at the higher ages, and the course which now obviously suggested itself was to carry it a little further. After careful thought I determined, for the mortality tables for ages at entry 20, 25, and 30, to incorporate with the lives who entered at these ages, and remained under observation at ages 70 to 80, those of the same present age who entered at age 35. A sufficient rectification would in this way be effected without an undue manipulation of the raw material, and the mortality tables would assume a character probably more in accordance with actual facts apart from accidental perturbations. For the three mortality tables for ages at entry 20, 25, and 30, the new values of p_x have therefore been prepared on this plan which are given in Table M; and they must be substituted for those which appear in the mortality tables as printed (vol. 19, pp. 399 and 400) for those ages at entry. They have been adopted in calculating the annuity values given in the succeeding Table N.

TABLE M.—*New Values of p_x for the Analyzed-Mortality Tables for Ages at Entry* 20, 25, *and* 30.

Present Age.	New Value of p_x.	Present Age.	New Value of p_x.
68	949389	75	908541
69	944531	76	897255
70	942060	77	885821
71	939102	78	871851
72	934171	79	865136
73	927104	80	854090
74	918540	81	842916

Table N.—*Annuities on Assured Lives.*

Age.	Age at Entry, 20.			Age.	Age at Entry, 25.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
20	21·250	19·529	18·026				
1	20·967	19·286	17·815				
2	20·722	19·077	17·635				
3	20·475	18·866	17·454				
4	20·249	18·674	17·290				
25	20·056	18·512	17·153	25	20·573	18·986	17·597
6	19·874	18·360	17·026	6	20·247	18·703	17·350
7	19·704	18·220	16·910	7	19·997	18·490	17·167
8	19·567	18·110	16·822	8	19·744	18·274	16·981
9	19·411	17·983	16·718	9	19·528	18·091	16·826
30	19·245	17·846	16·605	30	19·323	17·919	16·681
1	19·054	17·686	16·471	1	19·111	17·740	16·529
2	18·852	17·516	16·327	2	18·889	17·552	16·369
3	18·613	17·311	16·151	3	18·666	17·363	16·208
4	18·372	17·105	15·973	4	18·417	17·149	16·023
35	18·115	16·883	15·780	35	18·165	16·932	15·835
6	17·889	16·689	15·614	6	17·916	16·718	15·650
7	17·689	16·520	15·471	7	17·672	16·508	15·469
8	17·480	16·343	15·320	8	17·408	16·279	15·269
9	17·261	16·156	15·160	9	17·151	16·057	15·076
40	17·034	15·961	14·993	40	16·889	15·830	14·878
1	16·770	15·731	14·792	1	16·633	15·608	14·684
2	16·485	15·481	14·573	2	16·357	15·367	14·472
3	16·187	15·219	14·341	3	16·080	15·124	14·258
4	15·890	14·957	14·109	4	15·806	14·884	14·047
45	15·588	14·689	13·871	45	15·504	14·617	13·810
6	15·283	14·418	13·630	6	15·197	14·345	13·568
7	15·016	14·183	13·422	7	14·885	14·067	13·320
8	14·716	13·916	13·184	8	14·568	13·784	13·066
9	14·425	13·658	12·954	9	14·235	13·485	12·797
50	14·157	13·420	12·743	50	13·907	13·190	12·531
1	13·894	13·187	12·537	1	13·549	12·866	12·237
2	13·562	12·888	12·267	2	13·191	12·541	11·941
3	13·253	12·611	12·017	3	12·839	12·220	11·649
4	12·926	12·315	11·750	4	12·524	11·935	11·390
55	12·576	11·997	11·461	55	12·212	11·652	11·133
6	12·219	11·672	11·164	6	11·907	11·375	10·881
7	11·844	11·329	10·849	7	11·608	11·103	10·633
8	11·451	10·967	10·515	8	11·302	10·824	10·378
9	11·042	10·589	10·165	9	10·929	10·480	10·061
60	10·618	10·195	9·799	60	10·545	10.125	9·731
1	10·154	9·761	9·393	1	10·129	9·737	9·370
2	9·758	9·392	9·048	2	9·758	9·392	9·048
3	9·391	9·049	8·728	3	9·391	9·049	8·728
4	9·019	8·701	8·402	4	9·019	8·701	8·402
65	8·684	8·388	8·109	65	8·684	8·388	8·109
6	8·327	8·052	7·794	6	8·327	8·052	7·794
7	7·938	7·685	7·447	7	7·938	7·685	7·447
8	7·583	7·350	7·131	8	7·583	7·350	7·131
9	7·227	7·013	6·812	9	7·227	7·013	6·812
70	6·881	6·685	6·501	70	6·881	6·685	6·501
1	6·523	6·345	6·177	1	6·523	6·345	6·177
2	6·154	5·993	5·841	2	6·154	5·993	5·841
3	5·785	5·640	5·503	3	5·785	5·640	5·503
4	5·427	5·296	5·173	4	5·427	5·296	5·173

TABLE N.—*Annuities on Assured Lives*—(continued).

Age.	Age at Entry, 20.			Age.	Age at Entry, 25.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
75	5·085	4·968	4·857	75	5·085	4·968	4·857
6	4·765	4·660	4·560	6	4·765	4·660	4·560
7	4·470	4·375	4·285	7	4·470	4·375	4·285
8	4·198	4·112	4·031	8	4·198	4·112	4·031
9	3·942	3·865	3·792	9	3·942	3·865	3·792
80	3·693	3·624	3·558	80	3·693	3·624	3·558
1	3·454	3·392	3·333	1	3·454	3·392	3·333
2	3·220	3·165	3·112	2	3·220	3·165	3·112
3	3·008	2·959	2·911	3	3·008	2·959	2·911
4	2·820	2·776	2·733	4	2·820	2·776	2·733
85	2·653	2·613	2·575	85	2·653	2·613	2·575
6	2·505	2·469	2·435	6	2·505	2·469	2·435
7	2·338	2·306	2·276	7	2·338	2·306	2·276
8	2·197	2·169	2·143	8	2·197	2·169	2·143
9	2·019	1·996	1·974	9	2·019	1·996	1·974
90	1·787	1·769	1·751	90	1·787	1·769	1·751
1	1·508	1·494	1·480	1	1·508	1·494	1·480
2	1·163	1·153	1·144	2	1·163	1·153	1·144
3	·872	·865	·859	3	·872	·865	·859
4	·590	·585	·582	4	·590	·585	·582
95	·383	·380	·378	95	·383	·380	·378
6	·271	·269	·268	6	·271	·269	·268
7	·162	·161	·160	7	·162	·161	·160

Age.	Age at Entry, 30.			Age.	Age at Entry, 35.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
30	19·660	18·237	16·979				
1	19·335	17·955	16·733				
2	19·045	17·706	16·516				
3	18·755	17·454	16·298				
4	18·465	17·203	16·079				
35	18·194	16·969	15·876	35	18·595	17·342	16·218
6	17·926	16·737	15·675	6	18·234	17·025	15·938
7	17·665	16·512	15·480	7	17·915	16·746	15·693
8	17·397	16·280	15·278	8	17·605	16·476	15·456
9	17·122	16·040	15·069	9	17·304	16·213	15·226
40	16·838	15·792	14·851	40	17·011	15·957	15·002
1	16·549	15·539	14·628	1	16·710	15·693	14·770
2	16·252	15·278	14·397	2	16·402	15·422	14·531
3	15·945	15·007	14·157	3	16·085	15·142	14·283
4	15·632	14·730	13·910	4	15·760	14·854	14·027
45	15·312	14·445	13·656	45	15·427	14·558	13·763
6	14·987	14·155	13·396	6	15·090	14·257	13·494
7	14·655	13·857	13·129	7	14·750	13·952	13·221
8	14·332	13·568	12·869	8	14·413	13·650	12·950
9	14·006	13·275	12·605	9	14·076	13·348	12·678
50	13·693	12·993	12·351	50	13·751	13·056	12·415
1	13·380	12·711	12·096	1	13·418	12·755	12·143
2	13·058	12·420	11·832	2	13·076	12·446	11·862
3	12·727	12·120	11·569	3	12·723	12·125	11·570
4	12·386	11·809	11·276	4	12·365	11·798	11·272

TABLE N.—*Annuities on Assured Lives*—(continued).

Age.	Age at Entry, 30.			Age.	Age at Entry, 35.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
55	12·038	11·491	10·985	55	12·000	11·464	10·966
6	11·680	11·163	10·683	6	11·626	11·121	10·650
7	11·326	10·837	10·383	7	11·257	10·781	10·337
8	10·968	10·507	10·078	8	10·884	10·436	10·018
9	10·611	10·177	9·772	9	10·514	10·094	9·701
60	10·276	9·868	9·486	60	10·137	9·744	9·376
1	9·938	9·554	9·195	1	9·767	9·400	9·056
2	9·610	9·249	8·912	2	9·392	9·050	8·729
3	9·284	8·946	8·629	3	9·032	8·714	8·414
4	8·954	8·639	8·342	4	8·670	8·375	8·096
65	8·624	8·331	8·053	65	8·306	8·033	7·774
6	8·292	8·019	7·761	6	7·952	7·700	7·460
7	7·945	7·692	7·454	7	7·600	7·368	7·146
8	7·583	7·350	7·131	8	7·252	7·038	6·834
9	7·227	7·013	6·812	9	6·901	6·705	6·517
70	6·881	6·685	6·501	70	6·547	6·368	6·196
1	6·523	6·345	6·177	1	6·188	6·026	5·869
2	6·154	5·993	5·841	2	5·836	5·689	5·546
3	5·785	5·640	5·503	3	5·498	5·364	5·235
4	5·427	5·296	5·173	4	5·184	5·062	4·945
75	5·085	4·968	4·857	75	4·903	4·792	4·685
6	4·765	4·660	4·560	6	4·630	4·529	4·432
7	4·470	4·375	4·285	7	4·371	4·279	4·191
8	4·198	4·112	4·031	8	4·123	4·040	3·960
9	3·942	3·865	3·792	9	3·883	3·808	3·736
80	3·693	3·624	3·558	80	3·647	3·579	3·514
1	3·454	3·392	3·333	1	3·428	3·367	3·308
2	3·220	3·165	3·112	2	3·220	3·165	3·112
3	3·008	2·959	2·911	3	3·008	2·959	2·911
4	2·820	2·776	2·733	4	2·820	2·776	2·733
85	2·653	2·613	2·575	85	2·653	2·613	2·575
6	2·505	2·469	2·435	6	2·505	2·469	2·435
7	2·338	2·306	2·276	7	2·338	2·306	2·276
8	2·197	2·169	2·143	8	2·197	2·169	2·143
9	2·019	1·996	1·974	9	2·019	1·996	1·974
90	1·787	1·769	1·751	90	1·787	1·769	1·751
1	1·508	1·494	1·480	1	1·508	1·494	1·480
2	1·163	1·153	1·144	2	1·163	1·153	1·144
3	·872	·865	·859	3	·872	·865	·859
4	·590	·585	·582	4	·590	·585	·582
95	·383	·380	·378	95	·383	·380	·378
6	·271	·269	·268	6	·271	·269	·268
7	·162	·161	·160	7	·162	·161	·160

TABLE N.—*Annuities on Assured Lives*—(continued).

Age.	Age at Entry, 40.			Age.	Age at Entry, 45.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
40	17·255	16·182	15·212				
1	16·840	15·811	14·880				
2	16·470	15·482	14·586				
3	16·121	15·172	14·310				
4	15·776	14·865	14·036				
45	15·430	14·556	13·759	45	15·490	14·630	13·840
6	15·083	14·245	13·480	6	15·048	14·231	13·478
7	14·732	13·930	13·196	7	14·680	13·901	13·181
8	14·388	13·621	12·918	8	14·323	13·580	12·892
9	14·055	13·322	12·648	9	13·961	13·253	12·597
50	13·720	13·020	12·375	50	13·595	12·922	12·297
1	13·377	12·709	12·093	1	13·224	12·586	11·991
2	13·041	12·404	11·816	2	12·846	12·242	11·677
3	12·707	12·101	11·540	3	12·460	11·889	11·354
4	12·369	11·794	11·259	4	12·068	11·530	11·024
55	12·027	11·482	10·973	55	11·682	11·175	10·697
6	11·686	11·170	10·687	6	11·302	10·825	10·374
7	11·341	10·854	10·396	7	10·918	10·470	10·045
8	10·989	10·530	10·097	8	10·554	10·133	9·733
9	10·637	10·205	9·796	9	10·207	9·812	9·435
60	10·279	9·873	9·488	60	9·863	9·493	9·139
1	9·920	9·540	9·178	1	9·525	9·179	8·847
2	9·562	9·207	8·867	2	9·187	8·864	8·553
3	9·205	8·874	8·555	3	8·845	8·545	8·255
4	8·852	8·544	8·245	4	8·498	8·220	7·950
65	8·520	8·233	7·964	65	8·154	7·888	7·637
6	8·187	7·921	7·671	6	7·792	7·547	7·315
7	7·851	7·605	7·374	7	7·431	7·206	6·992
8	7·526	7·299	7·085	8	7·074	6·868	6·671
9	7·179	6·971	6·774	9	6·724	6·535	6·354
70	6·819	6·629	6·449	70	6·381	6·208	6·042
1	6·436	6·264	6·101	1	6·046	5·888	5·737
2	6·052	5·896	5·749	2	5·715	5·571	5·433
3	5·664	5·524	5·392	3	5·395	5·264	5·139
4	5·278	5·153	5·035	4	5·089	4·970	4·857
75	4·928	4·816	4·710	75	4·805	4·697	4·594
6	4·611	4·510	4·414	6	4·530	4·432	4·339
7	4·341	4·250	4·163	7	4·273	4·184	4·099
8	4·083	4·001	3·922	8	4·024	3·943	3·866
9	3·852	3·778	3·706	9	3·802	3·728	3·658
80	3·651	3·584	3·518	80	3·607	3·540	3·476
1	3·438	3·377	3·318	1	3·416	3·355	3·297
2	3·220	3·165	3·112	2	3·220	3·165	3·112
3	3·008	2·959	2·911	3	3·008	2·959	2·911
4	2·820	2·776	2·733	4	2·820	2·776	2·733
85	2·653	2·613	2·575	85	2·653	2·613	2·575
6	2·505	2·469	2·435	6	2·505	2·469	2·435
7	2·338	2·306	2·276	7	2·338	2·306	2·276
8	2·197	2·169	2·143	8	2·197	2·169	2·143
9	2·019	1·996	1·974	9	2·019	1·996	1·974
90	1·787	1·769	1·751	90	1·787	1·769	1·751
1	1·508	1·494	1·480	1	1·508	1·494	1·480
2	1·163	1·153	1·144	2	1·163	1·153	1·144
3	·872	·865	·859	3	·872	·865	·859
4	·590	·585	·582	4	·590	·585	·582
95	·383	·380	·378	95	·383	·380	·378
6	·271	·269	·268	6	·271	·269	·268
7	·162	·161	·160	7	·162	·161	·160

Table N.—*Annuities on Assured Lives*—(continued).

Age.	Age at Entry, 50.			Age.	Age at Entry, 55.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
50	14·255	13·531	12·863				
1	13·751	13·070	12·440				
2	13·304	12·661	12·066				
3	12·870	12·264	11·702				
4	12·462	11·891	11·359				
55	12·084	11·545	11·042	55	12·432	11·858	11·336
6	11·713	11·205	10·729	6	11·970	11·431	10·941
7	11·338	10·860	10·411	7	11·549	11·043	10·582
8	10·963	10·514	10·091	8	11·133	10·670	10·236
9	10·585	10·164	9·767	9	10·742	10·308	9·901
60	10·204	9·810	9·438	60	10·350	9·944	9·563
1	9·819	9·452	9·104	1	9·958	9·579	9·223
2	9·440	9·098	8·773	2	9·565	9·212	8·880
3	9·067	8·749	8·446	3	9·173	8·845	8·536
4	8·699	8·404	8·122	4	8·785	8·481	8·194
65	8·338	8·065	7·803	65	8·418	8·136	7·870
6	7·982	7·730	7·487	6	8·057	7·796	7·549
7	7·623	7·391	7·166	7	7·701	7·460	7·231
8	7·265	7·052	6·845	8	7·348	7·126	6·915
9	6·903	6·713	6·523	9	6·996	6·792	6·598
70	6·554	6·376	6·202	70	6·665	6·478	6·299
1	6·210	6·048	5·889	1	6·303	6·183	5·969
2	5·887	5·739	5·594	2	5·966	5·811	5·661
3	5·583	5·448	5·316	3	5·645	5·504	5·367
4	5·301	5·178	5·058	4	5·343	5·214	5·089
75	5·028	4·916	4·807	75	5·063	4·945	4·831
6	4·750	4·649	4·551	6	4·803	4·695	4·591
7	4·451	4·361	4·273	7	4·559	4·461	4·366
8	4·138	4·058	3·980	8	4·324	4·235	4·148
9	3·817	3·747	3·678	9	4·092	4·010	3·930
80	3·507	3·445	3·385	80	3·866	3·791	3·719
1	3·222	3·167	3·115	1	3·646	3·578	3·512
2	2·975	2·926	2·880	2	3·437	3·376	3·316
3	2·775	2·731	2·690	3	3·251	3·196	3·141
4	2·618	2·573	2·536	4	3·088	3·038	2·988
85	2·493	2·456	2·422	85	2·915	2·870	2·824
6	2·359	2·326	2·295	6	2·749	2·709	2·667
7	2·207	2·178	2·150	7	2·567	2·533	2·494
8	2·062	2·037	2·012	8	2·344	2·315	2·280
9	1·917	1·895	1·874	9	2·082	2·059	2·027
90	1·730	1·712	1·695	90	1·807	1·789	1·759
1	1·475	1·461	1·448	1	1·505	1·491	1·461
2	1·164	1·153	1·145	2	1·188	1·178	1·144
3	·872	·865	·859	3	·872	·865	·859
4	·590	·585	·582	4	·590	·585	·582
95	·383	·380	·378	95	·383	·380	·378
6	·271	·269	·268	6	·271	·269	·268
7	·162	·161	·160	7	·162	·161	·160

TABLE N.—*Annuities on Assured Lives*—(continued).

Age.	Age at Entry, 60.			Age.	Age at Entry, 65.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
60	10·874	10·441	10·028				
1	10·322	9·924	9·542				
2	9·868	9.499	9·144				
3	9·424	9·083	8·753				
4	9·002	8·686	8·380				
65	8·612	8·320	8·046	65	9·549	9·211	8·891
6	8·213	7·944	7·691	6	8·898	8·594	8·306
7	7·815	7·568	7·335	7	8·419	8·141	7·878
8	7·431	7·204	6·990	8	7·990	7·735	7·494
9	7·064	6·856	6·659	9	7·605	7·371	7·149
70	6·690	6·499	6·319	70	7·235	7·020	6·816
1	6·353	6·178	6·013	1	6·868	6·671	6·484
2	6·042	5·882	5·730	2	6·506	6·326	6·155
3	5·756	5·609	5·469	3	6·157	5·993	5·837
4	5·498	5·363	5·235	4	5·812	5·663	5·521
75	5·228	5·105	4·988	75	5·475	5·339	5·210
6	4·957	4·845	4·739	6	5·151	5·028	4·910
7	4·685	4·584	4·488	7	4·859	4·747	4·640
8	4·417	4·326	4·239	8	4·613	4·511	4·413
9	4·157	4·075	3·997	9	4·386	4·293	4·203
80	3·904	3·831	3·761	80	4·192	4·106	4·023
1	3·650	3·585	3·523	1	4·028	3·949	3·873
2	3·374	3·317	3·262	2	3·852	3·781	3·711
3	3·104	3·055	3·006	3	3·645	3·582	3·519
4	2·839	2·796	2·753	4	3·410	3·354	3·298
85	2·616	2·579	2·541	85	3·139	3·091	3·043
6	2·443	2·410	2·377	6	2·850	2·810	2·769
7	2·270	2·241	2·212	7	2·548	2·515	2·481
8	2·079	2·054	2·029	8	2·248	2·221	2·194
9	1·865	1·844	1·824	9	1·954	1·933	1·911
90	1·621	1·604	1·588	90	1·677	1·660	1·643
1	1·390	1·377	1·365	1	1·426	1·413	1·400
2	1·163	1·153	1·144	2	1·163	1·153	1·144
3	·872	·865	·859	3	·872	·865	·859
4	·590	·585	·582	4	·590	·585	·582
95	·383	·380	·378	95	·383	·380	·378
6	·271	·269	·268	6	·271	·269	·268
7	·162	·161	·160	7	·162	·161	·160

The change above mentioned in the values of p_x of course affects the annuity values, but only to a limited extent, and it was introduced principally for the sake of the policy values used in connection with the model office later on, which in certain cases would have taken anomalous forms had the original figures been adhered to. The annuities were calculated at 4 per-cent interest for both sets of values of p_x, and the effects of the change may best be shown by comparing the annuities at date of assurance, as follows:

Annuity: Interest 4 per-cent.

Age.	Original value of p_x.	Altered value of p_x.
20	18·058	18·026
25	17·642	17·597
30	17·009	16·979

The annuity values given in Table N have been directly deduced from the "Analyzed-Mortality tables", and the "Analyzed-Mortality tables" themselves were prepared by a process as rigid as the nature of the facts permitted from the "Mortality Experience" of the 20 companies. Beyond an inconsiderable grouping at the advanced ages—resorted to in order to obtain sufficient numbers to produce reliable results, and so planned as not to create any appreciable disturbance of the prevailing law of mortality—no artificial methods have been adopted in the calculations. We may therefore reasonably accept Table N as the best exponent yet extant of the values of annuities on assured lives. It is true that the years of life do not exactly coincide with the years of assurance, but overlap to the extent of six months: that is, the annuity on a life aged $x+n$, where x is the age at the head of the column, and $x+n$ the age at the side, is the annuity on a life now aged exactly $x+n$ on which a policy was effected $n-\frac{1}{2}$ years ago at the age $x+\frac{1}{2}$. This lack of correspondence is unavoidable from the way in which the experience was originally made up, but it is not worth considering as regards its effects on monetary values. For practical purposes we may reckon that the years of life and years of assurance are identical,— that a_{x+n} is an annuity on a life aged $x+n$ on which a policy was effected n years ago at age x. The only case on which doubts can arise as to the safety of this assumption is where $n=0$, and a_x represents an annuity at the moment of assuring. The tables have been calculated on the supposition that the deaths occurring during the year immediately succeeding medical examination, are exactly double those occurring in the first six months. This assumption is not altogether satisfactory, but it is difficult to devise a better one. In the first part of my paper I touched upon the point, and very briefly explained why I adhered to the course adopted by the Institute in preparing its tables, and treated year 0 as a complete year; and I have not since then seen any cause to alter my views. We have had in the interim the benefit of Mr. Sprague's mature experience, but so far from his recent paper having convinced me that an adjustment is necessary, I am rather confirmed in the opinion that it is unwise to

attempt a rectification of the figures at all. Mr. Sprague made very elaborate and highly instructive endeavours to find the value of an annuity at the moment of assuring, and the results he has arrived at may be worth the thought and labour he has bestowed upon them; but the corrections which he has found it necessary to introduce appear to me so arbitrary as to be dangerous, and it is evident that we must place our trust in the judgment of the computer and not in the facts themselves. In my reply at the close of the discussion which followed the reading of my own paper, I gave with a little more fulness some reasons for thinking that the rate of mortality for the first year of assurance is more approximately equal to that for the first six months than would at first sight be supposed. It is a conjecture liable to be disproved when a larger mass of observations has been made on this special point, but yet it appears probable that some diseases prove rapidly fatal and are as likely therefore to cause claims in the first six months after medical examination as in the second; that some are prolonged beyond a year; but that comparatively few last from six to twelve months, and cut off the lives assured during that portion of the first year which is not included in year 0. But it may be said that even in cases of acute diseases and comparatively sudden death, a few days generally elapse between the first seizure and the fatal end; and I have therefore out of curiosity recalculated the initial annuity for each age at entry, on the supposition that no deaths at all take place during the first month after passing the doctor, and that the deaths entered in the "Experience" under year 0 all fall in the succeeding five months. Making the further assumption that the number of deaths per month remains constant for the rest of the year, it follows that to obtain the total corrected number of deaths for year 0, we must multiply those registered in that year by 2·2, instead of by 2 as is usually done; or in other words we must increase the initial q_x of the "Analyzed-Mortality tables" by 10 per-cent. I say it with diffidence, but it appears to me probable that any further adjustment would be in excess, and that by the above described method we make the nearest approach to perfect accuracy of which the circumstances permit. The following are the corrected annuities as at date of assuring based on this hypothesis. It will be observed that the adjustment has had the most insignificant effects, in most cases barely reaching the second place of decimals. It has not been taken any account of in the succeeding investigations, but the unadjusted values have always been used.

Table O.—*Annuities as at Date of Assurance. Corrected Values.*

Age.	Value of Annuity.		
	3 per-cent.	3½ per-cent.	4 per-cent.
20	21·243	19·522	18·020
25	20·567	18·981	17·593
30	19·652	18·229	16·972
35	18·587	17·335	16·211
40	17·249	16·175	15·206
45	15·480	14·621	13·832
50	14·248	13·525	12·857
55	12·416	11·842	11·321
60	10·863	10·430	10·017
65	9·542	9·205	8·885

It is an easy step from the annuities of Table N to pass to the whole life premiums as at the moment of assuring. They are given at 3 per-cent interest in Table P. They have been placed alongside of the corresponding H^M premiums, and a column added of the percentages by which they differ from that standard. If these percentages showed the true pressure on the office of the cause which produces the difference between the two sets of premiums, they are not of sufficient magnitude to attract serious attention. Mr. Sprague in his late paper on this particular point, after explaining at great length the intricate process by which he reached it, dismissed it in one very brief paragraph. But in the discussion which followed the reading of his paper, I adverted to the fact that we must look deeper in order to ascertain the full effect of "selection" on premiums. It is well known, having been first pointed out by Mr. Sprague, that the mortality of assured lives has three marked peculiarities. It is during the first two or three years very light: after that, and throughout a great, and for assurance purposes the most important, period of life, it is unduly heavy: and in the later years it improves again. The number of policies on the books of an office decreases much more rapidly than if death alone were the cause, and it therefore follows that the number of assured during the years when the mortality is excessive, bears a larger proportion than in the column l_x from which the premiums are calculated to the number later on; and there is no source from which the company is recouped for the loss it thereby incurs. As an illustration, I have prepared Table Q, giving the premiums for age at entry 25 for term assurances of various durations. It will be observed that the percentages of excess over the corresponding H^M premiums are very considerable,—much greater

than in the case of whole life policies, and that not only are the percentages greater, but for forty years the actual differences are larger also. Other ages at entry would show similar results, and it should be noticed that the same cause operating among the older entrants, where the percentages in Table P are negative, tends to raise the true risk premium towards the H^M. It would therefore appear that the H^M premiums at the younger ages are decidedly too low, while those at the higher ages are not in a corresponding degree too high.

TABLE P.—*Annual Premiums per-cent. Whole of Life.* 3 *per-cent.*

Age.	H^M Premium.	Premium for Lives just insured.	Percentage greater (+) or less (−) than H^M Premium.	Age.	H^M Premium.	Premium for Lives just insured.	Percentage greater (+) or less (−) than H^M Premium.
20	1·427	1·582	+ 10·86	45	3·114	3·152	+ 1·22
25	1·625	1·722	+ 5·97	50	3·801	3·643	− 4·16
30	1·880	1·928	+ 2·55	55	4·725	4·532	− 4·08
35	2·193	2·191	− 0·09	60	5·987	5·509	− 7·98
40	2·589	2·576	− 0·50	65	7·705	6·567	−14·77

TABLE Q.—*Annual Premiums per-cent. Term Assurances.* 3 *per-cent. Age at Entry,* 25.

Term.	H^M Premium.	Premium for Lives just insured.	Percentage greater than H^M Premium.
10 years	·725	·828	14·21
20 „	·827	·984	18·98
30 „	·968	1·117	15·39
40 „	1·172	1·298	10·75
50 „	1·414	1·507	6·58

In Table R we have some examples of policy values derived from the new tables, and a comparison of them with those derived from the H^M and the combined H^M and H^{M5} tables respectively. It is hardly necessary to say much regarding them, as the figures do little more than confirm those of Mr. Berridge, vol. 19, p. 360. The most striking fact brought to light by Table R is, that what we may term the natural policy values are considerably below the combined H^M and H^{M5} values for young ages at entry, but gradually creep up upon them, and finally pass them at about age at entry 35 or 40. We have already seen that there is a tendency in experience tables as usually constructed to cause us to ask too low premiums for young lives, and now it further appears that we are also liable to pay too much for the policies when they

are surrendered. Circumstances seem curiously to conspire to render assurances on the lives of young persons unprofitable. The reserve for policies of less than five years' standing, as brought out by the new tables, is very much greater than the H^M, though it seldom reaches the combined H^M and H^{M5}; but the combination is indefensible as applied to so recent assurances, because it omits entirely the years of the experience during which alone they have been in existence. The bearing of this fact should not be lost sight of in considering the question, lately so much discussed, of the proper method of dealing with the expenditure and reserves for new business.

Table R—(see next page).

The chief aim of the paper hitherto has been to prepare the way for ascertaining what accumulations are rendered necessary to a life office by the peculiar mortality obtaining amongst the lives assured, and to show in tabular form the relative amounts of reserve created by the principal methods of valuation in common use. In order satisfactorily to complete the work, it is therefore necessary to have a hypothetical office as closely as possible resembling an average real company in the proportionate number of policies existing of every duration on lives of every age. To construct such an office we must know two things,—the law followed by the entrants of each age in going off the books of the company, and the proportionate numbers that assure at each age. Now we find in the "Mortality Experience" all that we require, always assuming, as we have done throughout, that every life included in it represented one whole-term policy. "Table of observations No. 1" is a complete record of the history of every policy in the 20 companies issued on a healthy male life. At the top of the column for each current year of age is given the total number of entrants at that age, and by passing down the column we learn the fate of every individual. From the same data was prepared the "Discontinuance table", given in vol. 19, p. 404, and the method pursued is fully detailed on p. 391. We are now concerned to know, not at what rate policies terminate from "discontinuance" only, but from all causes, death included. We might obtain the probabilities of the termination of policies by combining the probabilities of death and discontinuance, but the process would be tedious. It is more convenient to prepare a new table, and this has been done. Using the table of existing, discontinued, and died, which had been got ready for the discontinuance table above mentioned, the deaths and discontinuances were added together

TABLE R.—*Comparison of the Values of Policies for £100.*
Interest, 3 *per-cent.*

Years in Force.	Age at Entry.	Analyzed Experience.	H^M.	H^M & $H^{M(5)}$.	Age at Entry.	Analyzed Experience.	H^M.	H^M & $H^{M(5)}$.
5	20	5·366	4·360	6·827	45	11·492	10·228	11·269
10		9·011	9·440	10·915		23·093	21·091	21·830
15		14·090	14·996	16·232		34·124	32·287	32·833
20		18·948	21·119	22·085		44·488	43·243	43·684
25		25·447	27·987	28·833		55·240	53·858	54·089
30		31·879	35·353	36·103		64·797	63·471	63·684
35		38·984	43·175	43·708		72·062	71·420	71·530
40		47·784	51·238	51·631		77·847	77·468	77·619
45		56·476	59·128	59·409		83·099	83·488	83·743
50		64·580	66·772	66·938		91·613	91·471	91·846
5	25	5·794	5·311	6·855	50	14·231	12·100	12·924
10		11·162	11·121	12·414		26·555	24·573	25·180
15		17·077	17·523	18·533		38·787	36·777	37·212
20		23·497	24·705	25·589		50·482	48·601	48·858
25		30·900	32·406	33·190		60·485	59·309	59·546
30		38·757	40·585	41·142		70·456	68·164	68·286
35		46·484	49·016	49·426		77·108	74·901	75·069
40		55·111	57·265	57·559		82·104	81·607	81·891
45		63·468	65·258	65·431		90·934	90·500	90·917
50		71·793	72·496	72·656		...	...	...
5	30	7·096	6·135	7·501	55	15·500	14·189	14·880
10		13·659	12·897	13·963		29·884	28·074	28·568
15		21·046	20·481	21·415		42·935	41·526	41·818
20		28·882	28·614	29·443		54·862	53·708	53·977
25		36·893	37·252	37·840		63·773	63·781	63·920
30		45·421	46·156	46·589		70·853	71·446	71·637
35		53·417	54·868	55·178		79·102	79·075	79·398
40		61·854	63·309	63·492		89·704	89·192	89·666
45		70·547	70·953	71·122		...	...	...
50		77·285	77·274	77·361		...	...	...
5	35	8·084	7·203	8·340	60	19·050	16·180	16·757
10		16·167	15·284	16·278		35·237	31·857	32·198
15		24·721	23·948	24·831		47·549	46·053	46·367
20		33·657	33·151	33·777		58·700	57·792	57·954
25		43·164	42·636	43·098		69·547	66·725	66·947
30		52·508	51·918	52·249		77·927	75·615	75·991
35		61·485	60·910	61·106		88·353	87·405	87·957
40		69·875	69·054	69·234		...	...	...
45		76·285	75·788	75·881		...	...	...
50		81·357	80·912	81·040		...	...	...
5	40	9·997	8·708	9·779	65	21·936	18·703	19·109
10		19·365	18·045	18·996		38·620	35·640	36·014
15		28·639	27·962	28·637		50·782	49·645	49·838
20		38·214	38·183	38·681		60·764	60·301	60·567
25		47·850	48·186	48·542		74·623	70·908	71·357
30		57·168	57·876	58·087		86·890	84·973	85·633
35		67·527	66·652	66·846		...	...	...
40		74·522	73·909	74·009		...	...	...
45		79·989	79·430	79·568		...	...	...
50		84·738	84·926	85·159		...	...	...

for each year of assurance, and hence for each of the ten central ages at entry was deduced a table of "Exposed to risk", and "Terminated", in the manner explained on p. 18 of the "Mortality Experience", for the similar tables of "Exposed to Risk", and "Died", given in that volume. The probabilities of "termination" were then easily calculated, and all that remained was to choose a suitable radix for each central age at entry, and multiply continuously by the arithmetical complements of these probabilities, slightly graduated to remove the more glaring anomalies. The radixes which naturally presented themselves were the total number entering at the five ages which were combined to form each column of the table, because in using them it was at once secured that the proper proportion entered the hypothetical office at each age. In this way was prepared Table S, styled "Model Office". The numbers against year 0 are the entrants at the ages heading the columns, and below them are given the numbers that remain in each year of the duration of the policies. It is in fact a table of l_x for policies instead of lives, for each of ten ages at entry. If we assume an unvarying stream of entrants at each age assuring at the beginning of every year, the table shews us further the number of policies of each duration for each age at entry existing in a model office at the end of any given period of time; and on multiplying these numbers each by its corresponding policy value and taking the sum of the products, we finally arrive at the liability of the hypothetical company.

TABLE S.—*Model Office.*

Duration of Policy.	20	25	30	35	40	45	50	55	60	65	Duration of Policy.
0	8882	22631	26821	23462	17610	12044	7945	4473	2516	1087	0
1	8364	21883	25989	22774	17112	11784	7751	4365	2440	1059	1
2	7330	19938	24186	21217	15969	11002	7201	3996	2234	956	2
3	6633	18592	22853	20156	15129	10409	6807	3757	2076	891	3
4	6064	17537	21718	19207	14464	9907	6453	3583	1929	825	4
5	5644	16682	20778	18446	13891	9475	6149	3409	1801	764	5
6	5350	15961	20065	17763	13368	9099	5861	3232	1696	707	6
7	5092	15336	19382	17193	12906	8766	5597	3039	1592	653	7
8	4765	14619	18534	16463	12313	8280	5263	2816	1439	562	8
9	4571	14215	17936	15982	11927	8001	5045	2679	1329	511	9
10	4402	13820	17487	15579	11548	7724	4841	2541	1227	461	10
1	4245	13417	17016	15171	11188	7449	4645	2402	1129	415	1
2	4100	13081	16578	14754	10842	7171	4448	2266	1034	372	2
3	3965	12744	16127	14340	10495	6892	4252	2128	944	332	3
4	3839	12407	15744	13926	10157	6612	4057	1997	857	293	4
15	3722	12083	15358	13518	9823	6339	3853	1867	773	255	15
6	3613	11799	14975	13130	9484	6076	3646	1746	694	219	6
7	3512	11523	14581	12757	9138	5810	3451	1620	619	187	7
8	3418	11254	14214	12392	8810	5536	3257	1498	546	160	8
9	3329	10982	13856	12041	8507	5270	3068	1373	481	136	9

TABLE 8.—*Model Office*—(continued).

Duration of Policy.	20	25	30	35	40	45	50	55	60	65	Duration of Policy.
20	3246	10711	13500	11692	8195	5019	2878	1246	420	115	20
1	3167	10448	13143	11341	7883	4762	2687	1120	363	97	1
2	3090	10187	12783	10992	7583	4516	2493	997	312	80	2
3	3015	9970	12431	10646	7281	4280	2295	879	264	65	3
4	2944	9754	12070	10301	6969	4024	2094	769	220	51	4
25	2875	9538	11697	9949	6648	3748	1890	666	178	38	25
6	2808	9313	11347	9587	6319	3480	1689	571	139	27	6
7	2745	9089	10998	9220	5999	3215	1498	483	107	18	7
8	2687	8867	10634	8850	5665	2951	1316	402	81	12	8
9	2632	8644	10259	8466	5355	2687	1152	330	61	7	9
30	2580	8393	9890	8079	5047	2426	1005	268	45	3	30
1	2532	8128	9511	7690	4739	2171	860	215	31	1	1
2	2485	7871	9132	7306	4429	1921	718	170	21	...	2
3	2438	7605	8735	6900	4114	1683	583	135	13	...	3
4	2391	7343	8351	6480	3793	1453	461	105	7	...	4
35	2344	7091	7975	6086	3470	1235	355	81	3	...	35
6	2297	6846	7585	5709	3128	1039	272	60	1	...	6
7	2251	6606	7213	5323	2781	866	206	42	...	...	7
8	2205	6368	6839	4925	2446	717	153	28	...	...	8
9	2160	6136	6469	4510	2120	581	111	16	...	...	9
40	2114	5887	6110	4079	1809	462	80	7	...	...	40
1	2073	5646	5756	3659	1533	360	58	2	...	...	
2	2015	5418	5405	3249	1286	278	41	...	...	...	
3	1950	5160	5049	2855	1064	210	26	...	...	...	
4	1882	4899	4681	2483	863	157	15	...	...	...	
45	1805	4628	4300	2137	686	117	6	...	...	...	
6	1731	4361	3907	1813	535	86	2	...	...	...	
7	1661	4097	3506	1517	414	62	...	...	...	...	
8	1582	3826	3106	1255	312	39	...	...	...	...	
9	1502	3545	2708	1018	233	22	...	...	...	...	
50	1419	3255	2343	810	174	10	...	...	...	...	
1	1337	2958	2001	631	128	3	...	...	...	...	
2	1256	2655	1687	488	92	1	...	...	...	...	
3	1173	2352	1396	367	59	...	...	...	...	...	
4	1087	2052	1132	275	33	...	...	...	...	...	
55	998	1774	901	205	15	...	...	...	...	...	
6	907	1517	702	151	5	...	...	...	...	...	
7	814	1279	542	108	1	...	...	...	...	...	
8	721	1057	408	69	...	...	...	...	...	...	
9	629	858	307	39	...	...	...	...	...	...	
60	544	682	229	17	...	...	...	...	...	...	
1	465	532	168	5	...	...	...	...	...	...	
2	392	411	121	1	...	...	...	...	...	...	
3	324	310	77	...	...	...	...	...	...	...	
4	263	232	44	...	...	...	...	...	...	...	
65	209	178	19	...	...	...	...	...	...	...	
6	163	127	6	...	...	...	...	...	...	...	
7	126	91	2	...	...	...	...	...	...	...	
8	95	59	...	...	...	...	...	...	...	...	
9	71	33	...	...	...	...	...	...	...	...	
70	53	13	...	...	...	...	...	...	...	...	
1	39	3	...	...	...	...	...	...	...	...	
2	28	...	...	...	...	...	...	...	...	...	
3	18	...	...	...	...	...	...	...	...	...	
4	10	...	...	...	...	...	...	...	...	...	
75	4	...	...	...	...	...	...	...	...	...	

Besides using table S as the model office for valuation purposes, we can turn it to account in other interesting ways. For instance, we can find what may be termed the "expectation of existence" of policies in contradistinction to the "expectation of life" of persons. The following is a comparison of their respective values, the expectations of life being taken from the "Analyzed-Mortality tables" given with Part I of the paper, and all the expectations being curtate.

TABLE T.—*A comparison of the Curtate Expectations of Life of Persons* (e_x), *and the Curtate Expectations of Existence of Policies* (e'_x) *at the date of assuring.*

Age.	e_x	e'_x	Age.	e_x	e'_x
20	40·109	19·850	45	22·952	17·119
25	37·238	22·830	50	20·302	15·178
30	33·929	22·540	55	16·935	13·169
35	30·415	21·230	60	14·225	10·773
40	26·980	19·323	65	12·093	9·450

In Table V, placed at the end of the paper for convenience, are shown the reserves of the model office made by various tables of mortality at various rates of interest, and comparisons of the same with that by the combined H^M and H^{M5} tables at 3, 3½ and 4 per-cent interest respectively. It is the embodiment of very great labour, and I hope it may prove permanently useful. An ordinary net premium valuation has been made by 25 mortality tables and rates of interest in use in this country; and by the kindness of Mr. Walford I have been enabled to add one by the principal table of American experience, which may serve as a connecting link between valuations as practised here, and those in vogue across the Atlantic. The figures represent the net liability at the end of any quinquennium from 1 to 10, of a company which either divides no profits or pays away in cash any surplus realized, and which issues at the beginning of every year 127,471 policies of 1 each, distributed among entrants of different ages in the same proportions as the entrants bear to each other at the several ages in the 20 companies. The table is a very similar one to that given by Mr. Manly, vol. 14, p. 293, but there are two principal differences. In the first place his model office, though excellently devised, was prepared without the means being at hand to test whether it was properly constructed, while mine is built of blocks cut from the 20 offices' experience. Secondly, Mr. Manly assumes

new business to be transacted only at the beginning of each *quinquennium;* but I have introduced a refinement, and made the influx of policies to take place at the commencement of every *year.* The calculations have thereby been multiplied, but I trust it will be considered that the extra labour has not been thrown away. In presenting the table, it is intended to shew at a glance the ratio of departure of any method of valuation from a defined standard, but it is not meant to be asserted that that standard should be universally adopted in practice. The "Analyzed-Mortality table" is, from its very nature, the most likely to represent accurately at all ages the experience of companies; but as it can never be practically used in valuations, I have chosen, in deference to what I should conceive to be the wish of the profession, to institute comparisons with the combined H^{M} and H^{M5} tables at three rates of interest. The combination is for the first five years of insurance the same as the H^{M} alone, but for policies of longer standing it involves the H^{M} pure premium as at date of assuring, and the H^{M5} annuities and reversions as at date of valuing. Mr. Valentine, when he re-edited Mr. Manly's figures, vol. 18, p. 233, took the same standard as I have done, but even for the new business he combined the tables, a course which would never be followed in practice.

It is strikingly brought home to us, and not for the first time, that if we make a little additional allowance for policies of less than five years' duration, we cannot have a better valuation basis than the combination of the two great Institute tables. The reserves it gives are throughout almost identical with those here shown to be necessary, after an elaborate investigation into the effects of "selection" both for and against the companies. It supplies, it is true, an empirical method of valuation and not a scientific one, but I think that, notwithstanding, we may congratulate ourselves that by so easy a process we can arrive so nearly at perfection. It is however demonstrated that the reserves it brings out are none too high, and an answer is given to those who decry its use on account of the alleged surplus accumulations which it preserves. We must admit that it is unjust to present policyholders to keep back at their expense unneeded funds for the benefit of successors in whom they have no concern; but before we can say how much is surplus, we must ascertain clearly what reserves are necessary. If a valuation be made by a "true table of mortality" we have the result that the sum of the policy values, or, in other words, the reserve at the beginning of the year,

together with what may be not inappropriately termed the valuation-premiums then paid, accumulated at the rate of interest assumed, exactly amounts to the sum of the policy values at the end of the year, together with the claims which then fall to be settled. To produce this result it does not matter what premiums we bring into account as the valuation-premiums. They may be those usually termed the net premiums, or they may be larger or smaller. Whatever they are, if we value them at the rate of interest actually realized, by annuities based on the mortality actually prevailing, the above equation will hold. Now it will be found by trial that if we take a hypothetical company in which we assume the mortality to be that of the 20 offices, as displayed in the "Analyzed-Mortality tables", and value it by the annuities derived from the same tables, or by any other annuities giving, at the same rate of interest, equal reserves, we shall have neither surplus nor deficiency; whereas, if we use annuities which give on the average smaller policy values, we shall find that at the end of the year the funds are too small, and a sum must be provided from another source than the premiums valued and the interest assumed in order to make up the requisite amount. We, in fact, discover that the margin put aside for expenses and profits, or the interest realized in excess of that assumed in the valuations, must be partly drawn upon for the purpose of paying claims, and that the condition of the company is to that extent not so good as it was made to appear. Such is the state of all companies that employ a mortality table not sufficiently stringent. They may be unquestionably solvent; nay even, they may be among the most prosperous institutions in the country; but yet a certain amount of falsification has taken place in their returns, and their affairs have not been displayed altogether in a true light. It is, for instance, not at all uncommon to find in the reports of leading companies statements to the effect that they have made a net premium valuation by the Carlisle table at 3 per-cent interest; that *all* the loading has been reserved; and that the interest which the funds are sure to yield over and above 3 per-cent will also be an unfailing source of great profit. Much of this gloss is delusive. The figures here adduced prove that on the funds resulting from a Carlisle 3 per-cent valuation very nearly 4 per-cent interest must be realized in order to relieve the reserved loading from being taxed for claims; and it is not strictly right, even if the fault be committed in ignorance, to take credit, in glowing language, for the extra 1 per-cent as a source of bonus, and at the same time boast of the full percentage of the premiums laid aside. The Carlisle table, as

well as some others resembling it, gives at 3 per-cent interest a sufficient, but barely more than sufficient, reserve to companies making 4 per-cent on their investments; and it is only interest realized over and above 4 per-cent which will fall into the divisible surplus.

It is not pretended that the conditions of Table V will ever be exactly realized in practice. Certain assumptions—perhaps violent ones—have been made in the construction of the model office, and it may with great plausibility be questioned whether a real company is likely to come so near to it in character, as to find in its valuations a useful guide. The entrants are grouped as they were in the 20 companies, but each insurance institution has its own quality of business, and possibly does not secure its customers at precisely the same age as do its rivals. I do not however think that the average age at entry varies to any considerable extent in the different offices. It is 35·3 in the 20 companies, and companies probably will not find their experience in this respect so different as to weaken a comparison of their valuations with those in Table V. Variations in the rate of discontinuance are a more serious difficulty, but I am in hopes that they also will be found not to interfere to any very damaging extent with the usefulness of the table. I believe that the departure from the rate made use of in preparing the model office must be very great indeed, much greater than is likely to arise in practice, in order to prove a fatal barrier to its employment. The flow of business into the model office is uniform throughout its history; but with no existing company has this been the case. Some began with small things and have increased steadily year by year; some had a vigorous childhood and youth, but have reached a premature old age; yet another class, and perhaps the largest, have had many vicissitudes, at one period successfully extending their connections, and again barely holding their ground. Fortunately, it is easy to make a sufficient correction for this disturbing cause, and Table W has been introduced for the purpose. It shows the liability under policies existing in the model office in each quinquennium of insurance, and enables us by a few simple multiplications to make all the needful adjustment for fluctuations in the amounts of new business, how great soever they may have been. A numerical example will best illustrate its use. The figures are, of course, purely imaginary. It is called Example 1 for the sake of reference, as others are given later on.

Example 1. A company aged 40 years has transacted the following average annual amounts of new business in each quinquennium of its existence.

Quinquennium 1	£250,000		Quinquennium 5	£650,000
„ 2	370,000		„ 6	350,000
„ 3	250,000		„ 7	450,000
„ 4	400,000		„ 8	800,000

Its liabilities under a Carlisle $3\frac{1}{2}$ valuation are £1,970,000. What would they be under a combined H^M and H^{M5} valuation at 4 per-cent?

The new business in the 8th quinquenniun of the company is in the 1st of insurance; that in the 7th quinquennium of the company is in the 2nd of insurance; and so on. Therefore to obtain the liability of the model office under the altered conditions of the flow of policies, we must reverse the numbering of the quinquenniums against the company's new business, and write it thus:

1	£800,000	5	£400,000
2	450,000	6	250,000
3	350,000	7	370,000
4	650,000	8	250,000

And then we must multiply by the corresponding lines in Table W, under the headings Carlisle $3\frac{1}{2}$ per-cent and H^M and H^{M5} 4 per-cent respectively. We thus get the following relative liabilities, dropping four ciphers from each product:—

	Carlisle $3\frac{1}{2}$ per-cent.		H^M and H^{M5} 4 per-cent.
1	1647520	1	1677440
2	2086110	2	2338425
3	2296455	3	2469565
4	5127070	4	5410340
5	3438640	5	3578280
6	2144150	6	2207850
7	2905055	7	2975540
8	1643750	8	1682075
	21288750		22339515

By simple proportion we can now estimate the reserve of the real company, under an H^M and H^{M5} 4 per-cent valuation, taking the two above given sums and the actual Carlisle $3\frac{1}{2}$ per-cent reserve as the three known members:—

$$21,288,750 : 22,339,515 :: 1,970,000 : H^M \text{ and } H^{M5} \text{ 4 per-cent.}$$

Solving, we find that the H^M and H^{M5} 4 per-cent reserve is £2,067,235, or 4·94 per-cent in excess of the Carlisle $3\frac{1}{2}$ per-cent. If it were not desired to make any correction for the irregular

influx of business, we should obtain the members of the proportion at once from the column headed "40 years" in Table V.

527,440 : 552,202 : : 1,970,000 : H^M and H^{M5} 4 per-cent.

from which we get the H^M and H^{M5} 4 per-cent reserve £2,062,486, or 4·69 in excess of the Carlisle 3½ per-cent. It therefore appears that the adjustment effected by means of Table W is, after all, not of very great importance, though it should always be resorted to whenever practicable. It may be remarked that extreme figures have intentionally been selected for the example. It is not necessary for us to know the new sums assured in each quinquennium of the company under investigation. Any other figures giving the relative amounts of new business would do equally well.

Convinced, as I am, that a well-planned model office will prove, in judicious hands, a powerful engine in solving many problems, both theoretical and practical, I have given some thought to devise methods by which its use may be facilitated and extended. It is needless to say more regarding its application to ordinary net-premium valuations. Table V, which speaks for itself, and the above example explaining Table W, are sufficient. But there are other methods of valuation—good, bad, and indifferent—besides that usually called "net premium"; and I cannot conclude this paper better than by demonstrating formulas by means of which, from the net-premium valuations in Table V, others may be arrived at easily and conveniently.

Let us take first the valuations of the model office by the Analyzed Mortality tables, given in the first three lines of Table V. The tables of policy values used were prepared from the unadjusted annuities of Table N; and the liability, therefore, is not the same as would have been shown had the corrected annuities of Table O been employed, though the difference is so slight as to be hardly worthy of mention. But other actuaries may hold divergent views from mine on the treatment of year 0, and may perhaps be disposed to apply their own theories to my tables, and bring out another value for the initial a_x. If this annuity value be altered, while the succeeding ones remain the same, the liability of the model office will be changed too; and we now proceed to investigate in what manner and to what extent.

We have the general formula for the value of a policy—

$$_nV_x = \frac{a_x - a_{x+n}}{1 + a_x}.$$

Let a_{x+n} remain unaltered for all values of n, and let a_x become $a_x+\delta$, where δ may be either positive or negative; and let ${}_n\mathrm{V}'_x$ be the new policy value. Then we have

$$
\begin{aligned}
{}_n\mathrm{V}'_x &= \frac{a_x - a_{x+n} + \delta}{1 + a_x + \delta} \\
&= \frac{a_x - a_{x+n}}{1 + a_x + \delta} + \frac{\delta}{1 + a_x + \delta}.
\end{aligned}
$$

Let, now, $\delta = (1 + a_x)\gamma$

Then
$$
\begin{aligned}
{}_n\mathrm{V}'_x &= \frac{a_x - a_{x+n}}{1 + a_x} \cdot \frac{1}{1+\gamma} + \frac{\gamma}{1+\gamma} \\
&= {}_n\mathrm{V}_x\left(1 - \frac{\gamma}{1+\gamma}\right) + \frac{\gamma}{1+\gamma} \\
&= {}_n\mathrm{V}_x(1-\theta) + \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)
\end{aligned}
$$

when we write θ for $\frac{\gamma}{1+\gamma}$.

This expression does not depend upon the value of n, but holds for every policy value for age at entry x. The quantity γ may be positive or negative, but it will always be a small fraction, and its function, θ, will also be fractional, and positive or negative according to the sign of γ. When the initial annuity value is increased, γ is positive, and the policy values are decreased by a constant percentage and increased by the same percentage of unity. When the initial annuity value is diminished, γ is negative, and the policy values are increased by a constant percentage, and decreased by the same percentage of unity. If there be any number, R, of policies of any, or all durations, effected at age x, and if $\Sigma\mathrm{V}_x$ be the sum of their values based on the original initial annuity, and $\Sigma\mathrm{V}'_x$ the sum of their values based on the new initial annuity, then

$$
\Sigma\mathrm{V}'_x = \Sigma\mathrm{V}_x \times (1-\theta) + \mathrm{R}\theta \quad . \quad . \quad . \quad . \quad (2)
$$

In order to apply formula (2) it is necessary to have a summary of the model office shewing the number of policies, R, in force for each age at entry, and this is supplied in Table X. We should also have, if we wished to attain to mathematical exactness, a corresponding statement of their values, $\Sigma\mathrm{V}_x$, but there would be involved the printing of four pages of figures which possess in themselves no interest. We can make a sufficiently close approximation by finding an average value of θ for all ages at entry, in

a way to be explained later on, and then using it with the sum of the values of all the policies existing in the model office, which is given in Table V. Were the question one of sufficient consequence, a numerical example might here be inserted of the working of the formula; but as it is almost identical with that of more important formulas to be illustrated shortly, it is needless to cumber ourselves with calculations here. The use of the formula has, however, been practically tested, and it is satisfactory to know that the valuations of the model office are not seriously influenced by any such moderate difference in the initial annuity values as would be produced by adopting another mode of dealing with year 0. The liability of the model office at 3 per-cent interest after 50 years is, according to table V, 689,931. Had the initial annuity values given by Mr. Sprague in his late paper been substituted in the valuation for those of Table N, the liability would have been 687,268, or only ·39 per-cent less.

Formulas (1) and (2) are suited to the case where the initial annuity is changed in the expression for a policy value; but this seldom if ever takes place in practice. To change the premium valued,—a very common occurrence,—is really the same thing, and therefore a fresh investigation is not absolutely essential. By means of Orchard's tables the altered annuity might be found from the altered premium, and the above formulas might then be applied. It will, however, be convenient to deduce new expressions more directly bearing on the circumstances of the case.

Let P_x be the ordinary net premium, and ${}_nV_x$ the ordinary net policy value: P'_x the altered premium, and ${}_nV'_x$ the corresponding altered policy value: and let $P'_x = P_x + \delta$.

Then
$$\begin{aligned}
{}_nV_x &= (P_{x+n} - P_x)(1 + a_{x+n}) \\
{}_nV'_x &= (P_{x+n} - P'_x)(1 + a_{x+n}) \\
&= (P_{x+n} - P_x - \delta)(1 + a_{x+n}) \\
&= {}_nV_x - \delta(1 + a_{x+n}).
\end{aligned}$$

But from the ordinary equation for the value of a policy we have

$$(1 + a_{x+n}) = (1 - {}_nV_x)(1 + a_x).$$

Therefore
$${}_nV'_x = {}_nV_x - (1 - {}_nV_x)(1 + a_x)\delta;$$

and if we put $(1 + a_x)\delta = \kappa$,

then
$$\begin{aligned}
{}_nV'_x &= {}_nV_x - (1 - {}_nV_x)\kappa \\
&= {}_nV_x(1 + \kappa) - \kappa \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)
\end{aligned}$$

and $$\Sigma V'_x = \Sigma V_x(1+\kappa) - R\kappa \quad . \quad . \quad . \quad . \quad (4)$$

where R is any number of policies, and ΣV_x the sum of their values.

The quantity κ is positive or negative according to the sign of δ, but it is always fractional. We therefore see that when the premium valued is increased, the policy values are increased by a constant percentage, and diminished by the same percentage of unity; and when the premium valued is decreased, the policy values are diminished by a constant percentage, and increased by the same percentage of unity.

Formula (3) furnishes us with a method of preparing tables of policy values continuously, and it lends itself in an admirable manner for use with the arithmometer. If the values of policies of all durations be in the first place calculated for age at entry x, we can derive from them those for age at entry $x+1$. Our κ in this case is $(P_{x+1}-P_x)(1+a_x)$, and

$$\begin{aligned} {}_1V_{x+1} &= {}_2V_x(1+\kappa)-\kappa \\ {}_2V_{x+1} &= {}_3V_x(1+\kappa)-\kappa \\ &= {}_1V_{x+1}+\Delta\,.\,{}_2V_x(1+\kappa). \end{aligned}$$

We must place κ, taken negatively, on the slide of the machine, and $(1+\kappa)$ on its face. Multiplying $(1+\kappa)$ by ${}_2V_x$, the product is at the same time added to $-\kappa$, and forms ${}_1V_{x+1}$, and then continuing the multiplication of $(1+\kappa)$ by the differences of the column V_x, commencing with the difference of ${}_2V_x$, we have on the slide of the machine successively the values of ${}_2V_{x+1}$, ${}_3V_{x+1}$, &c. The same process may then be repeated for V_{x+2}, taking, of course, for κ, $(P_{x+2}-P_x)(1+a_x)$.

This method of forming tables of policy values is not, perhaps, more rapid than that described by Mr. Peter Gray, in the *Journal* (vol. 17, p. 255), though the differences of V_x are less than those of a_x; but formula (4) may be of great service in checking tables which have been otherwise calculated. If we have R policies, all taken out by persons at age x, but which have been in force for various periods, we have the formula

$$\Sigma_{n-1}V_{x+1} = \Sigma_n V_x(1+\kappa) - R\kappa \quad . \quad . \quad . \quad (5)$$

where n is an indefinite quantity, and not necessarily constant for the various policies included under the symbol of summation. To take an example from Mr. Hardy's *Valuation Tables*, p. 73—the sum of the column V_{40} on that page is 17·52789, and it

consists of 44 values of V_{40}, from ${}_1V_{40}$ to ${}_{44}V_{40}$. To find the sum of 44 values of V_{41}, from ${}_0V_{41}$ to ${}_{43}V_{41}$, we have the equation

$$\Sigma_{0\text{ to }43}V_{41}=\Sigma_{1\text{ to }44}V_{40}(1+\kappa)-44\kappa,$$

where $\qquad \kappa=(P_{41}-P_{40})(1+a_{40})=\cdot 014505.$

Performing the calculation, we have $\Sigma_{0\text{ to }43}V_{41}=17\cdot 14391$. The result of actual addition of the column in Mr. Hardy's book is 17·14393.

The difference in the premium valued may be any quantity we please. If, therefore, we take it negative and equal to the premium, we have the case where no premium at all is valued; that is, from the sum of any number of policy values of any periods of duration for a given age at entry, we can find the sum of the values of the corresponding reversions; or, in other words, we can divide the sum of the policy values into its component parts,—the sum of the values of the reversions, and the sum of the values of the premiums. Symbolically, the fact may be stated,

$$\Sigma A_{x+n}=\Sigma_n V_x(1+\kappa)-R\kappa \quad . \quad . \quad . \quad . \quad (6)$$

where $\kappa=-P_x(1+a_x)=-A_x$, and n, as before, is indefinite. Employing again the previous example, for age at entry 40, $\kappa=-A_{40}=-\cdot 379434$. $\Sigma_{1\text{ to }44}V_{40}=17\cdot 52789$, and the calculated value of ΣA_{x+n},—that is, the sum of A_x from A_{41} to A_{84}—is 27·57231. The sum taken from Mr. Hardy's book, pp. 66 and 67, is 27·57234.

I proceed now, in a few examples, to apply these formulas to the model office.

Example 2. A company aged 40 years uses the Carlisle 3 per-cent table, but brings out its liability by the formula ${}_nV'_x=A_{x+n}-P_{x+1}\,(1+a_{x+n})$. What would be its reserve under an ordinary net-premium valuation?

Our first operation, in every case, is to find what would be the reserve of the model office under the circumstances of the real company; and the second, to pass by simple proportion to the reserve sought for of the latter. The second step having been illustrated already in Example 1, may in future be omitted.

Formula (4) is adapted to the case in hand, and $\kappa_x=(P_{x+1}-P_x)(1+a_x)$. The number of policies, R, for each age at entry we get from Table X against quinquennium 8, and we may conveniently write it L_x. The following is a type of the calculation:

Age at Entry.	$(P_{x+1} - P_x)$	$\times (1 + a_x)$	$= \kappa_x$	$\kappa_x \times L_x$
20	·000372	22·694	·008442	1240·67
25	·000480	21·665	·010399	4707·30
30	·000497	20·556	·010216	5660·06
35	·000671	19·433	·013040	6193·22
40	·000780	18·143	·014152	4710·48
45	·000995	16·863	·016779	3436·98
50	·001621	15·303	·024806	2987·66
55	·002261	13·408	·030315	1785·68
60	·002419	11·491	·027797	753·47
65	·003568	9·917	·035384	363·46
			$\Sigma\kappa_x L_x =$	31838·98

As already stated, in order to secure perfect exactness the formula should be applied to each age at entry separately; but as the necessary tables are not printed, we must content ourselves with the average value of κ. This is evidently $\frac{\Sigma\kappa_x L_x}{\Sigma L_x}$, as each κ has thus given to it its due influence which is measured by the product $\kappa_x L_x$. The value of ΣL_x is found in the summary at the foot of Table X, and in this case is 2383021. In this way we get the average $\kappa = {\cdot}013361$. Inserting it in formula (4) we have

$$\Sigma V' = \Sigma V \times 1{\cdot}013361 - 2383021 \times {\cdot}013361$$
$$= 561755 - 31839$$
$$= 529916$$

= the reserve of the model office under the conditions of the real company.

I would here remark that ΣV is always taken from Table V, and that it is not necessary to calculate again $R\kappa$, that having already been done in finding the average value of κ. With reference to this average value, it is always found in precisely the same way, as also the average value of θ spoken of when discussing formula (2), and it need not therefore be particularly spoken of again.

Example 3. A company aged 35 years values Carlisle 3 per-cent premiums by Carlisle 3½ per-cent annuities. How far does its reserve fall short of that under a true Carlisle 3½ per-cent net-premium valuation?

In this case κ is formed by multiplying the difference between the Carlisle 3 per-cent and 3½ per-cent premiums into the 3½ per-

cent annuity-due, and the average value, which is ·019711, is found as before. Applying the formula, the reserve of the model office by the method of valuation under investigation is found to be 426,155, while that brought out by a Carlisle $3\frac{1}{2}$ per-cent net-premium valuation is, according to Table V, 461,690. The company is thus shown to make but small reserves—about the same as if it had valued by the Carlisle table at $4\frac{1}{4}$ per-cent interest.

This example meets the circumstances of a company valuing the premiums derived from one mortality table by the annuities of another—for instance, the Davies' Equitable net premiums by the H^M annuities. We should find κ by taking the differences between the two sets of premiums,—which might at some ages at entry be positive and at others negative,—and multiplying them into the annuities-due of the table the annuities of which are used in the valuation.

Example 4. The office premiums of a company of 50 years' standing consist of the Carlisle 4 per-cent net premiums, with a loading of 25 per-cent. If it were valuing by the H^M table at 4 per-cent, what percentage must be thrown off the value of the gross premiums in order to make a net-premium valuation?

Two processes are necessary here. We must first find the liability of the company under an H^M 4 per-cent gross-premium valuation. This is done by means of formula (4), forming κ by multiplying the differences between the office premiums and the H^M 4 per-cent net premiums into the H^M 4 per-cent annuities-due. The liability is 448,292. We must next find the value of the sums assured by the application of formula (6), κ for each age at entry being $-A_x$, and its average value being obtained as before. The value of the sums assured is 1,265,682. Deducting the reserve by a gross premium valuation given above, we have the value of the gross premiums, 817,390. Deducting from the value of the sums assured the reserve by a net-premium valuation, 625,308, given in Table V, we have the value of the net premiums 640,374; and it appears that to obtain the value of the net premiums we must throw off 21·66 per-cent from the value of the gross premiums. It is worthy of remark that 21·66 per-cent is not the average loading to be thrown off the gross premiums in order to get the H^M net premiums. We can find the average loading by multiplying the gross and net premiums respectively for each age at entry by the number of policies existing for that age. It will be found in this instance to be 21·23 per-cent of the gross premiums. The results are not the same of throwing off the *average loading* before

valuing the premiums, or the *average value of the loading* when the valuation has been completed. Other examples might easily be selected, in which the difference would be much greater than in the present one.

Example 5. A company makes a gross premium valuation, and throws off from the value of the gross premiums w per-cent, the percentage not being the same as that of the loading to the gross premiums. What is its reserve, as compared with a net premium reserve?

The question might be answered in two ways. We might throw w per-cent off the gross premiums, and then, using the remaining premiums, by means of formula (4) compare the reserves, much as we did in Example 3. This method is the shorter, and is correct where the loading is a constant percentage of the net-premiums by the table under which the valuation is conducted; but the remark appended to Example 4 shews that in other cases it might lead to fallacious results. It is better to proceed as in Example 4, by making a gross premium valuation; and finding the value of the sums assured, and thence ascertaining the *value* of the gross premiums from which to throw off the percentage w.

We have already seen that the model office is the type of a company which either divides no profits, or pays away in cash any surplus realized. It would not be advantageous to introduce other conditions. The circumstances of existing companies are in no other respect so diverse as in the amounts of profits they divide, and the methods of distribution they adopt. It would therefore be quite impracticable to devise a model, even approximately to represent them all. But in employing Table V, this fact must be borne in mind, and before any comparison is instituted between the reserves of the actual company and those of the model, all reversionary bonuses and reductions in premium must be thrown off, to be valued separately by the new table the probable financial effects of which are under consideration. The re-valuation of the bonuses is not a very serious matter. They are already classed according to the present age of the lives, and there are necessary only some sixty or seventy short multiplications into the respective values of A_x by the new table. In all companies the bonuses are no doubt kept apart in the classification of policies from the sums originally assured; but even were this not the case a simple device will enable us to resort to the model office. The liability of the model office can be divided by means of formula (6)

into its component parts,—the value of the sums assured and the value of the premiums; and the value of the premiums can be then substituted for the net liability in Example 1. Thus, for instance, if a company which had made large reversionary additions to its policies, and so blended them with the sums assured as to have rendered them undistinguishable, were desirous of knowing the effects of changing from the Carlisle to the H^M table, the value of the net premiums of the model office should be calculated by means of formula (6) for each table, and used with the value of the net premiums of the real company by the Carlisle table to find the value of the latter by the H^M. The value of the sums assured and bonuses of the real company by the H^M table should then be ascertained by actual valuation, and from it should be deducted the estimated value of the net premiums already found as above.

We have seen that for want of another auxiliary table we cannot attain to perfect accuracy in transforming the valuations of the model office by means of the formulas, but that we must content ourselves with an approximation. It becomes of importance to know how far the approximation may be relied on. Having the necessary table in manuscript, examples 2, 3, and 4 given by the approximate method were also worked out exactly. In example 2, the liability is brought out by the approximate method as 529,916. By the exact method it would have been 530,181, or only 265 greater in a sum of more than half a million. In example 3, the approximate method gives a liability of 426,155, and the exact 426,143, the difference being only 12. In example 4 the approximate method is put to a severe test, the quantity κ whose average is found having the largest value of which it admits. The value of the sums assured by that method is found to be 1,265,682, the true value being 1,262,660, or 3,022 less. There is thus an error of ·239 per-cent. In the same example the percentage to be thrown off the gross premiums is by the approximate method 21·66 per-cent, while by the exact method it is 21·71 per-cent, the approximation thus showing an error of ·230 per-cent. The tables are in manuscript, by which exact results can be obtained, and I shall be always happy to lend them to any gentleman wishing to use them.

One other important method of valuation, that called the "reinsurance" or "hypothetical", I had hoped to bring under control by means of formulas similar to those already discussed; but the connection between the "hypothetical" and the "net-premium" value of a policy has not proved suitable. In investi-

gating this subject however, some rather curious results were arrived at, which may be acceptable to the Institute.

Mr. Sprague, in vol. 11 of the *Journal*, p. 92, showed, by very beautiful mathematical reasoning, the circumstances of loading in which the net value of a policy, ${}_nV_x$, is greater than, equal to, or less than the hypothetical value, ${}_nV'_x$. He there proved, that where ϕ_x is the loading on the net premium,

$${}_nV_x > = \text{ or } < {}_nV'_x$$

according as $$\frac{\phi_x}{P_x+d} > = \text{ or } < \frac{\phi_{x+n}}{P_{x+n}+d},$$

and that consequently ${}_nV_x = {}_nV'_x$ when at all ages $\phi = k(P+d)$.

I was led to pursue Mr. Sprague's investigation further, and to find a general expression for V' in terms of V, irrespective of the age of the policy; from which it appears that when the loading is a constant quantity for all ages, or a constant percentage of the net premium, or a combination of a constant quantity, and a constant percentage, then the reciprocal of the hypothetical policy value is equal to the reciprocal of the net value algebraically increased by a constant percentage of itself, and decreased by the same percentage of unity; that is,

$$\frac{1}{{}_nV'_x} = \frac{1}{{}_nV_x}(1+c) - c.$$

The following is the process by which the formula was arrived at.

Let ${}_nV_x$ be the value of a policy by the net-premium method, and ${}_nV'_x$ its "hypothetical" value. Let P_x be the net premium, and ϕ_x the loading thereon, so that the office premium is $P_x + \phi_x$; and let $\phi_x = mP_x + \delta$, where m and δ are constant quantities for all values of x.

Then $$P_x + \phi_x = P_x(1+m) + \delta$$

$${}_nV_x = 1 - \frac{P_x+d}{P_{x+n}+d}$$

$${}_nV'_x = 1 - \frac{P_x(1+m)+\delta+d}{P_{x+n}(1+m)+\delta+d}$$

Therefore $${}_nV'_x - {}_nV_x = \frac{P_x+d}{P_{x+n}+d} - \frac{P_x(1+m)+\delta+d}{P_{x+n}(1+m)+\delta+d}$$

But $$P_{x+n} = \frac{P_x+d}{1-{}_nV_x} - d$$

Therefore $${}_n V'_x - {}_n V_x = (1 - {}_n V_x) - \frac{P_x(1+m) + \delta + d}{\left\{\dfrac{P_x + d}{1 - {}_n V_x} - d\right\}(1+m) + \delta + d}$$

and $${}_n V'_x = 1 - \frac{P_x(1+m) + \delta + d}{\dfrac{P_x + d}{1 - {}_n V_x}(1+m) + \delta - md}$$

$$= \frac{{}_n V_x (P_x + d)(1+m)}{(P_x + d)(1+m) + (1 - {}_n V_x)(\delta - md)}$$

$$= \frac{{}_n V_x}{1 + (1 - {}_n V_x)\dfrac{\delta - md}{(P_x + d)(1+m)}} \quad . \quad (7)$$

$$= \frac{{}_n V_x}{1 + c(1 - {}_n V_x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

when we write $c = \dfrac{\delta - md}{(P_x + d)(1+m)}$.

Taking now the reciprocal of each side of equation (8), we have the remarkable relation mentioned above,

$$\frac{1}{{}_n V'_x} = \frac{1}{{}_n V_x}(1+c) - c \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Going back to formula (7), we see, as it were pictorially, the connection between the hypothetical and the net-premium value of a policy. In the right-hand member of the equation, the loading affects only the one expression in the denominator $\dfrac{\delta - md}{(P_x + d)(1+m)}$, and by watching the consequences in this expression of any change in the method of loading the premiums, we can perceive how ${}_n V'_x$ is influenced. If the premiums are loaded by a percentage only, then $\delta = 0$, and the equation becomes

$${}_n V'_x = \frac{{}_n V_x}{1 - (1 - {}_n V_x)\dfrac{md}{(P_x + d)(1+m)}},$$

thus showing that, under these circumstances, ${}_n V'_x > {}_n V_x$. If, on the other hand, the amount of loading is equal at all ages, then m vanishes, and we have

$${}_n V'_x = \frac{{}_n V_x}{1 + (1 - {}_n V_x)\dfrac{\delta}{P_x + d}},$$

and we see that ${}_nV'_x < {}_nV_x$. When the constant addition to the premium is the same proportion of d that the constant percentage is of P_x, that is to say, when $\delta = md$, then the algebraical term of the denominator of the right-hand member of the equation disappears, and ${}_nV'_x = {}_nV_x$. In general, according as $\delta >= \text{or} < md$, the factor $\frac{\delta - md}{(P_x + d)(1+m)}$ is positive, equal to zero, or negative, and ${}_nV'_x <= \text{or} > {}_nV_x$. Thus, in another way, and with the policy values actually before us, the theorem is proved which was so admirably established by Mr. Sprague.

It may perhaps be not uninteresting to record in a few words the course pursued in preparing the chief of the tables which form so considerable a part of the paper, and the checks applied at the various stages to insure accuracy. The necessary data were extracted from the "Mortality Experience," entered on suitably ruled paper, and read over. The columns were then independently summed, and the totals carefully compared with those in the printed volume. The values of p_x were calculated in two ways; first, directly from the original facts, and secondly, by preparing q_x, and taking its arithmetical complement. The ungraduated column l_x was calculated by means of the arithmometer, and checked by logarithms. The expectations of life were prepared continuously by means of the arithmometer, and verified at intervals by addition of the column l_x. For the annuity tables, tables vp_x were first prepared on the arithmometer by a continuous method supplying in itself periodic checks, and these were used on the machine to bring out continuously the successive values of a_x, which were then checked at short intervals by means of the logarithmic tables given in Mr. Gray's "Tables and Formulæ." The multitude of tables of policy values required for the valuation of the model office were calculated and checked by the continuous method described by Mr. Gray, vol. 17, p. 255, some of them on the machine, and some by logarithms. The valuations of the model office were effected by means of Mr. J. T. Bottomley's convenient four figure logarithms; the logarithms of the number of policies existing in the model office being written on perforated cards. The check adopted was a very efficient one. Using the Carlisle table as an illustration:—the second differences of the columns for each age at entry at each rate of interest were taken out and compared. The second differences for, say, age at entry 20, at 3, 3½, and 4 per-cent, being placed along side each other. Even a small error could in this way be detected, and there was the further advantage that the

tables of policy values employed received an additional check, any error in them which might have previously escaped being at once made apparent. In fact one or two trifling mistakes in the tables of policy values were by this means discovered.

The arithmetical work was far heavier than anyone would imagine who has not attempted a task somewhat similar. It is sufficient to mention that for Table V alone, irrespective of the preparation of the preliminary tables of policy values, about twenty-five thousand entries had to be made in the table of logarithms. It is with great pleasure that I tender thanks to my friends Messrs. Robert Cross and James MacGowan, both of the Alliance Assurance Company, for the valuable aid they afforded me in the later stages of the calculations. Without their assistance the publication of the paper would have been considerably delayed.

TABLE V.—*Showing the Reserve of the Model Office by Various Tables of Mort[ality] at Various Rates of Interest, and a Comparison of each with the Reserve by [the] Combined* H^M *and* H^{M5} *tables at 3 per-cent,* $3\frac{1}{2}$ *per-cent, and 4 per-cent Interest.*

Table of Mortality and Rate of Interest.		Age of Office. 5 Years. Actual Reserve.	5 Years. Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest, 3 per-cent.	3½ per-cent.	4 per-cent.	10 Years. Actual Reserve.	10 Years. Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest, 3 per-cent.	3½ per-cent.
Analyzed-Mortality,	3 per-cent	28978	1199	1288	1382	87408	1054	1124
Do.	3½ „	27335	1131	1215	1304	82464	994	1061
Do.	4 „	25825	1069	1148	1232	77899	939	1002
Mr. Berridge's table,	3 „	26743	1107	1188	1275	83959	1012	1080
Combined H^M and H^{M5},	3 „	24166	1000	1074	1153	82951	1000	1067
Do.	3½ „	22503	931	1000	1073	77752	937	1000
Do.	4 „	20968	868	932	1000	72933	879	938
H^M,	3 „	24166	1000	1074	1153	78223	943	1006
Do.	3½ „	22503	931	1000	1073	73158	882	941
Do.	4 „	20968	868	932	1000	68483	826	881
Seventeen offices,	3 „	23814	985	1058	1136	77315	932	994
Do.	3½ „	22152	917	984	1056	72267	871	929
Do.	4 „	20636	854	917	984	67633	815	870
Davies's Equitable,	3 „	21923	907	974	1046	70656	852	909
Do.	3½ „	20389	844	906	972	65970	795	848
Do.	4 „	18991	786	844	906	61652	743	793
Carlisle,	3 „	22176	918	985	1058	71804	866	924
Do.	3½ „	20594	852	915	982	66952	807	861
Do.	4 „	19126	791	850	912	62463	753	803
English No. 2,	3 „	23327	965	1037	1113	75555	911	972
Do.	4 „	20267	839	901	967	66213	798	852
English No. 3,	3 „	23478	972	1043	1120	76009	916	978
Do.	3½ „	21891	906	973	1044	71183	858	916
Do.	4 „	20422	845	908	974	66710	804	858
Northampton,	3 „	20787	860	924	991	67358	812	866
American,	4 „	19726	816	877	941	65045	784	837

TABLE V—*(continued)*.

Table of Mortality and Rate of Interest.		Age of Office.							
		15 Years.				20 Years.			
		Actual Reserve.	Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest,			Actual Reserve.	Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest,		
			3 per-cent.	$3\frac{1}{2}$ per-cent.	4 per-cent.		3 per-cent.	$3\frac{1}{2}$ per-cent.	4 per-cent.
nalyzed-Mortality,	3 per-cent	165989	1025	1089	1157	256716	1010	1070	1132
Do.	$3\frac{1}{2}$ „	156902	969	1030	1093	243208	957	1014	1073
Do.	4 „	148478	917	974	1035	230652	908	961	1017
r. Berridge's table,	3 „	162324	1002	1065	1131	253033	996	1055	1116
mbined H^M and H^{M5},	3 „	161949	1000	1063	1129	254062	1000	1059	1121
Do.	$3\frac{1}{2}$ „	152388	941	1000	1062	239925	944	1000	1058
Do.	4 „	143492	886	942	1000	226728	892	945	1000
H^M,	3 „	153988	951	1010	1073	243870	960	1016	1076
Do.	$3\frac{1}{2}$ „	144630	893	949	1008	229960	905	958	1014
Do.	4 „	135947	839	892	947	216983	854	904	957
venteen offices,	3 „	152568	942	1001	1063	242062	953	1009	1068
Do.	$3\frac{1}{2}$ „	143246	885	940	998	228206	898	951	1007
Do.	4 „	134626	831	883	938	215512	848	898	951
vies's Equitable,	3 „	138869	857	911	968	219918	866	917	970
Do.	$3\frac{1}{2}$ „	130157	804	854	907	206859	814	862	912
Do.	4 „	122069	754	801	851	194574	766	811	858
rlisle,	3 „	141594	874	929	987	224965	885	938	992
Do.	$3\frac{1}{2}$ „	132565	819	870	924	211443	832	881	933
Do.	4 „	124181	767	815	865	198827	783	829	877
glish No. 2,	3 „	148890	919	977	1038	236073	929	984	1041
Do.	4 „	131520	812	863	917	210120	827	876	927
glish No. 3,	3 „	149651	924	982	1043	237055	933	988	1046
Do.	$3\frac{1}{2}$ „	140723	869	923	981	223749	881	933	987
Do.	4 „	132415	818	869	923	211323	832	881	932
orthampton,	3 „	132724	820	871	925	210515	829	877	928
merican,	4 „	130195	804	854	907	209319	824	872	923

TABLE V—(*continued*).

Table of Mortality and Rate of Interest.		Age of Office. 25 Years. Actual Reserve.	25 Years. Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest, 3 per-cent.	3½ per-cent.	4 per-cent.	30 Years. Actual Reserve.	30 Years. Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest, 3 per-cent.	3½ per-cent.
Analyzed-Mortality,	3 per-cent	352808	1002	1058	1116	446519	997	1050
Do.	3½ „	334989	952	1004	1059	425056	950	999
Do.	4 „	318442	905	955	1007	404968	905	952
Mr. Berridge's table,	3 „	349291	992	1047	1105	443649	991	1043
Combined H^M and H^{M5},	3 „	351989	1000	1055	1114	447659	1000	1052
Do.	3½ „	333507	948	1000	1054	425421	950	1000
Do.	4 „	316185	898	948	1000	404499	904	951
H^M,	3 „	340282	967	1020	1076	435007	972	1023
Do.	3½ „	322035	915	966	1019	412895	923	971
Do.	4 „	304936	866	914	964	392288	876	922
Seventeen offices,	3 „	338161	961	1014	1069	432576	966	1017
Do.	3½ „	319987	909	959	1012	410645	917	965
Do.	4 „	303185	861	·909	959	390259	872	917
Davies's Equitable,	3 „	307030	872	921	971	392837	877	923
Do.	3½ „	289756	823	869	916	371822	831	874
Do.	4 „	273468	777	820	865	351964	786	827
Carlisle,	3 „	315281	896	945	997	404834	904	952
Do.	3½ „	297409	845	892	941	383175	856	901
Do.	4 „	280691	797	842	888	362834	810	853
English No. 2,	3 „	329850	937	989	1043	422237	943	992
Do.	4 „	295639	840	886	935	380808	851	895
English No. 3,	3 „	330860	940	992	1047	423092	945	995
Do.	3½ „	313368	890	940	991	401976	898	945
Do.	4 „	296960	844	890	939	382067	854	898
Northampton,	3 „	294344	836	883	931	377827	843	887
American,	4 „	295786	840	887	935	381994	853	898

TABLE V—(*continued*).

Table of Mortality and Rate of Interest.	Age of Office.							
	35 Years.				40 Years.			
	Actual Reserve.	Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest,			Actual Reserve.	Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest,		
		3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
Analyzed-Mortality, 3 per-cent	530674	994	1043	1094	600575	992	1039	1088
Do. 3½ „	506262	948	995	1044	573912	948	993	1039
Do. 4 „	483327	905	950	997	548994	907	950	994
Mr. Berridge's table, 3 „	528925	991	1040	1091	599769	991	1038	1086
Combined H^M and H^{M5}, 3 „	533920	1000	1050	1101	605433	1000	1047	1096
Do. 3½ „	508711	953	1000	1049	578090	955	1000	1047
Do. 4 „	484919	908	953	1000	552202	912	955	1000
H^M, 3 „	520708	975	1024	1074	591921	978	1024	1072
Do. 3½ „	495722	928	974	1022	564778	933	977	1023
Do. 4 „	472134	884	928	974	539097	890	933	976
Seventeen offices, 3 „	517937	970	1018	1068	588803	973	1019	1066
Do. 3½ „	493042	923	969	1017	561757	928	972	1017
Do. 4 „	469789	880	924	969	536421	886	928	971
Davies's Equitable, 3 „	470826	882	925	971	536121	886	927	971
Do. 3½ „	446757	837	878	921	509793	842	882	923
Do. 4 „	423980	794	833	874	484835	801	839	878
Carlisle, 3 „	486379	911	956	1003	554348	916	959	1004
Do. 3½ „	461690	865	908	952	527440	871	912	955
Do. 4 „	438427	821	862	904	502028	829	868	909
English No. 2, 3 „	505987	948	995	1043	575616	951	996	1042
Do. 4 „	458804	859	902	946	524235	866	907	949
English No. 3, 3 „	506672	949	996	1045	576208	952	997	1043
Do. 3½ „	482653	904	949	995	550069	909	952	996
Do. 4 „	459925	861	904	948	525283	868	909	951
Northampton, 3 „	453231	849	891	935	517180	854	895	937
American, 4 „	461321	864	907	951	528450	873	914	957

Table V—*(continued)*.

Table of Mortality and Rate of Interest.		Age of Office. 45 Years. Actual Reserve.	45 Years. Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest, 3 per-cent.	3½ per-cent.	4 per-cent.	50 Years. Actual Reserve.	50 Years. Comparative Reserve, 1000 being assumed for the Reserve by the combined H^M and H^{M5} tables. Interest, 3 per-cent.	3½ per-cent.	4 per-cent.
Analyzed-Mortality,	3 per-cent	653849	991	1036	1083	689931	991	1035	1080
Do.	3½ "	625755	949	992	1037	661019	949	991	1035
Do.	4 "	599461	909	950	993	633919	910	951	993
Mr. Berridge's table,	3 "	653674	991	1036	1083	690071	991	1035	1081
Combined H^M and H^{M5},	3 "	659743	1000	1046	1093	696313	1000	1044	1090
Do.	3½ "	630986	956	1000	1045	666746	957	1000	1044
Do.	4 "	603721	915	957	1000	638680	917	958	1000
H^M,	3 "	646061	979	1024	1070	682535	980	1024	1069
Do.	3½ "	617508	936	979	1023	653185	938	980	1023
Do.	4 "	590447	895	936	978	625308	898	938	979
Seventeen offices,	3 "	642655	974	1019	1065	678943	975	1018	1063
Do.	3½ "	614189	931	973	1017	649659	933	974	1017
Do.	4 "	587466	890	931	973	622126	893	933	974
Davies's Equitable,	3 "	586315	889	929	971	620600	891	931	973
Do.	3½ "	558467	847	885	925	591851	850	888	927
Do.	4 "	532033	806	843	881	564557	811	847	884
Carlisle,	3 "	606196	919	961	1004	641252	921	962	1004
Do.	3½ "	577807	876	916	957	612002	879	918	958
Do.	4 "	550948	835	873	913	584284	839	876	915
English No. 2,	3 "	628547	953	996	1041	664202	954	996	1040
Do.	4 "	574375	871	910	951	608402	874	912	953
English No. 3,	3 "	629125	954	997	1042	664781	955	997	1041
Do.	3½ "	601584	912	953	996	636418	914	954	996
Do.	4 "	575423	872	912	953	609461	875	914	954
Northampton,	3 "	566593	859	898	939	600478	862	901	940
American,	4 "	580166	879	920	961	615436	884	923	964

Table W.—*Showing the Liability under Policies existing in the Model Office in each Quinquennium of Insurance.*

Quinquennium.	Analyzed-Mortality.			Quinquennium.	Combined H^M and H^{M5}.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
1	28978	27335	25825	1	24166	22503	20968
2	58430	55129	52074	2	58785	55249	51965
3	78581	74438	70579	3	78998	74636	70559
4	90727	86306	82174	4	92113	87537	83236
5	96087	91781	87790	5	97927	93582	89457
6	93716	90067	86526	6	95670	91914	88314
7	84155	81206	78359	7	86261	83290	80420
8	69901	67650	65667	8	71513	69379	67283
9	53274	51843	50467	9	54310	52896	51519
10	36082	35264	34458	10	36570	35760	34959

Quinquennium.	H^M.			Quinquennium.	Seventeen Offices.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
1	24166	22503	20968	1	23814	22152	20636
2	54057	50655	47515	2	53501	50115	46997
3	75765	71472	67464	3	75253	70979	66993
4	89882	85330	81036	4	89494	84960	80886
5	96412	92075	87953	5	96099	91781	87673
6	94725	90960	87352	6	94415	90658	87074
7	85701	82727	79846	7	85361	82397	79530
8	71213	69056	66963	8	70866	68715	66632
9	54140	52730	51350	9	53852	52432	51045
10	36474	35677	34861	10	36288	35470	34660

Quinquennium.	Davies's Equitable.			Quinquennium.	Carlisle.		
	3 per-cent.	3½ per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
1	21923	20389	18991	1	22176	20594	19126
2	48733	45581	42661	2	49628	46358	43337
3	68213	64187	60417	3	69790	65613	61718
4	81049	76702	72505	4	83371	78878	74646
5	87112	82897	78894	5	90316	85966	81864
6	85807	82066	78496	6	89553	85766	82143
7	77989	74935	72016	7	81545	78515	75593
8	65295	63036	60855	8	67969	65750	63601
9	50194	48674	47198	9	51848	50367	48920
10	34285	33384	32524	10	35056	34195	33336

Quinquennium.	Northampton,	English No. 2.		Quinquennium.	English No. 3.		
	3 per-cent.	3 per-cent.	4 per-cent.		3 per-cent.	3½ per-cent.	4 per-cent.
1	20787	23327	20267	1	23478	21891	20422
2	46571	52228	45946	2	52581	49292	46288
3	65366	73335	65307	3	73642	69540	65705
4	77791	87183	78600	4	87404	83026	78908
5	83829	93777	85519	5	93805	89619	85637
6	82983	92387	85169	6	92232	88608	85[illegible]
7	75904	83750	77996	7	83580	80677	7[illegible]
8	63949	69629	65481	8	69536	67416	[illegible]
9	49418	52931	50140	9	52917	51515	[illegible]
10	33885	35655	34027	10	35656	34834	[illegible]

TABLE X.—*Showing the Total Number of Policies for each age at Entry existing at the end of each Quinquennium.*

Quin-quennium.	Number of Policies existing for Age at Entry. 20	25	30	35	40	Quin-quennium.
1	34035	94632	115519	101800	76565	1
2	58215	168583	208923	184780	138632	2
3	78086	232315	289741	256489	191137	3
4	95204	288584	360867	318501	235271	4
5	110295	338481	422991	371730	271635	5
6	123747	382787	476119	415932	300020	6
7	135937	420825	519823	450394	320565	7
8	146964	452668	554039	474940	332849	8
9	156689	478419	579230	489323	338281	9
10	164584	497503	594800	495736	339949	10
	45	50	55	60	65	
1	52577	34361	19110	10480	4495	1
2	94447	60968	33417	17763	7389	2
3	128910	82223	44077	22500	9056	3
4	156621	98523	51560	25260	9873	4
5	177951	109982	55991	26597	10204	5
6	192710	116642	58045	27030	10271	6
7	201173	119619	58751	27105	10272	7
8	204838	120441	58904	27106	10272	8
9	205960	120587	58906	27106	10272	9
10	206179	120589	58906	27106	10272	10

Summary of above. Total for all Ages at Entry.

Quin-quennium.	Number of Policies.
1	543574
2	973117
3	1334534
4	1640264
5	1895857
6	2103303
7	2264464
8	2383021
9	2464773
10	2515624

Discussion.

The President (Mr. J. Hill Williams) said:—Gentlemen, when Mr. King was good enough to favour us in April last with the first part of his paper, some of us ventured to anticipate with considerable interest the second part, which involved only financial considerations. I think those who then formed that expectation, have not now been disappointed, if they have been able to follow this elaborate, ingenious, and interesting paper. The objections that have been sometimes made to the hypothetical selection of a model office, as it is called, seem to me to be rather overstrained. Worked out, as it has been by the author of this paper, it appears to me that the model office is founded most strictly on scientific principles. It is, in fact, founded on the law of averages, which is the foundation of the whole science of the actuary; and I think, although no one need expect to be able to trace the workings of a particular office by the working of this supposed model office, still it is a point of great interest to us to know what the combined working of all the insurance offices, under the circumstances in which they work together, would bring out; and this is clearly brought out in the paper now before us. This collection of facts, worked out as it has been so completely by Mr. King, and that by the help of many new and ingenious formulas, is very interesting; and it may be a matter of some moment to the members of the Institute generally to know that we may expect shortly to have another very large body of facts submitted to the public, from which very many interesting conclusions may be derived. I have been informed by a letter from Mr. Meech, the Secretary of the Chamber of Life Insurance in the United States, that the American life offices, to the number of twenty-five, have followed the example of the Institute of Actuaries, and are beginning to collect their own mortality experience. When we receive the result of their labours, we shall have many subjects of comparison to make with our own mortality table. In announcing the fact of this collection, Mr. Meech pays some graceful compliments to the Institute, expressing specially his own high appreciation of the *Journal*. He says that in its pages they have found laid down and treated in the most perfect and luminous manner all the subjects that were necessary to enable them to commence their gigantic work; and they particularly specify the labours of some distinguished members, Mr. Woolhouse, Mr. Sprague, and Mr. Makeham. At the same time it is evident they have no intention of following servilely the subjects that have been initiated in this country. With the energy which distinguishes that restless and enterprising nation, I have no doubt that members of the actuarial profession in the United States will look at many questions that interest us largely, from different points of view to ours, and it will be very interesting to us to witness the results of their labours. The meeting will be happy to hear any observations that any members may wish to make with reference to the paper just read, which gives food for a great deal of thought and consideration.

Mr. Manly said that the figures he gave nine years ago in what has now received the name of model office, were only approximately true; and his calculations were based on the assumption that new business was introduced only at quinquennial periods, but still, he

was not prepared at first to see the remarkable difference between his hypothetical office, and the office which has been prepared by Mr. King from established facts. Nevertheless, the comparisons between the use of the different tables are not so diverse as might have been expected. When he read in this paper that Mr. King intended to continue the name "model office", although not considering it a very felicitous phrase, he searched his own paper and found, to his great relief, that he had never used the term at all. He had always referred to it as the "assumed office". Mr. King's proposed office is far better entitled to the term "model office". At the same time, he did not think the directors would accept that as a "model office" which is not increasing its new business from year to year; he would therefore throw out the suggestion whether we might not adopt the term hypothetical office, instead of continuing the unfortunate name. He wished to point out in one line what seems to be simply a slip of the pen. Mr. King in discussing the use by an office of annuity values, and the effect of making a small reserve in the expectation of paying claims out of interest, says that "such is the state of all companies that employ too light a mortality table." That cannot be his intention, as about ten lines above he expresses himself, "whereas, if we use annuities which give on an average smaller policy values, we shall find at the end of the year the funds are too small." The term is not strictly accurate, and should be "companies using a mortality table which gives too small a reserve." His own table, when it was first prepared, had some little novelty in it, and produced some effects that we were not all prepared to see. That has served its purpose for nine years, and must now give place to what is more modern.

Mr. Berridge observed that Table P informs us that the H^M premiums at the younger ages are decidedly too low, but he was inclined to think that this must be a little overstated. The difference seems too large, and at age 20 it may have arisen from a paucity of figures at the later ages, which seems to have struck Mr. King himself in calculating the analyzed mortality for the younger ages, and possibly at the age 65 by his use of the facts as to the year 0. Mr. King has assumed that during the whole of the first year the same light mortality will appear as appears in the half-year immediately following examination; and possibly, in an annuity, the period of which is comparatively short, it may have influenced the value to an undue extent. Comparing his own annuity values with Mr. King's and Mr. Sprague's, he found that Mr. King states the initial annuities, or the annuities at the date of assurance, with greater divergence from the H^M table.

The following are the differences:—

Age.	King.	Sprague.	Berridge.
25	−·465	−·406	−·381
30	−·207	−·191	−·248
35	+·008	−·038	−·137
40	+·079	−·051	·000
45	−·103	−·163	−·020
50	+·359	+·218	+·169
55	+·338	+·172	+·046
60	+·638	+·094	+·029

These are serious differences. We must, therefore, not rush to the conclusion that we are going to make a heavy profit on old lives. He would now say a few words with regard to the peculiar variations of the analyzed mortality from the standard table, which is by Mr. King and Mr. Sprague attributed to the discontinuance of policies. Mr. King says that the variation is rather heavier than might have been expected from our previous ideas. He, Mr. Berridge, thought it probably arises, at least, in some degree, from the class of men who come to assure their lives. No doubt discontinuances have something to do with it; but, on the other hand, we get a sort of middle class coming to assure. The doctor rejects for us the bad lives, but the very best lives do not come in their full proportion, and of the men who do not come in there may be many who know themselves to be "shaky", which "shakiness" the doctor does not detect, and that "shakiness" turns up a few years afterwards in the shape of influence on the mortality experience. No man, probably, can look more than a few years forward—at any rate, the estimate he forms of himself is only for the few years in advance, and, therefore, it follows that those cases do not influence the remoter periods of life, but do influence the mortality within, perhaps, the next twenty years, and it seems probable that we get a class of men among whom there is an excess of mortality within a certain period.

Mr. Sutton had compared the various annuity values given in the paper. Mr. King's object was to separate from the younger lives at entry those selected lives that come in at older ages; in other words, to trace the young lives to the end on their own account. Mr. King has constructed annuity tables for ages 20, 25, 30, and so on. Taking these out and putting them side by side, there are two or three peculiarities worth noticing. For age 45 at entry, Mr. King's annuity is 15·49, whereas for present age 45, entered at 20,—that is after 25 years' assurance,—the annuity is greater. That appears to be rather opposed to what we should expect. Then at age 55, the annuity at entry is 12·43, and the annuity at the present age 55, entered at 20, after 35 years of assurance, is 12·58. Age 50, again, is very remarkable. They are less than those of later policies, and less than those of younger lives. That we may expect. Possibly the variation arises from the way in which Mr. King constructed the tables. He could not agree with Mr. King's remark as to surrender values. Mr. King says that his analyzed mortality tables give, generally speaking, for young ages at entry, smaller reserves than the ordinary tables do, and he says from that we are liable to pay larger surrender values than we ought. But even admitting that surrender values depend upon reserve values, when we look a little deeper into it, and notice the early years of assurance—in other words, when policies are more likely to be surrendered, things are the other way. Mr. King's figures bring out larger reserves than the H^M table. Mr. King in his model office, makes all persons who enter, no matter what their age, assure for the same amount. It is, however, doubtful whether that is a proper assumption to make. Then as to duplicate policies. There is no doubt you often have the same person come on the books after a first insurance is effected. In the Institute Twenty Offices' Experience, the duplicate policies are eliminated. It

is not quite certain that that is the best way of doing it. It is worthy of consideration whether a person effecting a second assurance ought not to be treated as a new entrant at his advanced age. If that had been done, it might have had a material effect on the figures employed in tracing the effect of selection. The question of duplicate policies is by no means an immaterial one. In looking at the Scotch Experience it is found that out of 115,000 policies 23,000 were duplicates. With regard to the mathematical demonstration at the end of the paper, he thought it would have been better if Mr. King had put it as a continuation of Mr. Sprague's in his paper in vol. xi. It would have involved hardly any alteration whatever, and would only have taken two lines instead of half a page.

Mr. Washburn (Boston, U.S.) had on two or three occasions listened to the valuable papers read in this place, and desired that these reports should be generally circulated among the actuaries on the other side. He thought an interchange of documents from one side of the Atlantic to the other would be beneficial to both parties, allied as they are by race and language, and connected so closely by steam communication. Holding these views, it had occurred to him that the members of the Institute would read with interest a valuable paper which had been lately published by the Life Assurance State Society, on the weights, under-weights, and over-weights of lives assured. It has been prepared with very great care by the medical department of the company, embraces about 23 years, and covers 30,000 lives, minutely examined with reference to their heights and weights. Having learnt from Mr. Walford that this subject had not been directly brought under the notice of the Institute, he had written home to New York and obtained several copies of the paper, and now had the pleasure of presenting a copy for the library, and three or four other copies for the use of the members of the Institute.

Mr. Bailey thought that Mr. Sutton had lost sight of a passage in Mr. King's paper, where he speaks of the mortality of assured lives having three marked peculiarities. "It is during the first two or three years very light; after that, and throughout a great, and for assurance purposes the most important period of life, it is unduly heavy; and in the latter years it improves again." If in the latter years it improves again, Mr. Sutton's remark is answered. He had also remarked that the value of an annuity by Mr. King's analyzed table at 55, is very nearly identical with the corresponding values for the entrants at age 20 after 35 years have elapsed. That there is reason to believe that after that period of time the lives on our books are probably better than the average was urged by Mr. Higham, sen., some years ago. Then as to the duplicate lives. When this Institute experience was collected, it was perhaps not sufficiently considered to what uses it might be applied. There was a great desire to eliminate the duplicate lives which had not been done before. As the materials are yet extant, the several cards relating to the policies on the same life were placed in one envelope, and these envelopes could be produced and analyzed if thought necessary. The duplicate lives consist of two classes, (1) the same man assuring at different periods of life, and (2)—what is a common case—assurances for a large sum effected at different offices at the same time. If, for instance, such a sum as

£100,000 were assured on the same life at the same time in 20 offices, when the life failed it would be counted as 20 deaths under the system of the old experience.

Mr. GORDON wished to put a question on one of the most interesting points of the paper, namely, with regard to the Tables P and Q. Mr. King says he has brought before us some short-term policies; and it should be noticed that the same cause operating among the older entrants where the percentages in Tables P and Q are negative, tends to raise the true risk premium towards the H^M. It would therefore appear that the H^M premiums at the younger ages are decidedly too low, while those at the higher ages are not in a corresponding degree too high. Now turning to the table it is seen that at the younger ages, 20, 25, and 30, the percentages by which it exceeds the H^M premium, are 10, 5, 2; while at 60, 65, it is less by 7 and 14. The extreme of the case of lower premium is greater than the extreme of the case of a higher premium. Why, then, does Mr. King say the one is decidedly too high, and the other is not correspondingly too low?

The PRESIDENT—Before asking the author to reply, I should like to make one observation in reference to the correspondence from the Chamber of Life Insurance of New York. It expresses the determination of the American actuaries to do that which has been alluded to—to value policies and amounts, and not lives. The subject is giving them a great deal of consideration, and they feel very much inclined to follow the example of the Institute, but they find the opinion of several actuaries is rather in favour of valuing policies and amounts, and not eliminating the duplicate policies.

Mr. KING in reply said that the way in which the Institute treats its younger members is very encouraging to them, and at the same time conducive to the interest of the profession. When he joined the Institute a few years ago, not knowing anyone in it, he feared it might be difficult to gain a footing, but he had found everyone very willing to help those who take an interest in these subjects, and his connection with the Institute had for the last few years been one of his greatest pleasures. Mr. Sutton's remark as to annuity values had been answered by Mr. Bailey. Mr. Sutton would find that the peculiarity to which he adverts is observable, not at one or two ages only, but it runs all through the tables, and is so general as to be a well marked law. Mr. Sprague had pointed out the same thing when he wrote a paper some years ago on the effect of selection amongst assured lives. In regard to the difference between the annuities as at date of assurance, brought out in this paper, and those of Mr. Berridge and of Mr Sprague, it is to be remembered that in Mr. Berridge's tables the "selection" was traced for only 20 years, and not for the whole of life, as in this paper; and this may at least in part account for the differences in the annuities. He did not think there is much danger of Mr. Manly's work of nine years ago being superseded. Only at one or two points does the present paper trench upon his. The expression "light mortality table", criticized by Mr. Manly, has special reference to the financial effects of the table, as the context of the passage shows. No one who has gone into the question of the influence of mortality on policy values can have failed

to discover how the reserves required by one table and those by another do not depend on the actual rate of mortality, but on the manner of its incidence at the different ages. He knew no paper that brings this fact out so admirably as Mr. Meikle's "Policy Life Lines", which shows very clearly the relation between mortality and policy values. With regard to the observations that had been made on Tables P and Q, Table Q is meant as a commentary on Table P. Table P gives the whole-life premiums, and Table Q gives the premiums for term assurances calculated for one age at entry. The same principle that operates at that age, will operate at all the others. The principle to be demonstrated is this, that the lighter mortality that takes place among those long assured, tends to reduce the whole-life premiums, as given by the tables, in an undue degree for all ages at entry. For if we take the term premium, and compare it with that by the H^M table, we find it considerably higher in proportion than the whole-life premium. I mean to express the fact that the loss incurred on the young entrants by charging the H^M premium, is greater than is apparent from the percentages of Table P, while that table also exaggerates the profit to be made on old entrants; so that the gain on one class of lives will not balance the loss on the other.

How to Wind up an Insolvent Life Insurance Company.

THE above question has at the present time become one of practical importance in the United States of America, and it will probably interest our readers to see the ideas that are prevalent on the subject on the other side of the Atlantic. We therefore reprint below, an article that has been contributed to the *Spectator* of New York and Chicago. The question was much discussed in England a few years ago, on the occasion of the failure of the *Albert* and *European* offices. Two papers on the subject appeared in the 16th volume of this *Journal:* Mr. Sprague's *On the Liquidation and Reconstruction of an Insolvent Life Insurance Company* (p. 229), and Mr. Bailey's *On Insolvency in Life Assurance Companies* (p. 389). In the 15th volume, p. 385, there is a letter from Mr. D. Pitcairn commenting on Mr. Bunyon's pamphlet *On the Liquidation of an Insolvent Life Office* (Laytons, 1870); but no other notice of this pamphlet has hitherto appeared in our pages. Attention being now drawn to the subject again by the failure of some American Life offices, one of which, the *Continental,* is said to have transacted a larger new business than any other company in the world, we have thought it desirable to place before our readers a fuller account of Mr. Bunyon's pamphlet than is

contained in Mr. Pitcairn's letter; and we therefore reprint, with the author's kind permission, the following passages from it.—Ed. *J. I. A.*

Extracts from the pamphlet "On the Liquidation of an Insolvent Life Office", by C. J. Bunyon, M.A., *Barrister-at-Law, and Actuary of the Norwich Union Life Insurance Society.*

The true principle of the liquidation of a life office is the application of a deficient fund so far as it will go in the solution of the liabilities to which it is subject. The principle of the Winding-up Act is to endeavour to ascertain the value of those liabilities, often incapable of valuation, and to divide the common fund in proportion to such assumed values. The Court of Chancery, then, can scarcely here do complete justice; and this is precisely the case in which Parliament should interpose, and supply increased powers and an amended form of procedure. Neither in this nor in any other case can Parliament interfere to alter the rights of individuals and form new contracts for them; but to supply deficient powers and regulate imperfect contracts, is among its commonest functions. The skill of the conveyancer in preparing the deed of settlement, might have anticipated every possible contingency in the history of a life office; and to supply omissions is here as reasonable as to give effect to an estate bill, or enact that deeds of settlement or mortgage effected after a certain date shall have powers supplied by implication.

The contract between the office and the insurer is in form in almost every case, that, in consideration of the premium paid by the latter, the funds and capital of the company shall be liable to the payment of the sum assured, together with such additions, in the case of profit policies, as may be made thereto in pursuance of the provisions of the deed of settlement. And it is generally provided that no shareholder shall be personally responsible for more than the unpaid part of the capital subscribed by him. In this contract, in a proprietary company —and the purely mutual insurance societies are few in number—there is, in legal language, no privity between the policyholders. They are not partners; they have no right to interfere in the management of the office. Their right is limited to the recovery of the sum assured, and the additions thereto when payable; and it is only when there is the prospect of an open breach of the contract—such as a proposal to transfer the funds in direct contravention of the provisions of the deed incorporated by reference in the policy—that they can act together in equity; and only then in the same way that any persons having similar interests can combine, by one or more of them filing their bill of complaint on behalf of themselves and all other persons similarly situated. They are beneficially interested in the invested capital of the company, but have no specific lien, no charge upon it, no right to interfere with it, no right of preference in respect of it, as against even the ordinary simple contract creditor of the office. They look upon the shareholders' capital, whether paid up or not, as a guarantee fund for the due performance of the contracts entered into with them, and with justice in the majority of instances; but it is doubtful

whether they can enforce the guarantee, except through the medium of the Winding-up Acts; and it is certain that, up to the present time, no bill has ever been sustained, of which the prayer was that a call upon the shareholders should be made on the ground of an estimated deficiency in the assets. Unless we can prove actual insolvency, the powers of a policyholder in a proprietary company are really limited to the right of remonstrance; and, except so far as they may be calculated to affect the business of the office, the shareholders may allow the policyholders' complaints to pass utterly unheeded. The question of bonus is usually thought one of the most important by the policyholder; but it has been considered that he cannot compel the shareholders to carry on the business to make bonuses for him, and there is no precedent for compelling them to cause valuations to be made, and a bonus to be declared, if they are unwilling to do so.

When a winding-up order has been made, the positions of the parties are greatly changed; the policyholder becomes a creditor, and the shareholder a contributory. The term creditor might have been thought inapplicable to the case of a policyholder during the existence of the life assured, but by the provisions of the winding-up clauses of "The Companies Act, 1862", "all claims against the company, present "or future, certain or contingent, ascertained or sounding only in "damages, are admissible in proof, a just estimate being made, "as far as possible, of their value." Acting upon a like provision in a previous Act, the Court directed an inquiry what assurances and annuities were in existence on the day when the company closed its business, and the then value of such assurances and annuities; and the owners were entitled to prove as upon an actuary's valuation of such liabilities. The company would then be wound up upon the payment of those values, so far as the assets would suffice, assisted by such calls upon the shareholders as could be made; first deducting the costs of the winding-up, including the charges of the liquidators, except so far as such costs and charges could be thrown on the shareholders alone.

This, however, may be scant justice to persons whose health will not allow them to insure in another office; while to others who may be able at once to effect new policies in a solvent company, it may give more than the values which their policies might otherwise realize, since an insurance company rarely purchases its policies at their full value, and at the auction mart they might be worthless.

The law, moreover, as we have stated it, by no means represents the feelings of the public, or the justice of the case, when the true principles of life insurance are considered. It cannot be said to be at fault, because it holds people to their contracts; but a broader view is taken of the case, and it is felt that the respective parties do not fill their true positions; that the written policy does not fully represent the real contract, and that there are, from the nature of the case, implied agreements unprovided for, if not at variance with the formal instrument.

The policy and the deed of settlement, professing to define the relations of the assurers and the assured, treat, as a rule, the shareholders as the one, and the policyholders as the other. But, useful as the share capital may be, either in the management or in protecting

the integrity of contracts entered into,—in other words, in acting as a guarantee fund,—it would be notoriously insufficient without the contributions of the policyholders, the due care and application of which, in consideration of the guarantee, has been confided to the shareholders. Underlying the whole contract, is the original idea of the Mutual Assurance office.

It is obvious that the liabilities of a life office vary in kind from those of almost any other company; and, being liabilities undertaken to be performed at future and distant dates, although susceptible of valuation with some accuracy in the mass, are in detail scarcely capable of valuation at all. For example, if the liabilities of an office are represented by one, five, or ten millions assured, an actuary can tell very nearly what sum of money, in addition to the annually accruing premiums, will eventually pay the claims; but in the case of single policies affected by the individual circumstances of the assured, circumstances of health, position, or even necessity, it becomes impossible to set a value upon them. Hence, to value the whole as an actuary would value them for a bonus, and admit individuals to prove for the separate values, does not meet the justice of the case. The Court might indeed endeavour to provide for all special cases by having a special valuation made whenever a life had become impaired by disease or extreme old age; but the expense of such a valuation would be immense; the outrage upon the feelings of invalids would be monstrous; and as the best lives could not be assumed better than the average of the tables, the increase of the shareholders' liability would be considerable.

To permit the policyholders to prove singly is to allow the liquidation to change the very nature of the contract. In the majority of offices the assured are not entitled, as of right, to surrender their policies at all, and certainly not to withdraw the full amount of their official values. To do this, especially at times of panic and discontent, would be to offer a premium to the better lives to withdraw from the concern, leaving only the aged and sickly to continue it. But if this is necessary in a company that is solvent, *à fortiori* it is so in one that is insolvent. The primary object, moreover, of life insurance being to make a provision for the widow and fatherless children, it is possible that a company might be so wound up as to provide, through its subscribed capital, the full value of its liabilities; and, if so, what reason can there be for allowing the liquidation to change the form of every contract, and, frustrating the good intentions of the assured, turn a provision for his family into a small sum of ready money? Again, at the risk of repetition, it must be observed that, when a man effects a life insurance, he seeks for a security as much against failing health as sudden death, against uninsurability for the future as well as any immediate necessity. There is no reason why this should be lost sight of in considering the case. The result is, that the only mode in which the liquidation can with justice be effected, is, permitting the liquidators to receive a proof in gross for the policyholders, as a body, on all their policies, and in winding up the concern according to the tenor of the contracts.

To the original projectors of life assurance, as now carried on in England, a deficiency appeared a thing perfectly possible and to be

provided against. The visitation of the plague in 1665 was not so distant but that the founders of the Equitable Society in 1765 deemed it necessary that it should be the subject of a special provision in their deed of settlement, that "if at any time the City of London should be "visited with the plague, or any contagious sickness, the court of "directors (if it should appear to them that the affairs of the society "required it) might reduce the payment of the respective sums of "money which should become due by reason of the deaths which "should happen in such time of public calamity, to any sum not less "than one-fourth part the sums due, giving credit for the remainder "in the books of the society, to be repaid with interest at 3 per-cent, "as soon as the affairs of the society would admit." Nor is this provision a singular one; in the case of the Norwich Union Life Office, founded 40 years later, it was repeated, and with this addition, that if at any time after a valuation the funds appeared insufficient to provide for the entire liabilities of the society, the majority of members present at a general meeting might reduce the sums payable to the respective claimants by virtue of policies of assurance.

In these cases such provisions must be considered rather as a monument of the farseeing providence of our predecessors, than as ever likely to be put in force. But they point out the true mode of administration of the funds of companies of this kind in case of need. A company under such regulations can never be insolvent and need never be wound up by the Court of Chancery.

It is submitted that to wind up an insolvent life office on the principle of a rateable reduction of claims and by the application of the common fund according to the tenor of the respective policies of assurance, is the only just mode of proceeding, and that which is required by the necessities of the case, by the rights of all parties, and by broad principles of equity underlying the nature of the institution itself. Let us then see how easily such a winding-up could be effected. During the panic which ensues upon the declaration of insolvency it would be almost impossible to say what would be the position of the company at the expiration of two or three years from that date. A very large number of policies would undoubtedly be discontinued, and on their lapse the liability in respect of them would cease. The common fund would not, on the whole, benefit from such withdrawals. In the more recent policies the present values of the premiums to be received would exceed the values of the liabilities on the sums assured, according to the methods of valuation commonly in use among insurance offices; and by the withdrawal of a large number of lives, of which the majority would be select lives, the rate of mortality among the residue would be injuriously affected. Nevertheless, the full effect of this operation would be ascertained within a comparatively short period; and the probability would be that the condition of the office would find its level at the end of three years, after which it would gradually improve. In the meantime, after the suspension of the payment of claims, funds would accumulate in the hands of the liquidators, and a great diminution of expenses ought to be immediately effected.

The proper plan would then be for the liquidators to pay interest at a low rate upon the claims matured by death; and so soon as a

valuation could be made, and which should annually be repeated, pay interest according to the estimated measure of the solvency of the concern. Thus, if upon a valuation it appeared that the company was solvent to the extent of 75 per-cent of its liabilities, £2. 5*s*. per-cent, or three-fourths of 3 per-cent, assuming 3 per-cent as the normal rate, should be the rate in use; if at the end of another year it appeared that the measure of solvency was 80 per-cent, the annual interest should rise to £2. 8*s*. per-cent; and this arrangement should continue during the liquidation.

When funds accumulated in the hands of the liquidators, and they were advised that they might be distributed, claimants, commencing with those earliest in point of date, should have the option of receiving payment of their claims according to the proportion of the sums assured for which the last valuation might have shown the company to be solvent.

The majority would be glad to receive their money subject to this necessary deduction; but those who were not pressed for money, and who had confidence in the working of the fund, would probably pass over their turns to the next claimants. So long as the capital remained in the hands of the liquidators, it would bear interest and might be made to do so at a higher rate than that which the fund would pay to claimants, so that a benefit would accrue from the refusal to accept what must be acknowledged to be a composition.

The expenses of liquidation, we repeat, need be but small. All liability to pay commissions would of course cease, since the responsibility in that respect must belong to the company which would be defunct, or with its animation at least suspended. An expensive establishment would not be required, and premiums would be paid direct to the office without deduction. A sufficient staff of clerks must be had, and the expense of an annual valuation must be borne, and the costs of supervision; but after the first expense of arrangement was incurred, little more than 1 per-cent on the income, the author imagines, need be expended in a large company.

Recent insurers in full health and vigour, who could readily insure elsewhere, might object to such a scheme, as by discontinuing to pay their premiums they would forfeit all claim to a dividend; but they would only be held to their bond according to the actual terms of their policies, and the loss of a dividend upon small surrender values, even if it were admitted that they were entitled to them, which we shall hereafter see is a question still open for decision, would be no very great hardship. On the other hand, those for whom life insurance is really primarily designed—the widows and children of deceased insurers—would not be reduced to destitution, as they seem now likely to be according to the present practice. They would, it is true, receive a smaller income than they had anticipated, upon the sums assured on their behalf, but still a provision relatively commensurate with the claims of their fellow sufferers. The greater part of those sums assured they would undoubtedly receive, *and it is by no means improbable that the office might ultimately work solvent;* but for them, at least, it would be far better that a very considerable reduction should be submitted to, rather than that the provision made for them should wholly fail.

Should the office work round into a solvent condition, the rights of those persons who had received less than the sums assured by their policies, would seem to require consideration, and there would be even a remote possibility that the shareholders might receive back some portion of their subscriptions; but it would probably be better, unless the latter could raise a sufficient fund to pay all claims in full and recommence business, that the account should be closed by the transfer of the remaining risks and funds to some solvent company. For this purpose large powers should be vested in the liquidators during the course of the liquidation, not only when solvency was attained, but at any prior time, with the consent of the majority of the assured, and after the exhaustion of the subscribed capital, to transfer the risks and funds to some solvent company in consideration of the issue by it of policies, if necessary of reduced amounts, to the assured. In such a case, equity would require that, as between the separate classes of profit and non-profit policies, the new insurances should be of the same denomination as those for which they were exchanged. It is, however, submitted that in any such arrangement the shareholders should have no voice. Having failed in their guarantee, they should not be allowed to make terms with the assured and exercise a right of veto as a means of evading their own responsibilities.

In the case of annuitants, it will not unfrequently be found, that they are, by virtue of the deeds of settlement of insurance companies, entitled to a priority, in which case they should undoubtedly be paid in full; but if they are not, they should receive regularly, during the liquidation, such a proportion of their annuities as might be indicated by the estimated measure of solvency of the concern. We should not then hear of appeals to the guardians of the poor, by annuitants brought suddenly by the failure of an insurance company from comparative affluence to poverty; and it is to be remembered that, in an annuity, time, or punctual payment, is of the essence of the contract, and that a protracted delay would, in the majority of cases, be a far greater injury to the annuitants, even if followed by payment in full, than the regular payment of a reduced annuity.

For the weak, the infirm, the sickly, the poor, the unprotected, the widow, the fatherless children, the mode of winding up for which we argue would undoubtedly be a benefit; but unless they are entitled to it as a matter of abstract justice, it would be worse than useless to contend for it. And here this question, like so many others connected with insurance, becomes a matter of actuarial calculation. It has been pointed out already that, in the case of impaired health in an insurer, the valuation of a policy would be beset with very great difficulty. Another question for solution would be the mode in which claims arising upon deaths happening during the liquidation should be dealt with; but the point now to be considered is that known to actuaries as the controversy respecting the valuation of gross or net premiums. It may be thus stated:—Since an insurance office is liable to fluctuations of mortality, to occasional and exceptional losses, and considerable annual expenses, it is necessary to charge premiums which shall be higher than those which would be required if it were certain that the rate of mortality in the company, and the rate of interest obtained on investments, would exactly coincide with

the tables of mortality and interest selected for the valuations, and that there would be no expenses or losses. This is generally arranged by making an addition to the scale of premiums assumed as likely to be sufficient without expenses, either in the shape of a rateable percentage or a rateable percentage and a fixed addition, or by assuming a higher rate of mortality and lower rate of interest than are likely to occur in practice. From this addition, technically termed the loading, the payment of commissions, expenses, and bonuses, is provided. This loading, except in relation to the rate of mortality and interest adopted in a valuation, has, however, no existence except in the mind of the actuary. It is his guide for determining what reserve should be made before a surplus is declared; and he rightly judges that, previous to a distribution, the whole of the value of the loadings on all the policies should be reserved, as otherwise he anticipates profits, which should be esteemed to belong to the future, and which on lapsed policies may not be received. The necessity for such a reservation is especially proved by observing that, without it, a company cannot justly invite new insurers to join it, charging them according to its published scale of premiums, of which on its existing policies an undue proportion has been lost or expended. This is a question of equity, as between the office and the public, and of sound insurance finance; but it lies wholly beside the insurance contract as contained in the policy. The reservation is made either by striking off from one-fifth to one-quarter of the premiums before valuation, or by setting aside a reserve equivalent to the computed value of such deductions. The result is the same by each method. As regards the individual policyholder, however, the value of his policy must depend upon this mode of valuation. If the full premium is valued, it will be some years before his policy attains a surrender value, it will even have a negative value in so far as the present value of his premium—increased (say) one-fifth beyond the assumed necessity of the case—exceeds the present value of the sum assured, the difference going to swell the reserve, or safety and profit-giving fund of the company. The question then, on the tender of proof by the policyholder of recent date, will be, in what way his policy is to be valued. Can he assume that the premium upon his policy is of a smaller amount than the policy declares? Is he entitled, on an insolvency, to assume that the office has on hand the full value of those loadings which were added to provide for contingencies which have occurred? Can he import this question into his contract, and say, "The office would "have required a higher premium at my present age than was charged "me some time ago, and I therefore claim to prove for the value of "the difference between those two charges"? Does not this last argument, the most specious that can be brought forward, assume a reason for fixing a value, which may be, and in fact is, wholly incorrect? He argues that because the company's prospectus asks a premium of say £4. 10*s*. to insure £100 at 49, therefore at that age, £100 payable at death is worth an annual premium of £4. 10*s*. paid in advance, which cannot be proved to be true. On the contrary, the mortality tables show it to be worth much less money. If, however, the policy is to be valued, like every other contingent property, for what it is actually worth, then a very large proportion of the policies become

valueless, and the entire reserve and assets of the company are devoted to the older insurers. This question is of extreme importance, as, if decided against the more recent insurers, it removes the principal difficulty in the way of the mode of winding up which we propose, for there is then no large class to contend with whose interests lie in the opposite direction. There can be little, if any, doubt but that this must be the legal decision, as may be deduced from the recent case of the petition to wind up the European Insurance Company, in which the Court declined to follow the view of the actuaries, whose arguments were based on this principle of net valuation. The actuary of the company had admitted that the loading had not been reserved; and, passing by the question in its theoretical form, the Vice-Chancellor remarked, "It did appear to me for a long time that it would be "necessary to add to the liabilities that further sum which the com-"pany's actuary in his cross-examination stated would have to be "added to them if the expenses were included, which seems to have "reached in the past year £70,000, a very large amount, which I am "told is likely to be diminished. During a considerable part of the "hearing, I was much impressed with that view; but I do not think "it is a sound one. If the company were content to say, 'We "'are willing to go on with our existing contracts only, rather than "'incur the expense of agents, offices, and the like; we are content to "'receive the premiums upon our existing policies, and to apply them "'in payments of our liabilities'; if they are minded to take that "course, and not to be competitors with other offices for fresh insur-"ances, I do not know why they need take £70,000 a year for "expenses, or even incur any large proportion of that sum." And again, "If I were to take upon myself to appoint a manager as "receiver of the premiums due from the policyholders, all the policy-"holders would have to pay their money—for that is the contract—"without putting the company to any further expense. Therefore I "do not think that there is that inaccuracy in the estimate of the "liabilities which was at first pressed upon me; and I cannot add to "it that three quarters of a million, or any great portion of it, in "testing the question of solvency."

The mode of winding up now contended for, seems then the only way in which the rights of the amalgamated policyholders can be preserved against their insurers, and it admits of a careful and accurate measurement of their respective liabilities. At the root of every liquidation, however effected, lies the question of the mode of valuation—that is, of the rates of interest and mortality to be adopted in estimating the contingent claims on immature insurances. This must be determined by the Court, with such actuarial assistance as it may think fit to summon to its aid. While, as a rule, in the common course of winding up a joint-stock company, it is necessary that proof of claims should be made by the creditors' solicitors, in a company where the claims were so numerous as in a large insurance office, the first step would be to call forth an army of attorneys, many thousands in number, each armed with his bill of costs to charge the unfortunate concern. To incur such an expenditure would of course be, in a measure, unnecessary; as no formal proof can be required when the books of the company correctly show the liabilities and the

creditors are to be found in the policy register; but it is to be remembered that policies of assurance as soon as granted may be the subject of settlement or mortgage by assignment or deposit, of which the company may have notice, but no proof, so that costs of this kind must be incurred, however desirous the liquidators may be to spare the assets.

Another great advantage in our scheme would be found in the ease and licence in the payment of calls which it would allow to the contributors. The money to be called up would be required as active interest-bearing capital at once, but its immediate payment would not necessarily be immediately required; hence time might be given and securities taken, which, in an ordinary winding-up could not be accepted, to the great benefit both of creditors and contributors.

The following is the paper contributed to the *Spectator*:—

How to Wind up a Life Insurance Company.

[From the *Spectator*, of New York and Chicago.]

That a life insurance company ought never to fail is an admitted truth. The scientific basis of the business is the most nearly perfect of human contrivances; and if the practical organization and management of it were as good, the familiar old saying that there is nothing in the commercial world which approaches a good life company in security, would be within bounds. But the theory of the business is not always equalled by the practice; and it once in a while happens that a life insurance company has to be wound up. How can this best be done?

Several ways have been tried. Reinsurance, in whole or in part, in some other company; the distribution of the funds under the supervision of a court; and the transfer of the trust to a state officer for distribution; have been the favorite plans. None of them has succeeded. That is to say, each of them has resulted in great waste, loss, distrust, and general sense of wrong among the insured. The fact is, that when a large fund, hitherto held and managed for a definite purpose, is diverted from that purpose, and suddenly thrown into new hands, all parties concerned are stimulated to embrace the last chance of obtaining something from the wreck; the ordinary restraints of habit and business principles are mainly removed; and the situation strongly resembles a general scramble for the spoils. As long as human nature is unchanged, this result is probable. If any satisfactory way of administering upon a defunct insurance company is devised, it must be by avoiding such a shock, and maintaining the continuity of the trust and of the business habits and principles which control it.

This consideration points to the simple and obvious rule, the only one which has never been tried in this business, and the only one which could ever have been thought of, had common sense controlled the matter; when a life company can no longer meet the legal standard of strength, and must stop issuing policies, let it at once cut off al'

the agencies, offices, and salaries, and other expenses which are not essential to the single purpose of caring for its investments and keeping its accounts; thus reducing its outlay at once to the lowest possible figures; and let it continue to pay death claims as long as its money holds out. The one thing which the law will have to provide is a new administration,—for certainly that which has wrecked the business ought not to continue in charge of it; and this can best be done through the State superintendent. If the law can be made so clear and definite in all points, as to enable him to assume and administer the trust in the cheapest way, without expensive litigation or delay, the result is sure to be far more just and satisfactory than any other method of "winding-up" ever tried.

It is easy to see what the result would be. Every policyholder, having full notice of the situation, would have his choice to continue payment of premiums or to forfeit his policy. In general, those who have little at stake in the company, having paid but few premiums, and being still insurable elsewhere, would drop out; and a rapid reduction of the estimated liabilities would take place. Experience shows that in almost every such case this reduction would be sufficient to restore even the technical solvency of the company, if administered with economy. But as the sick, impaired, and dying would remain, the average vitality of the company could not be so good as in a living institution; and its prestige being gone, it would be folly to begin new business again. Let the claims continue to be paid as they arise, and there would be little danger in ordinary cases but the last policyholder would have funds enough to pay his policy. The recuperative power of an association of this kind, from the loadings on premiums and the rapid accumulations of excess of interest, is so vast, that only an honest and careful application of surplus, as it is acquired, is needed to carry it successfully to the end.

In the worst case that can be supposed, the fund could but fall short of paying all the claims. It would pay until a few of the long-lived ones are left. As it should become obvious that these could not rely on their insurance, those which need it least would drop off. A full, plain statement of the condition of the fund, from time to time, would enable the policyholders to judge for themselves as to their own interests; and a steady process of natural selection would go on, leaving in the company at last only those whose claims it in the end could satisfy. That there might be a few cases of hardship and wrong is true; but the evil would certainly be insignificant, as compared with that which results from the wholesale waste, not to say the general plunder, that has sometimes been practised.

The principle we advocate is applicable not only to companies which are compelled to suspend business, but to others which, for any reason, may believe that they have fulfilled their mission. Let any life insurance company, however strong, resolve itself into a trust for the benefit of its existing members, and the science of the business affords a clear and complete way of winding up its work in equity. Paying its death claims as they fall due; accumulating its funds from year to year; declaring each year a dividend of its surplus, either in increased insurance or in remittance of premiums to the holders of policies; it will be certain, at the end of the best life of its existing

membership, to pay off its last obligation in full, and will be transferred to an immortality of honour on the pages of commercial history. The immense reduction of expenses which such a change in the form of its business would permit, would result in accumulations of profits, which cannot be accurately estimated beforehand, but which would certainly far transcend all known precedents. And one instance of such a magnificent completeness in the fulfilment of so great a financial trust, would prove an impressive demonstration of the truth and practical value of the science of life insurance, and would far outweigh all the distrust and apprehension which failure has ever brought upon it.

On this the editor remarks:—

A contributed paper, written by one of the most careful thinkers on the subject of life insurance, in this Number of the *Spectator*, suggests a method of treating impaired life insurance companies, which is obviously superior to some of the methods hitherto employed.

Its simplicity, and the entire absence of opportunity for the kind of deception and fraud usually committed by those who manipulate the assets of insolvent companies, are its most striking merits, and are certainly merits of a high order.

But we see a serious objection to the method suggested. As the writer substantially admits, it is quite possible the funds might "fall short of paying all the claims", and this would result in giving to some, those who died early, the full amount of their claims, while others, those who were long-lived, would get nothing, though they had paid much more than their fellows. Clearly, a method which subjects any of the policyholders to this risk, is not without serious fault; and we do not see how such a method could be defended from the point of view of equity.

But we publish this paper, as we shall be glad to publish others on the same subject, as it is a question on which light is now needed, and which should be freely discussed at this time, as there is likely to be urgent need of having all obtainable information and the best practical suggestions relating to it before the end of the present winter.

In connection with this subject, we think it will be useful to reprint the paper read by Mr. Sprague at the Belfast Meeting (1874) of the British Association, Section F (Economic Science and Statistics).

On the Causes of Insolvency in Life Insurance Companies, and the best means of detecting, exposing, and preventing it. By T. B. SPRAGUE, Esq., M.A., *Barrister-at-Law, and a Vice-President of the Institute of Actuaries.*

IF, a few years ago, a paper had been offerd to the British Association on the subject ov the insolvency of life insurance companies, it woud probably hav been met with the objection that

it was ov too theoretical a character. No life insurance company in the contry had ever been knoen to becom insolvent, and ther was no reason to anticipate that such a catastrofy woud ever occur. The well knoen remark ov the late Prof. De Morgan miht probably hav been quoted, that ther is nothing at all comparabl in point ov security with a well establisht and prudently conducted life insurance company. It was certainly within the range of possibility that a life insurance company miht becom insolvent, and be unabl to meet its liabilities; but the contingency was so improbabl that its discussion coud be ov very litl interest and ov no possibl practical use. The events ov recent years, and, in particular, the notorios failures ov the Albert and European, hav shoen that the contingency in question is not so improbabl as was supposed, and that companies transacting a large business, and establisht for many years, may becom insolvent. It will therefore not be without interest to review the circumstances connected with those failures, and trace the causes ov insolvency.

The possibl causes ov insolvency in life insurance companies ar ov varios kinds; but it will suffice to mention the four folloing as the principal ones.

1st. The company may charge insufficient premiums. In this case, insolvency is as certain to occur, as in the case ov a tradesman who sells his goods habitually under cost price. But in practis all British companies charge adequat premiums, and nothing more need be said under this hed.

2nd. The office may conduct its business reklessly, insuring a large number ov damaged lives at the ordinary rate, in which case the claims ar certain to be much abov the expectation, and bankruptcy must arrive sooner or later. This may hav contributed to recent failures, but only in a small degree.

3rd. Losses may be incurd thro' injudicios investments. The same remark applies to this, as to the last caus.

4th. The expenses ov conducting the business may be excessiv, and quite out of proportion to the profits. This last has been the real and efficient caus ov the insolvencies that hav occurd.

I cannot attemt on the present occasion to examin what amount ov expenditure constitutes extravagance. This must depend on a variety ov circumstances:—the magnitude ov the premiums charged by the company, the rate ov interest it realizes on its investments, and the amount ov the new business transacted;—and to quote any figurs without stating the qualifications under which they ar to be accepted, woud be wors than useless, and might be mischivos,

becaus misleading. It requires an actuary's special knolege and experience, to say what constitutes extravagance in any particular case; but when it exists, and is persisted in long enuf, ruin is only a question ov time. The case is very similar to that ov a tradesman, who has a good business, but habitually spends considerably more than the total profits ov the business. Such a man is certain to find his way sooner or later into the Bankruptcy Court. If, however, he is careful to keep up his credit with his bankers and other crediters, it may be many years before his career is cut short. Similarly with a life office; it may be hopelessly insolvent for years before it is compeld to close its doors. Its receits ar all immediat, but its liabilities ar mostly future; and, from the nature ov the business, it is easy for a life office to present to a superficial observer, the appearance ov prosperity, while it is actually insolvent and its position is yearly becoming more desperat. The incom may increas for a series ov years; the invested assets may also increas; but, if the business is extravagantly managed, the liabilities will increas still faster, and the deficiency gro in a geometrical ratio.

This peculiarity ov life insurance business is well seen in the case ov the amalgamations, which a few years ago wer so common. By the union ov two companies, a larger company was formd, with increast incom and assets; and the rate ov expenditure in the new company was perhaps less than in either ov the original ones. Here we seem to hav all the elements ov a mutually advantageos transaction; but it too often happend that each ov the original companies was really insolvent at the time, and that the deficiency was increast by the payment ov a large fee to the person who negotiated the amalgamation.

How then can insolvency in a life office be detected? In the case of the Albert, the business was carryd on until the company had not sufficient funds availabl to pay its current claims; and then one morning the astounding and wholly unprecedented intelligence was received, that the company had stopt payment. It had been suspected for years by som well informd persons, that the company was insolvent, but no proof coud be given ov the fact. The company had never publisht full accounts ov its receits and expenditure, but had selected for publication a few only ov the figurs; in particular, those relating to the large new business transacted, carefully suppressing all reference to the very large expenditure at which that business was obtaind. In the session ov Parliment folloing the stoppage ov the Albert, ther was pa

the very useful Act calld "The Life Assurance Companies Act, 1870", which requires all life offices to publish detaild accounts in a prescribed form. It then became impossibl for companies to conceal the facts relating to their real position; and one ov the first results ov the passing ov the Act was the winding-up ov the European Insurance Company by order ov the Court ov Chancery, the company being proovd insolvent, altho it had not, like the Albert, been unabl to pay the current claims. Having now complete and detaild accounts ov the receits and expenditure ov all companies, we ar abl in all cases to ascertain the rate ov expenditure; and when we see a company for a long series ov years indulging in excessiv expenditure, this is sufficient to create suspicion as to its state. It dos not in itself amount to proof; but in addition to the rate ov expenditure the returns under the Act giv us much further information respecting the financial position ov the life insurance companies.

Under the present state ov the law, it is possible for a company to pursue a cours of rekless and fraudulent extravagance, expending the money that shoud be carefully invested to provide for its future liabilities, until it becoms insolvent beyond all hope ov recovery. The parlimentary returns enabl skilld persons, conversant with figurs, and in particular those conversant with life insurance business, to draw their own conclusions; but the general public, including most intelligent and educated men, ar unabl to detect insolvency in this way. Can then any means be suggested by which the knolege ov the skilld actuary may be made availabl to warn the general public? It must be borne in mind that the insolvency ov a company cannot be proovd like a mathematical proposition, and that altho a company may now be insolvent, it is possibl that if it has a very large accession of new business, it may recover itself.

It is easy for an actuary, when furnisht with certain particulars ov the insurances, to advize a company as to the amount ov assets it is prudent to set aside to meet the liabilities under those policies; but even on this point actuaries will differ in opinion, and they woud probably differ still more if askt to giv their opinions as to the smallest amount ov assets consistent with solvency in any particular case. Perhaps the test that woud be most generally accepted as conclusiv proof ov insolvency, is the folloing. It appears from the returns alreddy mentiond that it is a common practis for companies, when striking a balance, to take credit for the policies recently issued, as assets, insted ov treating them as

liabilities. This is the same sort ov error as a tradesman woud commit, if, in making up his balance-sheet, he wer to take credit for the profit he hopes to make in future years from his regular customers. Ther can be no doubt that all such assets shoud be struk out ov the balance-sheet, both in the case ov the trader, and ov the company. The principl I woud lay down then is that if, when all such assets are struk out ov the account, the remaining assets ar less than the liabilities, the company is insolvent. The returns under the Act do not at present enabl us to ascertain to what extent policies ar taken credit for as assets in any particular account, and it is therefore at present impossibl to apply this test ov solvency without having further information than the parlimentary returns giv. It appears, therefore, that for the effectual detection ov insolvency in doutful cases, legislation is required. The Act shoud be amended, so as to require all companies to make a separat return ov the policies which ar reckond as assets in their balance-sheets. If this wer don, actuaries woud be abl to speak with confidence as to the position ov companies respecting which they ar at present in dout.

It is clearly to the advantage ov the public, that insolvent life offices shoud be exposed. If things ar calld by their riht names, such offices ar obtaining money under fals pretenses; for they receiv premiums in consideration ov a promis to pay certain sums at a future time, the greater part of which they will not be abl to pay. Shoud then the task of exposing such companies be left as at present to privat interest and personal motivs? or shoud it be entrusted to a public department? The inconvenience ov the present state ov things, is that any person who, having satisfied himself ov the insolvency ov a company, shoud be led by public spirit or other motivs to attemt to expose it, woud be liabl to an action for libel, which woud probably ruin him. The *Times* newspaper, having proclaimd the insolvency of the European life office, had an action for libel bro't agenst it, and was forst to apologize, besides paying the company's costs. It was subsequently proovd that the company was at the time hopelessly insolvent. On the other hand, ther ar objections to giving a public department authority to interfere at their discretion with the management ov trading companies, and it woud be difficult to lay down by Act ov Parliment a definit rule to gide their action. Perhaps the difficulty miht be best met by authorizing a public department—say the Board ov Trade—upon the application ov any person interested and upon a *primâ facie* case being made out by the applicant, to

hold an inquiry with closed doors; and if they find insolvency proovd, to report the result to Parliment—ov course requiring the applicant to pay the costs ov the investigation, if the company establisht its solvency.

While it is desirabl that insolvent companies shoud be exposed, and their operations stopt, it woud be even more desirabl to prevent, if practicabl, such mismanagement as will lead to insolvency in the future. It is however ov cours out ov the question to attemt to regulate by Act ov Parliment the rate ov expenditure at which a company shall conduct its business; and it is contrary to the spirit ov British legislation to interfere in any way with the internal management of trading companies. In the United States much more is don in this way than woud be tolerated in this contry. The law there even prescribes the particular kinds ov securities in which the funds ov the life offices shall be invested. Each company is required to make returns ov its liabilities, ov a much more minute character than is required here; and a reserv ov not less than a fixt amount, must be made by a company for its liabilities under each policy it has issued. Ther is in each ov the States a public official, whose duty it is to see that the provisions of the law ar complied with; and if he finds that any company's assets fall short ov the legal standard, he is authorized (or rather bound) to put the law in action agenst it, and hav it wound up. This law certainly has the effect ov preventing the scandals in America, which we hav seen in England*; but it may be douted whether this advantage is not too dearly purchast. The effect ov subjecting any business to such minute regulations, is to impede its natural helthy development; and it is better to leav the details ov the business to be settld by the parties interested. This was the principl ov the excellent Act past in 1870. The companies wer left at full liberty to manage their affairs as they tho't best; but the secrecy that formerly attacht to the proceedings ov so many, good and bad alike, is now effectually dispeld. Tho companies ar not yet compeld to make a proper reserv for their liabilities, yet, if they make an insufficient reserv, they can no longer conceal the fact. The public ar thus enabld to judge ov the solvency ov the company they have trusted, and to understand the full meaning and value ov the contracts they hav enterd into with it. This principl, it appears to me, may with advantage be carryd a step further. Let each company be required before issuing any policy

* This was written in 1874. Subsequent events have causd the statement in the text to be no longer true.

to state the principls on which its reserv is calculated, and let it be a part ov the contract that the reserv for each policy issued shall never be less than a certain specified amount. This woud enabl intending insurers to discover beforehand the nature ov the security offerd them; and woud probably do more than anything els to prevent future insolvencies.

Postscript.

At the present time the Life Assurance Companies Act requires each company to describe the principls upon which its valuation is made, the tabl or tabls ov mortality used in the valuation, and the rate or rates ov interest asumed; also the proportion ov the annual premium incom (if any) reservd as a provision for future expenses and profits. It woud be stil more satisfactory if the companies wer also required to sho how these principls ar aplied in particular cases; in other words, if they wer required to state distinctly what amount of reserv is made for varios specimen policies. This miht be very reddily don. The companies ar now required to giv specimens of bonuses alotted to policies for £100, efected at the respectiv ages 20, 30, 40, 50, and having been in force for 5 years, 10 years, and upwards, at intervals ov 5 years respectivly. If, in the same way, each company wer required to state the reserv that its method ov valuation makes for policies taken out at the several ages of 20, 30, 40, 50, 60, after they hav been in force for 1, 2, 3, 4, 5, &c., years, the information woud be ov very great use by shoing in what cases policies hav been taken credit for as assets. At present, in consequence ov the common adoption ov the method ov classifying together policies ov all durations on the lives ov persons ov the same present age, the fact ov the method ov valuation giving rise to negativ values in many cases, is offen conceald; and I think it not unlikely that the oficials of som companies miht be greatly surprised to lern that the rules of valuation they hav been pursuing with trustful confidence, lead to a result which they woud be the first to repudiate when it was plainly put before them.

The additional return sugested abov is in complete acordance with the principls ov the Life Assurance Companies Act, inasmuch as it woud stil leav the companies at liberty to value their liabilities as they tho't best, and woud only require them to sho by exampl, as well as verbal description, the method they hav actually adopted.

Married Women's Property Act.—Form of Procedure in appointing Trustee thereunder.

THE following order of the Chancery Division of the High Court of Justice, appointing a Trustee to receive payment of the sum assured by a policy effected under the Married Women's Property Act, will probably interest many of our readers. The Act provides that a Trustee may be appointed by the Court of Chancery or by the judge of the County Court of the district in which the insurance office is situated; and in the present case it was thought better for certain reasons to apply to the Court of Chancery rather than to the County Court Judge.

In another case which has come under our notice, the application was made to the judge in chambers—a less expensive process—and a similar order made. It must be admitted, however, that in either method of procedure the necessary expense attending the application will press heavily upon the claimants when the sum assured is very small. In order to avoid this expense, a clause is inserted in the policies of some offices, constituting the executors of the will trustees of the policy. When this is done, if the assured leave a will which is proved in the ordinary way, the policy moneys will be paid over without any expense being incurred; and even in the case, which sometimes happens, of the assured leaving a will but no property which would render it necessary to prove the will, the expense of obtaining probate would probably be a good deal less than that of getting a trustee appointed.

It may perhaps be open to question whether the office obtains a completely effectual discharge on paying to the executors as above, or whether it is under any obligation to see to the application of the policy moneys. The trusts to which the policy moneys are subject are specified in the policy itself; in fact, the policy is the original document constituting the trust and expressing its terms. It seems, therefore, that it should be retained by the trustees for their guidance in administering the trust; but it is usual, when a claim is paid, for the policy to be given up to the office, and we are informed that this has been done, as usual, in the case of several claims that have been paid in respect of policies issued under the Act. It has been further suggested to us as probable that the executors, having thus parted with the policy and received the policy moneys, have allowed these to fall into the general estate of the assured; and the question arises whether, in such a case, the office would not be liable to make good the loss that a

cestui que trust might thereby sustain. This is only one of many curious and interesting points that arise in the working of the Act; and we shall be glad at all times to receive contributions on the subject from those of our readers who may have actual experience in the matter.—ED. *J. I. A.*

In the High Court of Justice
Chancery Division.

______day the______day of
________1876.

Vice Chancellor__________.

In the matter of the trusts of the sum secured by a Policy of Assurance in the X. Life Assurance Society on the life of A. B. deceased

And In the Matter of the Married Women's Property Act 1870

And In the Matter of the Trustee Act 1850 and of the Act 15th and 16th Victoria Chapter 35 intituled An Act to extend the provisions of the Trustee Act 1850

Upon the Petition of J. B. Widow A. B. M. B. and C. B. Infants by the said J. B. their mother and next friend on the______of ________1876 preferred unto this Court and upon hearing Counsel for the Petitioners and for the said X. Life Assurance Society and upon reading the said Petition the following Affidavits all filed the ______of__________1876 that is to say an affidavit of the Petitioner J. B. and the Exhibits Nos. 1 and 3 therein referred to the said Exhibit numbered one being the Policy of Assurance in the Petition mentioned an Affidavit of C. D. an Affidavit of E. F. and an Affidavit of B. B. whereby the said B. B. expresses his willingness to become Trustee as hereinafter mentioned

This Count doth appoint the said B. B. of________in the County of________Trustee of the sum secured by the Policy of Assurance dated the______of________1871 effected pursuant to the Married Women's Property Act 1870 with the above mentioned X. Life Assurance Society on the life of A. B. the husband of the Petitioner J. B. and expressed to be made for the benefit of the wife of the said A. B for her separate use and of the children born and to be born of the marriage between them in such shares and proportions on such conditions and in such manner as the said A. B. might by any will or deed appoint

And it is ordered that it be referred to the Taxing Master to tax the costs of the Petitioners of this application as between Solicitor and Client and that such Trustee be at liberty to pay such Costs out of the sum to be received by him in respect of such Policy.

CORRESPONDENCE.

ON THE DETERMINATION OF AN AVERAGE LIFE OFFICE.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I was unavoidably absent from the reading of the second part of Mr. Sutton's paper at the Institute in April last, and thus lost the pleasure of hearing it and the interesting discussion which followed it. Had I been present, I should probably have taken part in the discussion, as I have lately given some attention to the subjects of which Mr. Sutton treated; but having been precluded from doing so, I trust you will now afford me the opportunity of making a few remarks.

The aim of Mr. Sutton appears to have been to prepare a hypothetical company, which might fairly represent an average office for purposes of valuation; and he has endeavoured to reach it in several distinct ways.

1st. By considering the lives existing at the close of the H^M observations; see Table 1 of his paper.

2nd. By summarizing the Board of Trade returns of 19 companies; see Table 5.

3rd. By dividing the Board of Trade Returns into the two classes, A and B, described in his paper, and examining each class separately; see Table 11.

4th. By investigating the mortality experience of the Scottish offices to 31 Dec. 1863; see Tables 12 and 13.

It will be observed that in all the four processes only the amounts remaining assured, or the policies existing at a given date, are taken into account: that is, the course which has been followed is analogous to forming a mortality table from an enumeration of the people living in a fluctuating population, without taking note of the deaths as well. The results are various hypothetical offices, which fairly represent each the average position of the group of companies from which it was formed at the date the observations closed. They show us the average condition of the respective groups, as regards *age and flow of new business* during the past, as well as in every other respect. But, from the nature of the method on which they have been constructed, it seems to me impossible to make experiments with them for companies of different ages, because the age of a company has a greater effect on the proportionate amounts assured at various periods of life than probably any other circumstance whatever; but of this fact Mr. Sutton has not made mention. Very likely, when Mr. Sutton's paper is concluded, we shall see that an average office such as he has suggested is perfectly adapted to his purpose: but for the object I had in view in my late paper, it would have been quite unsuitable. I wanted to illustrate the valuations of an average office, not of average age, but of a variety of definite ages; and although at first I thought of making use of the numbers "existing" in the Mortality Experience, as Mr. Sutton has done, I soon discovered the idea to be unworkable,

and was compelled to prepare mortality tables for policies, by calculating the ratios at each age for each age at entry of the "exposed to risk" and "terminated".

All the average offices constructed by Mr. Sutton represent companies of *average* age which have reached their present position by average rate of increase of new business. Both Mr. Manly's hypothetical company and that suggested by myself can be made to represent, on the other hand, a company of *any* age that may be named, which from the beginning has transacted yearly a uniform amount of new business; and in Table W of my paper I further introduced a means whereby correction can be made for fluctuations in the rate of influx of policies. My "model office", therefore, can be used at will to illustrate a company of any assigned age which has grown to its present position at any assigned rate or rates of progress.

The following table, No. 1, shows the effect exercised by the age of an office, doing a uniform annual amount of new business, on the proportion at risk at various present ages. It has been formed by summarizing Table S, "Model office" of my paper, and taking at various stages of the history of the company the percentages of assurances existing in each quinquennial period of life to the total assurances existing at all ages.

TABLE 1.

Showing the Proportions, in Companies of Various Ages, at risk at various Present Ages, on the assumption that the Total Amount at risk is 100.

Present Age of Lives.	Age of Office, in Years.										Present Age of Lives.
	19	24	29	34	39	44	49	54	64	74	
20 to 24	2·18	1·88	1·70	1·58	1·50	1·45	1·41	1·40	1·40	1·39	20 to 24
25 " 29	7·37	6·38	5·74	5·33	5·06	4·88	4·78	4·73	4·69	4·69	25 " 29
30 " 34	12·80	11·08	9·98	9·26	8·79	8·48	8·31	8·21	8·15	8·15	30 " 34
35 " 39	16·75	14·50	13·07	12·13	11·51	11·11	10·88	10·75	10·67	10·67	35 " 39
40 " 44	17·93	16·30	14·69	13·64	12·94	12·50	12·23	12·08	12·00	11·99	40 " 44
45 " 49	15·57	16·06	15·10	14·01	13·30	12·84	12·56	12·42	12·33	12·32	45 " 49
50 " 54	11·56	13·25	14·01	13·53	12·84	12·40	12·13	11·99	11·91	11·90	50 " 54
55 " 59	7·58	9·35	10·93	11·80	11·65	11·26	11·01	10·88	10·81	10·80	55 " 59
60 " 64	4·54	5·85	7·37	8·77	9·65	9·71	9·50	9·39	9·32	9·32	60 " 64
65 " 69	2·41	3·23	4·28	5·52	6·68	7·50	7·65	7·56	7·51	7·51	65 " 69
70 " 74	·96	1·46	2·05	2·84	3·76	4·68	5·36	5·53	5·49	5·49	70 " 74
75 " 79	·29	·51	·80	1·15	1·65	2·23	2·85	3·31	3·44	3·44	75 " 79
80 " 84	·06	·13	·23	·36	·53	·76	1·04	1·35	1·65	1·65	80 " 84
85 & over	...	·02	·05	·08	·14	·20	·29	·40	·63	·68	85 & over

The figures are sufficiently striking to make it worth while to ascertain the average ages of the companies in Mr. Sutton's Classes A and B, so that we may more easily judge with which columns his several average companies should be compared. Assuming that, taken one with another, companies are founded in the middle of the year of their establishment, the average age of the offices in Class A on

31 Dec. 1863—the date to which the returns of the 20 companies to the Institute were made up—was 34 years: and that of the offices in Class B, 40·7 years:* while the average age of the 20 companies taken together was 37·3 years. It is probably a fair assumption to make that on the average 10 years elapsed from 31 Dec. 1863 to the dates of the valuations from which Mr. Sutton's tables on Board of Trade data were constructed; so that at the date of those valuations, the offices in Class A would be on the average 44 years old, those in Class B 50·7 years, and all the offices combined 47·3 years. These are the average ages of the companies themselves, but not of their business. It is evident that if the annual amount of new business transacted by a company be progressively increasing, then the character of its existing business corresponds to that of a younger company whose new business has been stationary; and that the greater the rate of increase, the more markedly is this effect produced. Taken in the aggregate, the 20 companies have, no doubt, I think, been decidedly developing, but it is difficult to say to what extent the development practically reduces their average age. I do not attempt to estimate the amount of reduction, but merely point out that it unquestionably exists, and probably to an important extent.

In his Table 3, Mr. Sutton compares the position of the 20 companies, in regard to the proportions at risk at present ages, as at 31 Dec. 1863, and at the date of their first valuations deposited with the Board of Trade about 10 years later: and he seemingly attributes the discrepancies principally to a tendency of the average sum assured to increase with the age at entry. A glance at the above table shows that he has omitted a material circumstance which tells in the same direction, in ignoring the 10 years that elapsed between the two returns. Also his comparison in Table 11 of Classes A and B would seem to be similarly vitiated, but not quite to the same degree; for, as above shown, there is a difference of nearly 7 years in the average ages of the offices in the two classes;—a difference, too, that must have been exaggerated by the comparative rates of the flow of new business. Anyone inspecting the list of offices in each class must observe that the offices in Class A are those which, taken collectively, have grown rapidly; while the offices in Class B have been in the aggregate in this respect less progressive. For an illustration of this fact we need not go beyond Mr. Sutton's Table 8. The figures there displayed show that at the first valuation after the Act of 1870 came into operation, the companies in Class A had each on the average £6,518,470 of assurances existing, while those in Class B had each only £3,441,063: and that, although the offices in Class B were on the average 6·7 years older than those in Class A.

From the above considerations, it is evident that the comparisons Mr. Sutton has instituted between the average offices he devised and those of Mr. Manly and myself, are likewise untrustworthy. He has taken, for instance, my "model office" at its full age, 75 years, and placed it alongside of hypothetical companies, whose ages, it will be perceived, should be taken at not more, but rather considerably less,

* The age of the London Assurance Corporation is not taken at 143 years, but at only 75 years, which is the age at which the business becomes stationary of a company transacting a uniform amount annually.

than 47, 44, 51 and 37 years respectively. The consequence of his method of procedure has been to exaggerate the comparative amounts assured at the old ages in Mr. Manly's office and mine, at the expense of the young ages; and although he alleges, and probably with reason, that the amounts at the young ages in my office are too large, he thus makes them appear too small. Turning to his Table No. 15, it will be seen that, even after he has made corrections for the small amount of policies taken out at young ages, the amounts of assurances existing in his average office formed from the H^M experience, are at the young ages larger and at the old ages smaller than the corresponding amounts in my model office when it is taken as 75 years old. This result is exactly what, on the theoretical grounds above indicated, might have been anticipated. Mr. Sutton promises, on a future occasion, to examine the figures more closely, with a view to considering in detail the causes of the variations. I trust he will see his way to include in the investigation the effect of the age of an office on the sums remaining assured at the different periods of life.

In Table 13, Mr. Sutton has given a valuable statement of the proportions of sums assured effected at various ages in the Scotch Life Offices Mortality Experience for the year ending 31 Dec. 1863. It is very interesting to observe how little these proportions vary in different offices and at different times. My model office was formed from the mortality experience of twenty companies, English and Scotch, not for one year, but during the whole of their duration. It takes account of the numbers of the policies, and not of the amounts assured; but yet the percentages given in the following statement are remarkably close to those supplied by Mr. Sutton. The variations displayed would not exercise any appreciable effect on the comparative valuations of an average company by different tables.

TABLE NO. 2.

Showing Proportions of Sums Assured at Various Ages.

Ages at Entry.	Scotch Offices, by Mr. Sutton.*	20 Offices, as shown in my Model Office.
20–24	7·47	6·97
25–29	17·05	17·76
30–34	19·33	21·04
35–39	19·34	18·41
40–44	14·68	13·81
45–49	10·77	9·45
50–54	5·89	6·23
55–59	3·20	3·51
60–64	1·74	1·97
65–69	·53	·85

* The figures in this column are borrowed from Mr. Sutton's Table No. 13. There appears to be a discrepancy between his Tables Nos. 13 and 16. The column in the latter table headed "Scotch Life Office Mortality Experience for year ending 31 Dec. 1863," would seem from the former table to relate to assurances *existing* at the end of 1863, and not to those *effected* in that year.

Mr. Sutton is no doubt right in his conclusion that the aver amount of assurance effected on each life, increases with the ag

the entrants; but I think he has possibly given to the increase too high a ratio; at least, the above Table (No. 2) would seem to teach us so. His figures relate to *sums assured;* mine to *policies;* yet the differences are not great. Had I known where to find data to work upon, I should probably have taken account of the unequal amounts of the policies at different ages at entry in constructing my "model office"; and it may be worth while to point out that, by reducing the proportionate amounts of assurances effected at the younger ages, we should have increased the already large reserves demanded by the "Analyzed Mortality Tables", in comparison with those required by each of the other mortality tables included in Table V of my paper.

I remain,

Yours obediently,

London, June 1877. GEORGE KING.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.—Session 1876-7.

First Ordinary Meeting, Monday, 27 *November* 1876.

The President (Mr. J. Hill Williams), in the Chair.

The following gentlemen were elected:—

Associates.

Alfred Carter.
St. John Cottingham.
Henry William Des Vœux.
Harry Reginald Harding.
Samuel Johnson.
Alexander George Mackenzie.
John Corbet McBride.
Frederick Piper.
Thomas James Pound.
William John Price.
Thomas Pringle.
Charles Herbert Edmund Rea.

Mr. J. M. Templeton then read a paper "On Mutual Life Assurance; its Aims and Objects, and the Means of attaining them."

The following gentlemen took part in the discussion which followed:— The President, Messrs. Baden, Bailey, Macfadyen, Manly, Ambrose Smith, and Walford.

Second Ordinary Meeting, 18 *December* 1876.

The President (Mr. J. Hill Williams), in the Chair.

The following gentlemen were elected:—

Fellows.

Charles James Lucas.
David Hutchinson MacGregor.
David Johannes Anthony Samot.

Associates.

Eric Mackay Carter.
Henry Robertson Cockburn.
James Graham.
George Trevelyan James.
John Lighton.
Henry John Lockwood.
Andrew Robinson.
Edmund H. Wells.

Mr. G. W. Berridge read a paper by Mr. T. B. Sprague "On the Premiums for the Insurance of Recently Selected Lives."

The following gentlemen, viz., Messrs. Bailey, Berridge, George King, Sutton, Walford, and Woolhouse, took part in the discussion which followed.

Third Ordinary Meeting, 29 *January* 1877.

The President (Mr. J. HILL WILLIAMS), in the Chair.

The following gentleman was elected an Associate:—

W. H. Spiller.

The following was announced as the result of the Examinations for 1876:—

FIRST YEAR'S EXAMINATION.

Twenty-four gentlemen presented themselves for this Examination, two retired, and three passed, in the following order of merit:—

1.—R. Wilding.
Eq. { A. Robinson.
H. J. Rothery.

SECOND YEAR'S EXAMINATION.

Eight gentlemen presented themselves for this Examination, and four passed, as follows:—

1.—G. F. Hardy.
2.—J. Blakey.
3.—R. C. Sayce.
4.—C. J. Harvey.

THIRD YEAR'S EXAMINATION.

Five gentlemen presented themselves for this examination. One passed, namely:—

J. Sorley.

Mr. C. D. Higham then read a paper "On the True Measure of the Death Strain on the Funds of a Life Assurance Society."

The following gentlemen took part in the discussion which followed:— The President, Messrs. R. P. Hardy, Justican, Macfadyen, A. Smith, and Walford; also the Hon. S. H. Washburne, a visitor.

Fourth Ordinary Meeting, 26 *February* 1877.

The President (Mr. J. HILL WILLIAMS), in the Chair.

The following gentlemen were elected Associates of the Institute:—

Thomas Lidbetter. | David Strang.
Wilfrid Walter White.

Mr. W. Sutton then read a paper entitled "A Comparison of various Methods of Graduation of a Mortality Table considered in reference to the Valuation of the Liability of an Average Life Office under its Assurance Contracts."

In the discussion which followed, the President, Messrs. Bailey, Berridge, Finlaison, P. Gray, Hardy, Lazarus, and Walford, took part.

Fifth Ordinary Meeting, 26 *March* 1877.

The President (Mr. J. HILL WILLIAMS), in the Chair.

The following gentleman was elected an Associate of the Institute:—

Joseph Wallis Bartlett.

Mr. George King then read a paper "On the Mortality amongst Assured Lives, and the requisite Reserves of Life Offices. Part II. Financial."

The following gentlemen took part in the discussion which followed:— The President, Messrs. Bailey, Berridge, Gordon, Manly, Sutton, Walford, and the Hon. S. H. Washburne, of Boston, U.S.A., a visitor.

INSTITUTE OF ACTUARIES.

Income and Expenditure for the Year ending 31 March 1877.

Dr.

1876-7.	£ s. d.	£ s. d.
Amount of Funds on 31 March 1876—		
Messenger Legacy Fund	270 5 10	
Hardy Memorial Fund	236 2 5	
Brown Prize Fund	236 7 11	
General Fund	1,569 4 6	
		2,312 0 8
Annual Subscriptions—		
Town Fellows	242 11 0	
Country "	58 16 0	
Town Associates	291 18 0	
Country "	90 6 0	
		683 11 0
Subscription compounded		21 0 0
Mortality Experience Sales		1 11 6
Life Tables Sales		5 15 6
Examination Fees		8 8 0
Dividend and Interest		65 6 7
		£3,097 13 3

Cr.

1876-7.	£ s. d.	£ s. d.
Rent	100 0 0	
Salaries	154 2 6	
Journal	157 10 6	
Stationery	68 6 6	
Ordinary Meetings	16 19 6	
Postage	17 12 3	
House Expenses	24 6 0	
Miscellaneous Petty Expenses	2 5 8	
		541 2 11
Tutor		31 10 0
Examination Expenses		16 14 6
Library		3 14 4
Amount of Funds on 31 March 1877—		
Messenger Legacy Fund	276 10 10	
Hardy Memorial Fund	242 0 11	
Brown Prize Fund	244 6 1	
General Fund	1,741 13 7	
		2,504 11 6
		£3,097 13 3

Balance Sheet, 31 March 1877.

Dr.

	£ s. d.	£ s. d
Messenger Legacy Fund (£211. 1s. 10d. Consols), cost	203 17 8	
Unappropriated Dividends	72 13 3	
		276 10 11
Hardy Memorial Fund (£200 Consols), cost	179 14 6	
Unappropriated Dividends	62 6 5	
		242 0 11
Brown Prize Fund (£200 Metropolitan Railway Stock)	200 0 0	
Unappropriated Dividends	44 6 1	
		244 6 1
General Fund		1,741 13 7
		£2,504 11 6

Cr.

	£ s. d.	£ s. d.
Consols, £1,100 Stock		1,008 19 3
£700 Metropolitan Railway 4 per-cent Guarantee Stock		700 0 0
Cash on Deposit at Union Bank		300 0 0
Cash at London and Westminster Bank	471 0 10	
" in hand	8 16 5	
		479 17 3
Arrears of Subscriptions:—		
Country Fellows	7 7 0	
Town Associates	2 2 0	
Country "	6 6 0	
		15 15 0
		£2,504 11 6

Examined and found correct, 9 May 1877.

(Signed) { Thos. J. Searle, W. Gage, Francis W. White, } *Auditors.*

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On certain Methods of Valuation. *By* W. T. Gray, A.I.A. *of the General Reversionary and Investment Company.*

THE proper method of estimating the liabilities of an office under its policies, is a question of so much importance that numerous essays on the subject have at various times been contributed to the pages of the *Journal* by some of the most eminent members of the Institute, whilst Mr. Manly, in his *Messenger Prize Essay,* and Mr. Valentine in his extension of the same, and more recently Mr. Geo. King, in his paper in vol. xx of the *Journal,* have furnished us with valuable information as to the financial results arrived at by the employment of different data in valuations. The subject has been so extensively treated in these papers, that I should not have ventured to refer to it again, but for the employment recently in some quarters of a method of valuation producing certain anomalies hitherto but little noticed in the pages of the *Journal,* and I therefore beg to submit for the consideration of its readers the results of an investigation showing the effects of the use of that method on the reserves and profits of an office. The method I desire to draw attention to is that advocated by Mr. Templeton in his paper read before the Institute on the 27th November 1876, (vol. xx, p. 77) wherein he

recommends that, when an office proposes to change the table or rate of interest hitherto employed for estimating its liabilities, the annuities and reversions deduced from the newly adopted data should be used, but the pure premiums derived from the original table or rate of interest should be those brought into account.

This method of valuation might well be designated the "mixed" method; for under it we may have premiums deduced at a certain rate of interest from one mortality table, valued by annuities obtained from the same table at another rate, or even by annuities from another table at the same, or a different, rate of interest. So that if P'_x be such premium, we have for the value of a policy at the end of n years

$$_nV'_x = A_{x+n} - P'_x(1+a_{x+n})$$

instead of the hitherto generally accepted formula,

$$_nV_x = A_{x+n} - P_x(1+a_{x+n}),$$

where P_x is the pure premium derived from the same table and at the same rate of interest as A_{x+n} and a_{x+n}. Therefore, according as $P'_x > = < P_x$, the reserve will be less, equal, or greater than that derived from the net-premium method. It may not perhaps here be out of place to point out that since

$$P'_x = \frac{1}{1+a'_x} - (1-v),$$

we have, when the quantities are all at the same rate of interest,

$$_nV'_x = 1-(1-v)(1+a_{x+n}) - \left\{\frac{1}{1+a'_x} - (1-v)\right\}(1+a_{x+n})$$
$$= 1 - \frac{1+a_{x+n}}{1+a'_x}.$$

This relation, however, will not otherwise obtain.

Mr. J. J. W. Deuchar, in his interesting paper on Negative Policy Values, has shown that where P'_x, the premium brought into account, is greater than P_{x+n}, the pure premium at the assured's present age according to the data used in the valuation, the value of the policy will be negative; and bearing this in mind, it will be easy, on comparing the premiums according to different tables and rates of interest, to see when negative values will occur if the mixed method be employed. Since the valuation of a premium different from the net premium may be regarded as an anticipation, if the premium valued be greater than the net, or a postponement, if it be less, of a portion of the loading, a comparison of the rates of

premium obtained from different tables may be advantageous in considering the method of valuation in question. In the following table I have compared the premiums obtained at the same rate of interest only, as these are the ones most likely to be substituted in a valuation when an office proposes to pass from one table to another; but the valuation of three per-cent premiums by the annuities of the same table at a higher rate of interest is not without precedent in the Reports of some offices.

TABLE A.—*Rates of Premium for the Insurance of* 100, *obtained from different Tables of Mortality.*

Table.	PREMIUM AT		PER-CENT OF H^M PRM.		PREMIUM AT		PER-CENT OF H^M PRM.	
	3 per-cent.	4 per-cent.	3 per-cent.	4 per-cent.	3 per-cent.	4 per-cent.	3 per-cent.	4 per-cent.
	Age 20.				Age 30.			
EIII	1·620	1·445	113·5	116·1	2·084	1·882	110·9	112·8
EII	1·585	1·410	111·1	113·3	2·039	1·837	108·5	110·1
EI	1·597	1·423	111·9	114·3	2·054	1·855	109·3	111·1
D	1·525	1·351	106·9	108·5	1·995	1·803	106·1	108·0
C	1·494	1·319	104·7	105·9	1·952	1·755	103·8	105·2
O	1·473	1·295	103·2	104·0	1·905	1·697	101·3	101·7
H^M	1·427	1·245	...	...	1·880	1·669	...	...
S	1·464	1·279	102·6	102·7	1·936	1·723	103·0	103·2
	Age 35.				Age 40.			
EIII	2·404	2·188	109·6	111·1	2·810	2·582	108·5	109·8
EII	2·352	2·136	107·3	108·5	2·747	2·518	106·1	107·1
EI	2·355	2·142	107·4	108·8	2·731	2·502	105·5	106·4
D	2·289	2·083	104·4	105·8	2·647	2·428	102·2	103·2
C	2·233	2·022	101·8	102·7	2·599	2·375	100·4	100·9
O	2·210	1·987	100·8	100·9	2·605	2·368	100·6	100·7
H^M	2·193	1·969	...	...	2·589	2·352	...	...
S	2·265	2·038	103·3	103·5	2·682	2·442	103·6	103·8
	Age 50.				Age 60.			
EIII	4·020	3·770	105·8	106·4	6·164	5·898	103·0	103·2
EII	3·924	3·670	103·2	103·6	6·131	5·867	102·4	102·7
EI	3·873	3·615	101·9	102·1	6·129	5·863	102·4	102·6
D	3·744	3·508	98·5	99·0	5·461	5·202	91·2	91·0
C	3·622	3·364	95·3	95·0	5·790	5·532	96·7	96·8
O	3·835	3·578	100·9	101·0	6·025	5·755	100·6	100·7
H^M	3·801	3·542	...	...	5·987	5·715	...	...
S	3·963	3·702	104·3	104·5	6·291	6·018	105·1	105·3

Here EI, EII, EIII, denote the English Tables I, II, and III, respectively; D, Griffith Davies's Equitable Experience; C, the Carlisle; O, the 17 Offices' Experience; H^m, the Institute H^M table; and S, the premium that would be valued by Mr. Sprague in certain cases—namely, the net premium for the next higher age to that at entry.

As however it is usual in practice to value by the formula

$$A_{x+n}-P_x(.5+a_{x+n}),$$

under which it is not so easy to see when negative values arise, I append a table showing the values of policies of various durations, where the 4 per-cent net premiums according to the English Tables I, II, III, Davies's Equitable (D), Carlisle (C), 17 Offices (O), and H^M tables are valued by H^M 4 per-cent annuities by this formula; and thinking that a comparison of these values with those obtained by employing the method recommended "as justifiable in certain cases", by Mr. Sprague, in his paper in vol. xv, p. 411, might be of interest, I have added the values when the net premiums (S), at the next higher age to that at entry, according to the H^M table, also at 4 per-cent, are valued by the same formula. In this latter instance I have given no value for the reserve at the end of the first half year, as it is not derived from the formula above stated, but is simply the amount required to cover the "unexpired current risk."

In all cases I have assumed the assured to attain their office age six months after entry, so that the reserve for a policy issued on a life of 30 next birthday, would at the end of that time be

$$A_{30}-P'_{30}\ (.5+a_{30}).$$

TABLE B.—*Values of Policies by the Mixed, Net, and Mr. Sprague's Methods of Valuation.*

Age at Entry.	Premium valued.	Years in Force.					
		½	2½	4½	7½	17½	37½
20	E_{III}	−3·207	− 1·834	− ·386	+ 2·100	11·910	40·320
	E_{II}	−2·540	− 1·175	+ ·263	2·733	12·478	40·701
	E_{I}	−2·787	− 1·418	·023	2·499	12·268	40·560
	D	−1·411	− ·060	1·361	3·803	13·439	41·345
	C	− ·798	+ ·544	1·957	4·383	13·960	41·694
	O	− ·343	·993	2·400	4·815	14·348	41·954
	H^m	+ ·622	1·945	3·338	5·730	15·168	42·504
	S	...	1·302	2·704	5·111	14·614	42·132
30	E_{III}	−2·910	− ·864	+ 1·306	4·791	18·653	53·728
	E_{II}	−2·122	− ·092	2·061	5·519	19·273	54·074
	E_{I}	−2·434	− ·398	1·762	5·231	19·027	53·937
	D	−1·528	+ ·490	2·630	6·068	19·740	54·335
	C	− ·683	1·317	3·440	6·848	20·405	54·706
	O	+ ·346	2·326	4·426	7·799	21·214	55·158
	H^m	·835	2·805	4·894	8·250	21·598	55·372
	S	...	1·870	3·980	7·369	20·848	54·953
40	E_{III}	−2·427	+ ·596	3·827	8·941	27·902	67·066
	E_{II}	−1·425	1·568	4·766	9·830	28·602	67·377
	E_{I}	−1·170	1·816	5·006	10·056	28·778	67·456
	D	− ·013	2·938	6·091	11·082	29·587	67·816
	C	+ ·808	3·735	6·861	11·810	30·160	68·070
	O	·926	3·848	6·970	11·914	30·242	68·106
	H^m	1·176	4·091	7·205	12·136	30·417	68·182
	S	...	2·727	5·887	10·889	29·436	67·748
50	E_{III}	−1·215	+ 3·215	7·814	14·930	39·104	76·524
	E_{II}	+ ·093	4·463	9·002	16·281	39·880	76·808
	E_{I}	·808	5·140	9·654	16·625	40·306	76·962
	D	2·215	6·490	10·930	17·800	41·140	77·270
	C	4·083	8·275	12·629	19·365	42·250	77·675
	O	1·299	5·615	10·098	17·034	40·595	77·070
	H^m	1·771	6·065	10·526	17·428	40·875	77·172
	S	...	4·071	8·628	15·679	39·635	76·719
60	E_{III}	+1·032	7·274	13·452	22·627	51·075	93·205
	E_{II}	1·341	7·564	13·722	22·867	51·225	93·220
	E_{I}	1·380	7·600	13·756	22·898	51·244	93·222
	D	7·969	13·765	19·501	28·020	54·435	93·553
	C	4·682	10·690	16·635	25·464	52·843	93·388
	O	2·456	8·607	14·693	23·734	51·765	93·276
	H^m	2·858	8·983	15·044	24·046	51·959	93·296
	S	...	6·164	12·417	21·704	50·500	93·145

Consideration of this table will show that if the "mixed" method, as here exhibited, be employed by an office adopting the common and convenient course of grouping the policies for valuation according to the date of birth of the lives assured, the total sum assured being

multiplied once for all by the reversion, and the total of the "premiums for valuation" by the annuity, then, unless the policies issued during the last five years be excluded and valued separately, a very considerable amount of negative values must be included, —a course that cannot under any circumstances be too strongly condemned. We are also led to the conclusion that the estimated liability of an office would, in all cases exemplified in the foregoing table, fall very far below the amount obtained by a pure premium valuation, in spite of the fact that for some of the higher ages at entry the Carlisle and Davies's Equitable premiums are less than the H^M, and the liability consequently greater, for the policies effected at these periods would be insufficient in amount to counterbalance the deficiency in the reserve for entrants at the younger ages.

It was originally my intention to have tested the extent of the variation by carrying the valuations of Messrs. Manly and Valentine a step further; but on reconsideration I abandoned the idea, as according to Mr. Manly's model office no policies are valued until they have been five full years in force, by which time all the policies would have attained to positive values, and the evil effects of the mixed method, as regards negative values, would to a very considerable extent have been concealed. I have therefore thought it advisable to construct a new model,* with the improved data which the mortality experience collected by the Institute has furnished. For this purpose I have utilized the rates of discontinuance in the H^M observations which I had calculated and roughly graduated some time ago. The method of calculation has been so clearly set forth by Mr. George King, in his paper in the 19th volume of the *Journal*, that I need not further refer to it than to state that I took the central age next birthday, instead of last birthday, as he has done, as a multiple of 5, so that my rates for age 40 were deduced from the experience of lives entering at 38, 39, 40, 41, 42 next birthday, whilst Mr. King's were obtained from those entering at 39, 40, 41, 42, 43. As a rule my rates are slightly higher than his, in consequence of the younger life to which mine are applicable, the rate of discontinuance decreasing as the age at entry increases, but the difference is so small that I do

* This investigation was commenced before Part I (vol. xix) of Mr. Geo. King's paper had appeared, and the greater part of the following calculations was completed before his "model" in Part II was published; otherwise I should not have thought it worth while to offer another, as the facilities for comparing the effect of employing different methods and data in valuations are thereby to some extent reduced.

not think it worth while to reproduce them here. The few discontinuances occurring after the 30th year of assurance were disregarded.

Taking the H^M rate of mortality for the deaths, and the adjusted rates of discontinuance for the withdrawals, I have supposed the case of an office issuing annually 3,000 policies for £100 each (this large number of entrants being assumed in order to obtain a continuous experience for all ages at entry), the lives entering at the ages 20, 25, 30, 35, 40, 45, 50, 55, and 60, in the proportions found in the Mortality Experience by taking the numbers there given as joining at the ages 18 to 22 for 20, 23 to 27 for 25, and so on, and dividing the total entrants in each group by all the entrants between 18 and 62.

Having thus determined the number of entrants for each age, assuming the discontinuances to take place at the end of the year of assurance, and the deaths on the average in the middle of the year, if

${}_0R$	= the number of entrants,		
${}_1R, {}_2R \ldots\ldots$	= the renewals in the 2nd, 3rd	year of	assurance,
${}_0d, {}_1d, {}_2d$	= the deaths in the 1st, 2nd	,,	,,
${}_0w, {}_1w, {}_2w$	= the withdrawals ,,	,,	,,
${}_0q, {}_1q, {}_2q$	= the rate of mortality ,,	,,	,,
${}_0s, {}_1s, {}_2s$	= the rate of discontinuance	,,	,,

we have $\quad {}_0d = {}_0R \times {}_0q,$

$$ {}_0w = ({}_0R - \tfrac{1}{2}{}_0d)_0s = {}_0R(1 - \tfrac{1}{2}\,.\,{}_0q)_0s. $$

Putting $\quad (1 - \tfrac{1}{2}q)s = t,$

we have $\quad {}_0w = {}_0R \times {}_0t.$

And
$$ {}_1R = {}_0R - {}_0d - {}_0w $$
$$ = {}_0R(1 - {}_0q - {}_0t) = {}_0R \times {}_0r, $$

where r is the probability of renewal.

Similarly,
$$ {}_2R = {}_1R(1 - {}_1q - {}_1t) = {}_1R \times {}_1r $$
$$ = {}_0R \times {}_0r \times {}_1r. $$

And generally, using logarithms,

$$ \log {}_nR = \log {}_0R + \log {}_0r + \log {}_1r + \log {}_2r + \ldots\ldots $$

The renewals in each year of assurance for each age at entry were therefore calculated by means of log r in the same way as the living in a mortality table from log p, and then log R + log q gives log d, the deaths in the year, and log R + log t gives log w, the discontinuances. It is not, however, absolutely necessary to calculate these latter, since the renewals and deaths in each year being known, we have

$${}_n\mathrm{R} = {}_nd + {}_nw + {}_{n+1}\mathrm{R}.$$

Therefore $${}_nw = {}_n\mathrm{R} - ({}_nd + {}_{n+1}\mathrm{R}).$$

In the actual calculations, the number of deaths to accrue has been slightly over-estimated; for I have taken ${}_0q_x$ at q_x and ${}_1q_x$ at q_{x+1}, which is not true, since the assured enter on the average six months before attaining their office age x, and consequently ${}_0d_x$ should be ${}_0\mathrm{R}_x \times \frac{q_{x-1}+q_x}{2}$; ${}_1d_x = {}_1\mathrm{R}_x \times \frac{q_x+q_{x+1}}{2}$, and ${}_0w_x = {}_0\mathrm{R}_x\left(1 - \tfrac{1}{2} \cdot \frac{q_{x-1}+q_x}{2}\right){}_0s_x$. The additional strain on the funds of the office is however compensated for in the premiums charged to the assured, and the facility obtained for calculating the renewals is obvious. As the valuations were supposed to take place in the middle of the year of assurance, I have assumed the number of policies then in force as equal to the number of renewals at the beginning of the year, less one-half the deaths; or when these latter were an uneven number $(2n+1)$, I supposed n to take place before the valuation and $n+1$ thereafter.

Table C (see pp. 318, 319).

In view of the large number of calculations that appeared necessary for making a valuation for the various rates of premium referred to in Table A, I originally limited my intention to testing the results of valuing English No. 1 and Carlisle premiums at 4 per-cent, and comparing the reserves with those required by the net and Mr. Sprague's methods; but on making various experiments for checking the first calculations, I discovered a method which has enabled me to extend the investigation to the other two English tables, Davies's Equitable, and the Seventeen Offices, and at the same time to save myself considerable labour. I did not think it worth while to include the Northampton rates, as the premiums even at 4 per-cent by this table are so high that they must be looked upon as containing in themselves a portion of

the loading, whilst the others have all at various times been regarded as true tables, and the pure premiums derived from them as covering the risk only.

For facility of reference, I shall hereafter denote an office valuing by the Institute H^M 4 per-cent annuities, the pure premiums at the same rate of interest derived from the 3 English, Davies's Equitable, Carlisle, Seventeen Offices, and H^M tables, as Offices (EIII), (EII), (EI), (D), (C), (O), and (H^m) respectively; and an office valuing according to Mr. Sprague's method the premium at next higher age to that at entry, as Office (S); and when reference is made to an office valuing a premium $P_{(1)}$, $P_{(2)}$, as Offices (P_1), (P_2).

If now we have m offices (P_1) (P_2) (P_3) ...... (P_m), where the premium $P_{(1)} > P_{(2)} > P_{(3)} \ldots\ldots > P_{(m)}$, the reserves they will make for a policy issued on the same life x, when the assured attains the age of $x+n$, the next premium falling due 6 months hence, will be:

$$\text{For } (P_1) \ldots\ldots A_{x+n} - P_{(1)}(.5 + a_{x+n}),$$
$$\text{,, } (P_2) \qquad A_{x+n} - P_{(2)}(.5 + a_{x+n});$$

and so on for the others; and the amount reserved by (P_2) will always be greater than that by (P_1) by the difference of the above values, or

$$(P_{(1)} - P_{(2)})(.5 + a_{x+n}).$$

This sum may be large in the early years of the policy, but will diminish as it grows older, until it is reduced to $(P_{(1)} - P_{(2)}) \times .5$, when the assured attains the last age in the H^m table, on the assumption that one-half of the assured living at this age will pay their premiums before their policies become claims. We see then that when once the rate of premium brought into account in the valuation is fixed, an office (P_t) will never reserve less than office (P_{t-1}) or more than office (P_{t+1}), but will always have in hand an amount between the two.

If, now, we suppose that there are N policies, for 1 each, issued at age x, in force at age $x+n$, the respective reserves will be:

$$\text{Office } (P_1) \qquad N\{A_{x+n} - P_{(1)}(.5 + a_{x+n})\}$$
$$\text{,, } (P_2) \qquad N\{A_{x+n} - P_{(2)}(.5 + a_{x+n})\}$$
$$\text{,, } (P_3) \qquad N\{A_{x+n} - P_{(3)}(.5 + a_{x+n})\}$$
$$* \quad * \quad * \quad * \quad * \quad *$$
$$\text{,, } (P_m) \qquad N\{A_{x+n} - P_{(m)}(.5 + a_{x+n})\}$$

Table C.

Age at Entry.

Year of Assurance.	20 R.	20 d.	20 w.	20 N	25 R.	25 d.	25 w.	25 N	30 R.	30 d.	30 w.	30 N
0	150	1	18	150	484	3	43	483	639	5	43	637
1	131	1	14	131	438	3	30	437	591	5	30	589
2	116	1	10	116	405	3	22	404	556	4	25	554
3	105	0	8	105	380	3	18	379	527	5	19	525
4	97	1	5	97	359	2	15	358	508	4	17	501
5	91	1	4	91	342	3	11	341	482	4	14	480
6	86	0	4	86	328	3	10	327	464	4	13	462
7	82	1	2	82	315	2	9	314	447	4	11	445
8	79	0	3	79	304	3	7	303	432	5	7	430
9	76	1	2	76	294	2	7	293	420	4	7	418
10	73	1	2	73	285	3	5	284	409	4	7	407
1	70	0	2	70	277	2	5	276	398	4	5	396
2	68	1	1	68	270	3	3	269	389	4	6	387
3	66	0	2	66	264	2	4	263	379	5	4	377
4	64	1	1	64	258	3	3	257	370	4	4	368
15	62	0	2	62	252	2	3	251	362	4	5	360
6	60	1	0	60	247	3	2	246	353	5	3	351
7	59	0	2	59	242	3	2	241	345	5	3	343
8	57	1	0	57	237	2	3	236	337	5	3	335
9	56	1	0	56	232	3	2	231	329	5	3	327
20	55	0	2	55	227	3	2	226	321	5	3	319
1	53	1	0	53	222	3	2	221	313	5	3	311
2	52	0	1	52	217	3	1	216	305	5	2	303
3	51	1	0	51	213	3	2	212	298	6	2	295
4	50	1	0	50	208	3	1	207	290	5	3	288
25	49	0	1	49	204	4	1	202	282	6	2	279
6	48	1	0	48	199	3	2	198	274	6	2	271
7	47	0	1	47	194	3	1	193	266	6	2	263
8	46	1	0	46	190	4	1	188	258	6	2	255
9	45	1	0	45	185	3	2	184	250	6	2	247

Year of Assurance.	35 R.	35 d.	35 w.	35 N	40 R.	40 d.	40 w.	40 N	45 R.	45 d.	45 w.	45 N
0	576	5	34	574	451	5	27	449	307	4	18	305
1	537	5	23	535	419	4	18	417	285	3	12	284
2	509	5	20	507	397	4	16	395	270	4	10	268
3	484	5	17	482	377	4	12	375	256	4	8	254
4	462	4	15	460	361	5	10	359	244	4	6	242
5	443	5	11	441	346	4	9	344	234	3	7	233
6	427	4	11	425	333	4	8	331	224	4	4	222
7	412	5	8	410	321	5	6	319	216	4	5	214
8	399	4	8	397	310	4	7	308	207	4	3	205
9	387	5	6	385	299	5	4	297	200	4	4	198
10	376	4	6	374	290	4	5	288	192	4	3	190
1	366	5	5	364	281	5	4	279	185	4	2	183
2	356	5	5	354	272	5	3	270	179	4	2	177
3	346	5	4	344	264	5	3	262	173	4	3	171
4	337	5	4	335	256	5	3	254	166	5	1	164
15	328	5	4	326	248	5	3	246	160	5	1	158
6	319	5	3	317	240	5	2	238	154	5	1	152
7	311	6	3	308	233	6	2	230	148	5	1	146
8	302	5	3	300	225	6	2	222	142	5	1	140
9	294	6	1	291	217	6	1	214	136	6	1	133
20	287	6	2	284	210	6	2	207	129	5	1	127
1	279	7	2	276	202	6	2	199	123	6	0	120
2	270	6	2	267	194	7	1	191	117	6	0	114
3	262	7	1	259	186	7	1	183	111	6	1	108
4	254	7	2	251	178	7	1	175	104	6	0	101
25	245	7	1	242	170	7	1	167	98	6	1	95
6	237	7	2	234	162	8	1	158	91	6	0	88
7	228	8	1	224	153	8	0	149	85	6	1	82
8	219	8	1	215	145	7	1	142	78	6	0	75
9	210	9	1	206	137	8	1	133	72	7	0	69

50 R.	50 d.	50 w.	50 N	55 R.	55 d.	55 w.	55 N	60 R.	60 d.	60 w.	60 N	Year of Assurance.
206	3	13	205	119	3	7	118	68	2	4	67	0
190	3	7	189	109	2	4	108	62	2	2	61	1
180	4	6	178	103	3	3	102	58	2	2	57	2
170	3	5	169	97	2	4	96	54	2	2	53	3
162	3	5	161	91	2	2	90	50	2	1	49	4
154	3	4	153	87	3	2	86	47	2	1	46	5
147	4	3	145	82	2	2	81	44	2	1	43	6
140	3	3	139	78	3	2	77	41	2	1	40	7
134	3	3	133	73	3	1	72	38	2	1	37	8
128	4	1	126	69	3	1	68	35	2	0	34	9
123	3	3	122	65	3	0	64	33	2	1	32	10
117	4	1	115	62	3	1	61	30	2	0	29	1
112	4	1	110	58	2	1	57	28	2	0	27	2
107	4	1	105	55	3	1	54	26	2	1	25	3
102	4	1	100	51	3	0	50	23	2	0	22	4
97	4	1	95	48	3	0	47	21	2	0	20	15
92	5	0	90	45	3	1	44	19	2	0	18	6
87	4	1	85	41	3	0	40	17	2	0	16	7
82	4	1	80	38	3	0	37	15	2	0	14	8
77	5	0	75	35	4	0	33	13	2	0	12	9
72	5	0	70	31	3	0	30	11	1	0	11	20
67	4	0	65	28	3	0	27	10	2	0	9	1
63	5	0	61	25	3	0	24	8	1	0	8	2
58	4	1	56	22	3	0	21	7	2	0	6	3
53	5	0	51	19	2	1	18	5	1	0	5	4
48	5	0	46	16	2	0	15	4	1	0	4	25
43	4	1	41	14	2	0	13	3	0	0	3	6
38	4	0	36	12	2	0	11	3	1	0	3	7
34	5	0	32	10	2	0	9	2	0	0	2	8
29	4	0	27	8	2	0	7	2	1	0	2	9

1	44	1	0	44	178	4	0	174	[illegible]	8	0	[illegible]	192	9	0	[illegible]	[illegible]	8	0	118	58	6	0	55	[illegible]	4	0	[illegible]	5	1	0	5	...	1
2	43	1	0	43	172	4	0	170	227	8	0	223	183	9	0	179	112	8	0	108	52	6	0	49	18	3	0	17	4	1	0	4	...	2
3	42	1	0	42	168	4	0	166	219	8	0	215	174	10	0	169	104	9	0	100	46	5	0	44	15	3	0	14	3	0	0	3	...	3
4	41	0	0	41	164	5	0	162	211	9	0	207	164	9	0	160	95	9	0	91	41	6	0	38	12	2	0	11	3	1	0	3	...	4
35	41	1	0	41	159	4	0	157	202	8	0	198	155	10	0	150	86	8	0	82	35	5	0	33	10	2	0	9	2	1	0	2	...	35
6	40	1	0	40	155	5	0	153	194	9	0	190	145	10	0	140	78	8	0	74	30	5	0	28	8	2	0	7	1	1	0	1	...	6
7	39	1	0	39	150	5	0	148	185	10	0	180	135	10	0	130	70	8	0	66	25	4	0	23	6	1	0	6	...				...	7
8	38	1	0	38	145	6	0	142	175	9	0	171	125	10	0	120	62	8	0	58	21	4	0	19	5	1	0	5	...				...	8
9	37	1	0	37	139	5	0	137	166	10	0	161	115	11	0	110	54	7	0	51	17	3	0	16	4	1	0	4	...				...	9
40	36	1	0	36	134	6	0	131	156	9	0	152	104	10	0	99	47	7	0	44	14	3	0	13	3	1	0	3	...				...	40
1	35	1	0	35	128	6	0	125	147	10	0	142	94	10	0	89	40	6	0	37	11	3	0	10	2	1	0	2	...				...	1
2	34	1	0	34	122	6	0	119	137	11	0	132	84	10	0	79	34	6	0	31	8	2	0	7	1	1	0	1	...				...	2
3	33	2	0	32	116	6	0	113	126	10	0	121	74	9	0	70	28	5	0	26	6	1	0	6	...				...				...	3
4	31	1	0	31	110	7	0	107	116	11	0	111	65	9	0	61	23	5	0	21	5	1	0	5	...				...				...	4
45	30	2	0	29	103	6	0	100	105	10	0	100	56	8	0	52	18	4	0	16	4	1	0	4	...				...				...	45
6	28	1	0	28	97	7	0	94	95	10	0	90	48	7	0	45	14	3	0	13	3	1	0	3	...				...				...	6
7	27	1	0	27	90	7	0	87	85	10	0	80	41	7	0	38	11	2	0	10	2	1	0	2	...				...				...	7
8	26	1	0	26	83	6	0	80	75	9	0	71	34	6	0	31	9	2	0	8	1	1	0	1	...				...				...	8
9	25	2	0	24	77	7	0	74	66	9	0	62	28	6	0	25	7	2	0	6	...				...				...				...	9
50	23	1	0	23	70	7	0	67	57	8	0	53	22	5	0	20	5	1	0	5	...				...				...				...	50
1	22	2	0	21	63	7	0	60	49	8	0	45	17	4	0	15	4	1	0	4	...				...				...				...	1
2	20	1	0	20	56	6	0	53	41	7	0	38	13	3	0	12	3	1	0	3	...				...				...				...	2
3	19	2	0	18	50	6	0	47	34	6	0	31	10	2	0	9	2	1	0	2	...				...				...				...	3
4	17	1	0	17	44	6	0	41	28	6	0	25	8	2	0	7	1	1	0	1	...				...				...				...	4
55	16	2	0	15	38	6	0	35	22	4	0	20	6	2	0	5	...				...				...				...				...	55
6	14	1	0	14	32	5	0	30	18	4	0	16	4	1	0	4	...				...				...				...				...	6
7	13	2	0	12	27	4	0	25	14	3	0	13	3	1	0	3	...				...				...				...				...	7
8	11	1	0	11	23	5	0	21	11	3	0	10	2	1	0	2	...				...				...				...				...	8
9	10	1	0	10	18	3	0	17	8	2	0	7	1	1	0	1	...				...				...				...				...	9
60	9	2	0	8	15	3	0	14	6	2	0	5	...				...				...				...				...				...	60
1	7	1	0	7	12	3	0	11	4	1	0	4	...				...				...				...				...				...	1
2	6	1	0	6	9	2	0	8	3	1	0	3	...				...				...				...				...				...	2
3	5	1	0	5	7	2	0	6	2	1	0	2	...				...				...				...				...				...	3
4	4	1	0	4	5	1	0	5	1	1	0	1	...				...				...				...				...				...	4
65	3	0	0	3	4	1	0	4	...				...				...				...				...				...				...	65
6	3	1	0	3	3	1	0	3	...				...				...				...				...				...				...	6
7	2	0	0	2	2	1	0	2	...				...				...				...				...				...				...	7
8	2	1	0	2	1	1	0	1	...				...				...				...				...				...				...	8
9	1	1	0	1	...				...				...				...				...				...				...				...	9

and the differences in amount will be

$$\begin{array}{lll} (P_2) \text{ over } (P_1) & N(P_{(1)}-P_{(2)})(.5+a_{x+n}) \\ (P_3) \quad ,, \quad (P_2) & N(P_{(2)}-P_{(3)})(.5+a_{x+n}) \\ * \quad * \quad * & * \quad * \quad * \\ (P_m) \quad ,, \quad (P_{m-1}) & N(P_{(m-1)}-P_{(m)})(.5+a_{x+n}). \end{array}$$

The total of these differences is

$$N(P_{(1)}-P_{(m)})(.5+a_{x+n}),$$

which is the excess of the amount reserved by the office valuing the lowest premium over that valuing the highest.

If then (P_1) reserve a sum S_1, and (P_m) a sum S_m, (P_2) will reserve

$$S_1+(S_m-S_1)\frac{N(P_{(1)}-P_{(2)})\,(.5+a_{x+n})}{N(P_{(1)}-P_{(m)})(.5+a_{x+n})}$$

$$=S_1+(S_m-S_1)\cdot\frac{P_{(1)}-P_{(2)}}{P_{(1)}-P_{(m)}}.$$

For (P_3) we have

$$S_1+(S_m-S_1)\left\{\frac{P_{(1)}-P_{(2)}}{P_{(1)}-P_{(m)}}+\frac{P_{(2)}-P_{(3)}}{P_{(1)}-P_{(m)}}\right\},$$

and for (P_t)

$$S_1+(S_m-S_1)\left\{\frac{P_{(1)}-P_{(2)}}{P_{(1)}-P_{(m)}}+\frac{P_{(2)}-P_{(3)}}{P\quad-P_{(m)}}+\ldots\ldots+\frac{P_{(t-1)}-P_{(t)}}{P_{(1)}-P_{(m)}}\right\};$$

or if the value for (P_t) alone be wanted:

$$S_1+(S_m-S_1)\cdot\left(\frac{P_{(1)}-P_{(t)}}{P_{(1)}-P_{(m)}}\right).$$

From this we see that, taking any age at entry, no matter what the number of rates of "premium for valuation" may be, we have only to make the necessary calculations for the two extreme rates of premium, the amounts reserved for the intermediate rates being found by interpolation; and as the formula for this is independent of the duration of the policies, it is sufficient when it is proposed to make quinquennial valuations to find the values for the "interpolated" offices from the total amount reserved by the "extreme" offices for the quinary group.

As an example of the working of the formula, let us take age at entry 30, the extreme rates being English No. 3 and H^M. In col. (1) we have the duration of the policies, in col. (2) the values of the same supposed to be for 100 each for a mixed English No. 3

valuation, and in col. (3) the pure premium values for H^M, in col. (4) the number of policies for valuation, in cols. (5) and (6) respectively the products of (2) and (3) by (4) :—

TABLE D.

Duration of Policies. (1)	V' Eng. No. 3. (2)	V H^m (3)	N (4)	V' × N (5)	V × N (6)
½ years	−2·910	·835	637	−1853·5	531·9
1½ „	−1·901	1·806	589	−1119·6	1063·7
2½ „	− ·864	2·805	554	− 478·6	1554·0
3½ „	+ ·204	3·833	525	+ 107·1	2012·3
4½ „	+1·306	4·894	501	+ 654·2	2451·9
				−3451·7	7613·8
				+ 761·3	2690·4
				−2690·4	10304·2

Premium. (7)		Δ (8)	$\frac{\Delta}{\Sigma\Delta}=c$ (9)	$c(\Sigma VN-\Sigma V'N)$ (10)	Reserve. (11)
EIII	1·8817	·0270	·1271	1309·9	−2690·4
EI	1·8547	·0177	·0833	858·7	−1380·5
EII	1·8370	·0337	·1587	1634·9	− 521·8
D	1·8033	·0479	·2255	2323·8	+1113·1
C	1·7554	·0320	·1507	1552·4	3436·9
S	1·7234	·0264	·1243	1280·7	4989·3
O	1·6970	·0277	·1304	1343·8	6270·0
H^m	1·6693	...	...	...	7613·8
		·2124	...	10304·2	...

Arranging the premiums in descending order of magnitude in col. (7), we have in col. (8) the excess of each over the next in succession, and in col. (9) the proportion of this difference to the total differences, being the excess of the English No. 3 premium over the H^M. Multiplying this proportion by the difference 10304·2 in the above reserves, we have in col. (10) the amount to be added to the English No. 3 reserve to obtain that for English No. 1, and the amount to be added to this to obtain the English No. 2, and so on; and in col. (11) the actual reserves for each rate of premium. The quantity $\frac{\Delta}{\Sigma\Delta}$ in col. (9) is the constant to be

multiplied by the difference in the reserves of H^m and EIII for policies issued at age 30, for all subsequent periods of duration, in groups of 5 years.

The values above given for the first quinquennium are the reserves made when negative values are included, and the amount quoted for Office S is therefore incorrect, as it is one of the special features of Mr. Sprague's suggested method of valuation that negative values should never under any circumstances be allowed, but the correction for the policies in their first year is easily made afterwards. It will be observed that when the total reserve for policies of all durations at any age at entry is obtained for the two extreme rates of premium, the total for the intermediate rates can be found by the formula; and by thus checking the gross reserves for each age at entry and summing them, we have a check on the partial summations for each quinquennium, and also on the carriage of each partial value to its proper place, which is of considerable assistance, as the positions of the various offices are continually changing in consequence of the crossing in the premiums at different ages at entry.

As further on I have considered the effect of excluding the negative values where they occur, and making the requisite reserve for the unexpired risk, the policies for the first five years were valued separately for each rate of premium. In this case, however, the final check by the foregoing formula is no longer obtainable; but as the negative values are limited to policies of less than five years' duration, the reserves will always be in excess of the corresponding value when the negative values are retained, by a quantity equal to the difference in the reserves for the first quinquennium.

As one of the objects of this investigation was to test the effect of the mixed method of valuation on the profits of an office, or rather the amounts represented to be divisible as such, I have calculated the actual receipts and disbursements on the assumption of the foregoing experience, taking the gross premiums charged as follows:—

Age.	Premium.	Age.	Premium.	Age.	Premium.
	£ s. d.		£ s. d.		£ s. d.
20	2 0 0	35	2 18 0	50	4 10 0
25	2 4 0	40	3 6 0	55	5 12 0
30	2 10 0	45	3 18 0	60	6 18 0

and $4\frac{1}{2}$ per-cent* as the rate of interest at which the average of the funds at the beginning and end of the year can be accumulated. For this purpose, however, it was necessary to re-arrange the deaths. Assuming the financial year to run from 1st January to 31st December, and the premiums to fall due on 1st July, at the close of the first financial year an office would have in hand the premiums and interest less the expenses and the claims which I have taken at the number to accrue, as previously mentioned, before the date of valuation on 31st December; whilst in the second financial year we have, besides the income and expenses, the remainder of the claims falling in during the first year of assurance, and the first part of the claims for the second year of assurance, and so on for other years; the surrenders coming into the same financial year as the second part of the claims for the same year of assurance. I have allowed throughout one-third of the premiums paid as a surrender-value after payment of three premiums. The rate of expenditure was probably the most difficult point of all to determine. To have assumed it at 50 per-cent of the first premium, and 7 or 8 per-cent of the renewals, was insufficient for the early years; and that standard once departed from, it was difficult to take it up afterwards. In consequence of the expense of obtaining business, and the heavy proportion that even a moderate amount of preliminary expenses bears to a small premium income, it would appear from the few accounts of young companies I have seen that an office can do little more than pay all its expenses and make a net-premium valuation out of its income for the first quinquennium. Starting with this view, the rates of expenditure I have assumed allow of a small surplus at the first valuation, and taking 13 per-cent of the total premium income as the average rate amongst offices doing £300,000 a-year new business, I have gradually reduced the rates until this minimum is arrived at, and thenceforward taken the rate as constant.

If, then, ${}_0A \,.\, {}_1A \ldots\ldots$ be the amount of funds at the beginning of the 1st, 2nd ...... year between the dates of valuation, ${}_0P$, ${}_1P \ldots\ldots {}_0C$, ${}_1C \ldots\ldots {}_0S$, ${}_1S \ldots\ldots {}_0E$, ${}_1E \ldots\ldots {}_0I$, ${}_1I \ldots\ldots$ the premiums, claims, surrenders, expenses, and interest, in those

* This manner of stating the rate of interest is in accordance with Messrs. Malcolm and Hamilton's Report to the Board of Trade, but the same results would be arrived at by assuming an effective rate of 4·6036 per-cent on the amount of funds at the beginning of the year, and a half year's interest at the same rate on the balance of premium income over total outgo.

years respectively; and i, one-half the rate of interest realized on the average of the funds at the beginning and end of the year, we have

$$ {}_1A = {}_0A + {}_0I + {}_0P - {}_0C - {}_0S - {}_0E. $$

Putting $B = P - (C + S + E)$, or the difference between premium income and total outgo, and observing that

$$ {}_0I = ({}_0A + {}_1A)i, $$

we have

$$ \begin{aligned} {}_1A &= {}_0A + ({}_0A + {}_1A)i + {}_0B \\ &= {}_0A \cdot \frac{1+i}{1-i} + \frac{1}{1-i} \cdot {}_0B, \end{aligned} $$

Similarly,

$$ \begin{aligned} {}_2A &= {}_1A \frac{1+i}{1-i} + \frac{1}{1-i} \cdot {}_1B \\ &= \left({}_0A \frac{1+i}{1-i} + \frac{1}{1-i} \cdot {}_0B\right) \frac{1+i}{1-i} + \frac{1}{1-i} \cdot {}_1B \\ &= {}_0A \left(\frac{1+i}{1-i}\right)^2 + \frac{1}{1-i} \left\{ {}_0B \cdot \frac{1+i}{1-i} + {}_1B \right\}. \end{aligned} $$

And generally,

$$ {}_tA = {}_0A \left(\frac{1+i}{1-i}\right)^t + \frac{1}{1-i} \left\{ {}_0B \left(\frac{1+i}{1-i}\right)^{t-1} + {}_1B \left(\frac{1+i}{1-i}\right)^{t-2} + \ldots\ldots {}_{t-1}B \right\}. $$

It will be observed that ${}_0A$ is the reserve made at the last valuation (including any profit that may not have been divided), and ${}_0A \left(\frac{1+i}{1-i}\right)^t$ the amount of this at the rate of interest realized at the date of the following valuation; whilst the remaining part of the expression, representing the accumulation of the balance of premium income over outgo in each of the years between the dates of valuation, will be constant for all the offices hereafter referred to. In some years B would be positive, in others negative; if the total for the term be positive, it must be added to ${}_0A \left(\frac{1+i}{1-i}\right)^t$, and if negative, deducted, to obtain the gross amount of funds at the date of valuation; and the difference between this amount and the reserve then made will be the profit for the period. The value of this constant for each quinquennium is given in column K.

TABLE E.

Age of Office. (1)	Premium Income. (2)	No. of Claims of 100 each. (3)	Surrenders. (4)	EXPENSES. Per-cent of Premium Income. (5)	EXPENSES. Amount. (6)	Premium Income less Total Outgo. 2−(3+4+6) (7)	K. (8)
1	9,381·	12	Nil.	50·	4,690	3,491	
2	18,029·	42	”	45·	8,113	5,716	
3	26,168·	72	”	40·	10,467	8,501	
4	33,858·	101	338	35·	11,850	11,570	
5	41,166·	129	714	30·	12,350	15,202	48,417
6	48,149·	155	1,091	29·	13,963	17,595	
7	54,838·	185	1,475	28·	15,355	19,508	
8	61,258·	211	1,867	27·	16,540	21,751	
9	67,427·	240	2,255	26·	17,531	23,641	
10	73,368·	269	2,624	25·	18,342	25,502	120,128
11	79,105·	298	2,934	24·	18,985	27,386	
12	84,640·	327	3,296	23·	19,467	29,177	
13	89,993·	356	3,588	22·	19,798	31,077	
14	95,172·	386	3,874	21·	19,986	32,712	
15	100,165·	416	4,214	20·	20,033	34,318	172,528
16	104,992·	448	4,457	19·5	20,473	35,262	
17	109,653·	478	4,739	19·	20,834	36,280	
18	114,152·	514	4,947	18·5	21,118	36,687	
19	118,487·	547	5,185	18·	21,328	37,274	
20	122,664·	583	5,424	17·5	21,466	37,474	204,961
21	126,677·	618	5,571	17·	21,535	37,771	
22	130,537·	654	5,796	16·5	21,539	37,802	
23	134,239·	690	5,975	16·	21,478	37,786	
24	137,794·	728	6,115	15·5	21,359	37,520	
25	141,185·	765	6,307	15·	21,178	37,200	211,029
26	144,424·	804	6,510	14·6	21,086	36,428	
27	147,512·	842	6,677	14·2	20,947	35,688	
28	150,455·	880	6,884	13·8	20,763	34,808	
29	153,250·	918	7,034	13·4	20,536	33,880	
30	155,903·	957	7,163	13·	20,267	32,773	195,152
31	158,401·	997	7,319	13·	20,592	30,790	
32	160,770·	1,038	Constant.	Constant.	20,900	28,751	
33	163,008·	1,078	...	...	21,191	26,698	
34	165,121·	1,118	...		21,466	24,536	
35	167,111·	1,158	...	...	21,724	22,268	150,299
36	168,974·	1,199	...	...	21,966	19,789	
37	170,716·	1,238	...	...	22,193	17,404	
38	172,334·	1,279	...	...	22,403	14,712	
39	173,838·	1,318	...	...	22,599	12,120	
40	175,228·	1,355	...	...	22,780	9,629	83,906
41	176,510·	1,393	...	...	22,946	6,945	
42	177,686·	1,431	...	...	23,099	4,168	
43	178,756·	1,468	...	...	23,238	1,399	
44	179,723·	1,503	...	...	23,364	− 1,260	
45	180,601·	1,535	...	...	23,478	− 3,696	9,824

TABLE E—*continued.*

Age of Office.	Premium Income.	No. of Claims of 100 each.	Surrenders.	Expenses. Per-cent of Premium Income.	Expenses. Amount.	Premium Income less Total Outgo. 2−(3+4+6)	K.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
46	181,387·	1,570	...	...	23,580	− 6,512	
47	182,091·	1,598	...	...	23,672	− 8,700	
48	182,719·	1,627	...	...	23,753	−11,053	
49	183,273·	1,654	...	...	23,825	−13,271	
50	183,762·	1,680	...	...	23,889	−15,446	− 60,538
51	184,184·	1,703	...	...	23,944	−17,379	
52	184,552·	1,726	...	...	23,992	−19,359	
53	184,865·	1,745	...	...	24,032	−20,986	
54	185,134·	1,764	...	...	24,067	−22,652	
55	185,361·	1,780	...	...	24,097	−24,055	−116,294
56	185,549·	1,796	...	...	24,121	−25,491	
57	185,704·	1,807	...	...	24,142	−26,457	
58	185,833·	1,818	...	...	24,158	−27,444	
59	185,939·	1,827	...	...	24,172	−28,252	
60	186,022·	1,836	...	...	24,183	−29,080	−152,902
61	186,088·	1,844	...	...	24,191	−29,822	
62	186,138·	1,849	...	...	24,198	−30,279	
63	186,177·	1,854	...	...	24,203	−30,745	
64	186,208·	1,858	...	...	24,207	−31,118	
65	186,229·	1,861	...	...	24,210	−31,400	−171,818
66	186,244·	1,864	...	...	24,212	−31,687	
67	186,257·	1,865	...	...	24,213	−31,775	
68	186,265·	1,867	...	...	24,214	−31,968	
69	186,271·	1,868	...	...	24,215	−32,063	
70	186,273·	1,870	...	...	24,216	−32,262	−179,116
71	186,273· Constant.	1,871 Constant.	...	...	24,216 Constant.	−32,362 Constant.	−181,493 Constant.

I have not carried the investigation further than for one rate of interest, but on comparing the percentages given in Mr. Geo. King's admirable paper on Net Premium Reserves (vol. xx), it will be seen that the difference between the standard he assumes and the reserves by other tables increases with the rate of interest; so that, for instance, a 3 per-cent 17 Offices valuation is nearer to a 3 per-cent H^{M}, than a 4 per-cent valuation by the first table is to the like value by the latter; the variation, with the exception of the Carlisle and Davies's Equitable Tables, being unimportant. But in mixed valuations, such as are the subject of this paper, we practically deal with only one table and varying percentages of the net premiums derived therefrom; and, as in Table A, we see that this percentage increases with the rate of interest, it will be fair to assume that the reserves for 3 per-cent mixed valuations would

approximate somewhat more closely to the 3 per-cent H^M values than those at 4 per-cent.

In the following Table F are given the quinquennial reserves and profits for offices valuing the 4 per-cent premiums referred to in Table A by the H^M 4 per-cent annuities, the offices marked (*a*) including the negative values arising from the method of valuation, whilst those marked (*b*) exclude them, and in addition make for all policies having negative values a reserve to cover the unexpired current risk, which I have taken at $\frac{rq_x}{2}\left(1+\frac{i}{2}\right)$.

TABLE F (see pp. 328, 329).

On comparing the different reserves brought out, one is immediately struck with the great effect produced by the mixed method of valuation on the liability of a young office, that for (EIII*a*) being as little as 5·08 per-cent of the H^M value at the first investigation, increasing to 50·85 per-cent at the second, and 67·49 per-cent at the third, but subsequently approximating more slowly, until it eventually reaches at the 14th valuation 89·59 per-cent, at which, under the conditions assumed, it would remain constant, the difference in the amounts reserved being 165,684. The great jump in the percentages at the first and second valuations arises from the negative values appearing only in policies of less than 5 years' duration, and further from the difference in the values of policies by the mixed and net methods decreasing as they get older. In fact, at the first valuation the difference in the amounts reserved by offices (EIII*a*) and (H^m) is 45,139; at the second, (H^m) adds 111,050 to its previous reserve, and (EIII*a*) 78,227; the difference being here reduced to 32,823; whilst at the third valuation, the balance in favour of (H^m) is 25,368 only; and so on, the difference in the amounts added at each subsequent valuation decreasing, but the difference in the gross total always increasing. A similar state of things occurs in all the other cases, the jumps in the percentages being greatest where the smallest reserve is made at the commencement.

The difference in the amounts reserved by (EII*a*) and (EI*a*) is very small throughout. The English No. 1 premiums being higher than those by the No. 2 table, for ages at entry 20 to 35, and less for 40 to 60, the amounts reserved by (EI*a*) is less for the younger and more for the older group; and this crossing in the premiums enables (EI*a*) to make a triflingly larger reserve at the first valuation, but at the second the balance is the other way,

TABLE E—*continued.*

Age of Office. (1)	Premium Income. (2)	No. of Claims of 100 each. (3)	Surrenders. (4)	Expenses. Per-cent of Premium Income. (5)	Expenses. Amount. (6)	Premium Income less Total Outgo. 2−(3+4+6) (7)	K. (8)
46	181,387·	1,570	...	...	23,580	− 6,512	
47	182,091·	1,598	...	...	23,672	− 8,700	
48	182,719·	1,627	...	...	23,753	−11,053	
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50	183,762·	1,680	...	...	23,889	−15,446	− 60,538
51	184,184·	1,703	...	...	23,944	−17,379	
52	184,552·	1,726	...	...	23,992	−19,359	
53	184,865·	1,745	...	...	24,032	−20,986	
54	185,134·	1,764	...	...	24,067	−22,652	
55	185,361·	1,780	...	...	24,097	−24,055	−116,294
56	185,549·	1,796	...	...	24,121	−25,491	
57	185,704·	1,807	...	...	24,142	−26,457	
58	185,833·	1,818	...	...	24,158	−27,444	
59	185,939·	1,827	...	...	24,172	−28,252	
60	186,022·	1,836	...	...	24,183	−29,080	−152,902
61	186,088·	1,844	...	...	24,191	−29,822	
62	186,138·	1,849	...	...	24,198	−30,279	
63	186,177·	1,854	...	...	24,203	−30,745	
64	186,208·	1,858	...	...	24,207	−31,118	
65	186,229·	1,861	...	...	24,210	−31,400	−171,818
66	186,244·	1,864	...	...	24,212	−31,687	
67	186,257·	1,865	...	...	24,213	−31,775	
68	186,265·	1,867	...	...	24,214	−31,968	
69	186,271·	1,868	...	...	24,215	−32,063	
70	186,273·	1,870	...	...	24,216	−32,262	−179,116
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TABLE F (see pp. 328, 329).

On comparing the different reserves brought out, one is immediately struck with the great effect produced by the mixed method of valuation on the liability of a young office, that for (EIIIa) being as little as 5·08 per-cent of the H^M value at the first investigation, increasing to 50·85 per-cent at the second, and 67·49 per-cent at the third, but subsequently approximating more slowly, until it eventually reaches at the 14th valuation 89·59 per-cent, at which, under the conditions assumed, it would remain constant, the difference in the amounts reserved being 165,684. The great jump in the percentages at the first and second valuations arises from the negative values appearing only in policies of less than 5 years' duration, and further from the difference in the values of policies by the mixed and net methods decreasing as they get older. In fact, at the first valuation the difference in the amounts reserved by offices (EIIIa) and (H^m) is 45,139; at the second, (H^m) adds 111,050 to its previous reserve, and (EIIIa) 78,227; the difference being here reduced to 32,823; whilst at the third valuation, the balance in favour of (H^m) is 25,368 only; and so on, the difference in the amounts added at each subsequent valuation decreasing, but the difference in the gross total always increasing. A similar state of things occurs in all the other cases, the jumps in the percentages being greatest where the smallest reserve is made at the commencement.

The difference in the amounts reserved by (EIIa) and (EIa) is very small throughout. The English No. 1 premiums being higher than those by the No. 2 table, for ages at entry 20 to 35, and less for 40 to 60, the amounts reserved by (EIa) is less for the younger and more for the older group; and this crossing in the premiums enables (EIa) to make a triflingly larger reserve at the first valuation, but at the second the balance is the other way,

Table F.

Office.	Reserve.	Per-cent of H^M Value.	Profit.	Per-cent of H^M Office.	Reserve.	Per-cent of H^M Value.	Profit.	Per-cent of H^M Office.	Reserve.	Per-cent of H^M Value.	Profit.	Per-cent of H^M Office.
	Age of Office.											
	5 Years.				10 Years.				15 Years.			
EIII *a*	2,418	5·08	45,999	5348·7	80,645	50·85	42,511	201·67	214,552	67·49	58,973	110·68
„ *b*	18,410	38·71	30,007	3489·2	96,637	60·93	46,548	220·82	230,544	72·53	63,010	118·26
EII *a*	13,790	29·00	34,627	4026·4	100,229	63·19	37,169	176·32	240,400	75·63	57,651	108·20
„ *b*	23,240	48·87	25,177	2927·6	109,679	69·15	39,554	187·64	249,850	78·60	60,037	112·68
EI *a*	13,818	29·06	34,599	4023·1	100,213	63·18	37,221	176·57	240,222	75·57	57,809	108·50
„ *b*	24,406	51·32	24,011	2791·9	110,801	69·86	39,893	189·25	250,810	78·90	60,482	113·52
D *a*	30,330	63·78	18,087	2103·1	128,299	80·89	29,813	141·43	276,829	87·09	56,378	105·81
„ *b*	35,206	74·03	13,211	1536·1	133,175	83·97	31,044	147·26	281,705	88·62	57,607	108·12
C *a*	41,506	87·28	6,911	803·6	147,720	93·13	24,389	115·70	302,689	95·22	54,839	102·92
„ *b*	43,202	90·85	5,215	606·4	149,416	94·39	24,817	117·72	304,385	95·75	55,268	103·73
O *a*	41,671	87·62	6,746	784·4	148,455	93·60	23,860	113·19	304,425	95·77	54,024	101·40
„ *b*	41,975	88·26	6,442	749·1	148,759	93·79	23,937	113·55	304,729	95·86	54,101	101·54
H^m	47,557	...	860	...	158,607	...	21,080	...	317,882	...	53,281	...
S	33,194	69·80	15,223	1770·1	132,544	83·56	29,155	138·31	283,084	89·05	55,438	104·05
	20 Years.				25 Years.				30 Years.			
EIII *a*	388,861	75·97	84,798	92·97	586,582	80·97	111,445	87·32	788,518	84·14	141,252	85·45
„ *b*	404,853	79·10	88,835	97·39	602,574	83·18	115,481	90·48	804,510	85·85	145,289	87·89
EII *a*	419,552	81·97	86,479	94·81	620,783	85·69	115,680	90·64	825,183	88·06	147,419	89·18
„ *b*	429,002	83·81	88,864	97·42	630,233	87·00	118,066	92·51	834,633	89·07	149,803	90·62
EI *a*	419,041	81·87	86,768	95·13	619,959	85·58	115,864	90·78	824,024	87·93	147,546	89·26
„ *b*	429,629	83·94	89,439	98·06	630,547	87·04	118,536	92·87	834,612	89·06	150,219	90·87
D *a*	461,918	90·24	89,736	98·38	667,341	92·12	122,181	95·73	874,525	93·32	156,384	94·60
„ *b*	466,794	91·20	90,967	99·73	672,217	92·79	123,411	96·69	879,401	93·84	157,615	95·35
C *a*	492,789	96·27	91,251	100·04	701,971	96·90	126,213	98·89	911,816	97·30	162,464	98·28
„ *b*	494,485	96·60	91,679	100·51	703,667	97·13	126,641	99·22	913,512	97·48	162,892	98·54
O *a*	495,815	96·87	90,400	99·11	706,404	97·51	125,570	98·39	917,600	97·92	162,231	98·14
„ *b*	496,119	96·93	90,476	99·19	706,708	97·55	125,646	98·44	917,904	97·95	162,307	98·19
H^m	511,852	...	91,214	...	724,426	...	127,631	...	937,095	...	165,307	...
S	470,572	91·94	88,914	97·48	678,487	93·66	121,873	95·49	887,974	94·76	156,894	94·91
	35 Years.				40 Years.				45 Years.			
EIII *a*	977,963	86·25	150,852	84·12	1,141,060	87·69	167,617	82·89	1,265,804	88·57	178,049	82·08

D *a*	1,067,329	94·13	178,198	93·78	1,232,508	94·88	186,088	93·02	1,356,408	95·04	194,974	95·47
„ *b*	1,072,205	94·56	179,429	94·43	1,237,384	95·06	189,818	93·68	1,363,282	95·39	196,204	98·06
C *a*	1,106,387	97·58	185,845	97·80	1,272,662	97·77	196,847	97·35	1,399,170	97·90	204,498	96·99
„ *b*	1,108,083	97·73	186,273	98·03	1,274,358	97·90	197,404	97·56	1,400,866	98·02	205,058	97·19
O *a*	1,113,319	98·19	186,155	97·97	1,280,452	98·37	197,742	97·79	1,407,502	98·48	205,922	97·67
„ *b*	1,113,623	98·21	186,231	98·01	1,280,756	98·39	197,818	97·83	1,407,806	98·50	205,999	97·70
H^m	1,133,866	...	190,022	...	1,301,719	...	202,206	...	1,429,216	...	210,842	...
S	1,082,679	95·49	179,692	94·56	1,249,274	95·97	190,547	94·23	1,376,086	96·28	198,292	94·05
	50 Years.				55 Years.				60 Years.			
EIII *a*	1,348,932	89·11	175,785	81·52	1,395,528	89·40	177,539	81·24	1,416,597	89·53	178,219	81·10
„ *b*	1,364,924	90·17	179,822	83·39	1,411,520	90·43	181,576	83·09	1,432,589	90·54	182,255	82·94
EII *a*	1,389,045	91·76	185,564	86·06	1,435,757	91·98	187,547	85·82	1,456,866	92·08	188,331	85·70
„ *b*	1,398,495	92·39	187,948	87·17	1,445,207	92·59	189,933	86·91	1,466,316	92·68	190,716	86·79
EI *a*	1,387,043	91·63	185,184	85·88	1,433,709	91·85	187,089	85·61	1,454,801	91·95	187,327	85·47
„ *b*	1,397,631	92·33	187,857	87·12	1,444,297	92·53	189,761	86·83	1,465,389	92·62	190,505	86·69
D *a*	1,442,098	95·27	198,591	92·10	1,488,937	95·39	200,810	91·89	1,510,088	95·44	201,709	91·79
„ *b*	1,446,974	95·59	199,822	92·67	1,493,813	95·70	202,040	92·45	1,514,964	95·75	202,940	92·35
C *a*	1,483,162	97·99	208,579	96·73	1,530,124	98·03	211,050	96·58	1,551,316	98·05	212,063	96·50
„ *b*	1,484,858	98·10	209,008	96·93	1,531,820	98·14	211,479	96·77	1,553,012	98·16	212,481	96·69
O *a*	1,491,781	98·55	210,395	97·57	1,538,865	98·59	213,102	97·52	1,560,098	98·60	214,228	97·40
„ *b*	1,492,085	98·57	210,472	97·61	1,539,169	98·61	213,179	97·55	1,560,402	98·62	214,305	97·52
H^m	1,513,738	...	215,631	...	1,560,935	...	218,531	...	1,582,212	...	219,754	...
S	1,460,279	96·47	202,552	93·93	1,507,344	96·57	205,172	93·89	1,528,576	96·61	206,274	93·87
	65 Years.				70 Years.				Reserve less than H^M Office.	Office (*b*) in excess of Office (*a*).	Profit less than H^M Office.	Office (*b*) in excess of Office (*a*).
EIII *a*	1,423,865	89·58	178,420	81·04	1,425,648	89·59	178,442	81·02	165,684	15,992	41,800	4,036
„ *b*	1,439,857	90·58	182,457	82·88	1,441,640	90·59	182,478	82·85	149,692		37,764	
EII *a*	1,464,145	92·11	188,573	85·65	1,465,931	92·12	188,603	85·63	125,401	9,450	31,639	2,385
„ *b*	1,473,595	92·71	190,958	86·74	1,475,381	92·71	190,988	86·72	115,951		29,254	
EI *a*	1,462,074	91·98	188,058	85·42	1,463,859	91·99	188,083	85·40	127,473	10,588	32,159	2,672
„ *b*	1,472,662	92·65	190,730	86·63	1,474,447	92·66	190,755	86·38	116,885		29,487	
D *a*	1,517,379	95·46	201,992	91·75	1,519,167	95·47	202,037	91·73	72,165	4,876	18,205	1,230
„ *b*	1,522,255	95·77	203,223	92·31	1,524,043	95·77	203,267	92·29	67,289		16,975	
C *a*	1,558,617	98·06	212,387	96·47	1,560,407	98·06	212,443	96·46	30,925	1,696	7,799	428
„ *b*	1,560,313	98·16	212,815	96·66	1,562,103	98·16	212,871	96·65	29,229		7,371	
O *a*	1,567,410	98·61	214,592	97·48	1,569,201	98·61	214,661	97·47	22,131	304	5,581	77
„ *b*	1,567,714	98·63	214,669	97·51	1,569,505	98·63	214,738	97·50	21,827		5,504	
H^m	1,589,538	...	220,159	...	1,591,332	...	220,242	...	...	...	...	...
S	1,535,889	96·62	206,636	93·86	1,537,681	96·63	206,704	93·85	53,651	...	13,538	...

and gradually increases, until at the final valuation it is 2,072, or ·13 per-cent of the H^M reserve.

The crossings that take place in the Carlisle and Davies's Equitable premiums with the 17 Offices and H^M premiums (P_x and P_{x+1}), do not at any time interfere with the rank taken by the offices valuing those premiums.

When, however, we come to exclude the negative values, where these occur, and to retain a sum to cover the unexpired risk, considerable crossing in the reserves takes place, the reason for which may be gathered from Table G, where are given the amounts of positive and negative values, and unexpired risk reserves at the first valuation for each set of premiums.

Table G.—*Showing the Positive and Negative Values, and Unexpired Risk Reserves for each set of Premiums.*

Office.	Positive Values. (1)	Negative Values. (2)	Unexpired Risk reserve. (3)	Reserve. Office (a). (1)−(2). (4)	Office (b). (1)+(3). (5)	Difference being excess of (b) over (a). (5)−(4)=(2)+(3).
EIII	15,146	12,728	3,264	2,418	18,410	15,992
EII	21,057	7,267	2,183	13,790	23,240	9,450
EI	22,091	8,273	2,315	13,818	24,406	10,588
D	33,696	3,366	1,510	30,330	35,206	4,876
C	43,071	1,065	631	42,006	43,702	1,696
O	41,771	100	204	41,671	41,975	304
H^m	47,557	Nil.	Nil.	47,557		...
S	32,146	"	1,481	33,627		...

In spite of the large addition made to (EIIIa), (EIIIb) is better than (EIa) and (EIIa) at the first valuation only; (EIb) is better than (EIIa) throughout, and only falls below (EIIb) at the sixth quinquennium, by which time the natural increase in the reserves of (EIIa) over (EIa) exceeds the larger amount added to the latter to get rid of the negative values. Offices (Da) and (Db) maintain their rank throughout, whilst (Cb), which is in excess of (Oa) and (Ob) for the first and second valuations, sinks at the third below both. All the offices valuing the English premiums are below office S throughout, as also office (Da); (Db) is in excess for the first two valuations only, and the (C) and (O) offices throughout.

In order further to compare the effects of the mixed method of valuation with the more common pure premium method, I have calculated from Mr. Geo. King's paper the percentages to his H^M 4 per-cent reserves of the net values at the same rate of interest for the 17 Offices, Carlisle, Davies's Equitable, and English Tables 2 and 3. It is probable that the effect of estimating the next premium

as just due, or due six months hence, or even the variation in the assumed ages of the assured at the date of valuation, will not, as the two models are otherwise so similar, cause any material difference in the proportions found to exist; and the comparison instituted in the following table may therefore be taken as throwing additional light on the all-important question of data in valuations.

TABLE H.—*Showing the Percentage of Net and Mixed Valuations to a* H^M *Net-Premium Valuation—Interest 4 per-cent.*

Table of Mortality.	Net. (1)	Mixed. (2)	Difference. (1)−(2)	Net. (1)	Mixed. (2)	Difference. (1)−(2)
	1st Quinquennium.			2nd Quinquennium.		
17 Offices . .	98·42	87·62	10·80	98·76	93·60	5·16
Carlisle . .	91·22	87·28	3·94	91·21	93·13	− 1·92
Davies's Equitable	90·57	63·78	26·79	90·03	80·89	9·14
English No. 2 .	96·66	29·00	67·66	96·69	63·19	33·50
" " 3 .	97·40	5·08	92·32	97·41	50·85	46·56
(Office S) . .	...	69·80	...	...	83·56	...
	3rd Quinquennium.			4th Quinquennium.		
17 Offices . .	99·03	95·77	3·26	99·32	96·87	2·45
Carlisle . .	91·35	95·22	− 3·87	91·63	96·27	− 4·64
Davies's Equitable	89·80	87·09	2·71	89·67	90·24	− ·57
English No. 2 .	96·75	75·63	21·12	96·84	81·97	14·87
" " 3 .	97·41	67·49	29·92	97·39	75·97	21·42
(Office S) . .	...	89·05	...	...	91·94	...
	5th Quinquennium.			6th Quinquennium.		
17 Offices . .	99·43	97·51	1·92	99·48	97·92	1·56
Carlisle . .	92·05	96·90	− 4·85	92·49	97·30	− 4·81
Davies's Equitable	89·68	92·12	− 2·44	89·72	93·32	− 3·60
English No. 2 .	96·95	85·69	11·26	97·07	88·06	9·01
" " 3 .	97·38	80·97	16·41	97·40	84·14	13·26
(Office S) . .	...	93·66	...	...	94·76	...
	7th Quinquennium.			8th Quinquennium.		
17 Offices . .	99·50	98·19	1·31	99·50	98·37	1·13
Carlisle . .	92·86	97·58	− 4·72	93·12	97·77	− 4·65
Davies's Equitable	89·80	94·13	− 4·33	89·93	94·68	− 4·75
English No. 2 .	97·18	89·62	7·56	97·24	90·67	6·57
" " 3 .	97·41	86·25	11·16	97·44	87·66	9·78
(Office S) . .	...	95·49	...	...	95·97	...
	9th Quinquennium.			10th Quinquennium.		
17 Offices . .	99·50	98·48	1·02	99·49	98·55	·94
Carlisle . .	93·31	97·90	− 4·59	93·44	97·99	− 4·55
Davies's Equitable	90·11	95·04	− 4·93	90·28	95·27	− 4·99
English No. 2 .	97·28	91·35	5·93	97·30	91·76	5·54
" " 3 .	97·46	88·57	8·89	97·47	89·11	8·36
(Office S) . .	...	96·28	...	...	96·47	...

Briefly summarized the results are as follows:—

17 *Offices.*—The net reserve improves 1 per-cent in 30 years and then remains stationary, whilst the mixed improves 10 per-cent in the same time, and continues thenceforward to approach nearer to the H^M net value. The difference between the net and mixed reserves decreases, at first rapidly, and then more slowly, but the balance would always be in favor of the net reserve.

Carlisle.—The net reserve improves slowly throughout, gaining 2·22 per-cent in 50 years; but the mixed, which is below the net at the first valuation, is in excess at the second, and rapidly improves, gaining 10·71 per-cent in the 50 years. After the first quinquennium, then, a Carlisle mixed valuation requires a larger reserve than one by the net premium method.

Davies's Equitable.—The net falls off ·90 per-cent in the first four valuations, and then slowly improves; and at the end of 50 years is ·29 per-cent less than at the first valuation. The mixed rapidly improves, and passes the net at the fourth valuation, and at the tenth is 4·99 per-cent in excess of the net. Hence a Davies's Equitable mixed valuation requires, after the third quinquennium, a higher reserve than the net.

English No. 2.—The net improves slowly, the mixed rapidly, but the balance is always in favor of the net.

English No. 3.—The net is almost stationary, whilst the mixed rapidly improves; still the balance is always in favor of the former.

A valuation on the principle of office S rapidly improves; it is always below the 17 Offices net and mixed values, but is better than a Carlisle or Davies's Equitable net valuation at and after the fourth quinquennium; is in excess of a Davies's mixed throughout, but below a Carlisle mixed; apparently it is always below the English 2 and 3 net values, but much superior throughout to a mixed valuation by either table.

Turning again to Table F, we see that in consequence of the small reserve made by (Eiii*a*) at the first valuation, the amount represented to be divisible as profit is very large. I do not mean to suggest that any actuary would advise the distribution of so large a proportion as 95 per-cent of the funds in hand; but bearing in mind the high rates of expenditure assumed, the example clearly shows the extent to which wasteful expenditure, or other unfavorable circumstances in starting a company, may be concealed

by employing a specious and unsound method of valuation. Moreover, the effect on the reserves and profits would be still more marked if the new business increased year by year; but I should observe that a reduction in the expenses, by adding a constant quantity to the profits of all the offices, would materially reduce the percentages. The smaller reserves made by the offices valuing by the mixed or Mr. Sprague's methods, allow of a larger distribution for the first three quinquenniums only; at the fourth, all, except a trifling and final excess for the two Carlisle offices, are below the (H^m), (EIII*a*), with 24 per-cent less reserve, making a distribution of 7 per-cent less profit; and thenceforward as the percentages of reserve increase, those of profit decrease.

According to the conditions laid down in Mr. Sprague's paper, before referred to, the policies in Office (S) would rank for the purpose either of surrender or of distribution of bonus, as if they had been taken out at age $(x+1)$. Assuming, then, that a policy on which three premiums had been paid, be surrendered for one-third of two premiums; by the time the surrender values in Table E attained a constant amount, a saving of £775 per annum would be effected under this head, and the rate of bonus to the assured entitled to participate would for some time after the fourth quinquennium be in excess of that allotted to entrants in the same year in Office (H^m); but as the benefits granted would thus be different in the two offices, a further comparison (into which I have not thought it advisable to go) would have to be made, in order to test the relative merits of the two systems.

Considering the very wide limits between which the reserves and profits vary at the first valuation, one cannot but see that an increase or falling off in the new business transacted during any quinquennium would have an important bearing on the reserves and profits brought out at a valuation by the methods in question. In order to throw some light on this point, I have supposed no new business, and double new business, to have been transacted during the five years, 6–10, or 26–30, or 51 to 55, the rates of expenditure remaining in all cases as previously assumed, and the results are shown in Tables I, K, and L.

TABLE I.

Office.	Reserve.		Percentage of H^M Reserve.		
	No New Business.	Double New Business.	No New Business.	Ordinary New Business.	Double New Business.
	Years 6 to 10.				
EIII *a*	78,227	83,063	70·44	50·85	40·29
„ *b*	78,227	115,047	70·44	60·93	55·81
EII *a*	86,439	114,019	77·84	63·19	55·31
„ *b*	86,439	132,919	77·84	69·15	64·48
EI *a*	86,395	114,031	77·80	63·18	55·31
„ *b*	86,395	135,207	77·80	69·86	65·59
D *a*	97,969	158,629	88·22	80·89	76·95
„ *b*	97,969	168,381	88·22	83·97	81·68
C *a*	106,214	189,226	95·65	93·13	91·79
„ *b*	106,214	192,618	95·65	94·39	93·43
O *a*	106,784	190,126	96·16	93·60	92·23
„ *b*	106,784	190,734	96·16	93·79	92·52
H^m	111,050	206,164	...	...	...
S	99,350	165,738	89·46	88·56	80·39
	Years 26 to 30.				
EIII *a*	786,100	790,936	88·37	84·14	80·33
„ *b*	786,100	822,920	88·37	85·85	83·58
EII *a*	811,393	838,973	91·22	88·06	85·21
„ *b*	811,393	857,873	91·22	89·07	87·13
EI *a*	810,206	837,842	91·08	87·93	85·09
„ *b*	810,206	859,018	91·08	89·06	87·24
D *a*	844,195	904,855	94·90	93·32	91·90
„ *b*	844,195	914,607	94·90	93·84	92·90
C *a*	870,310	953,322	97·84	97·30	96·82
„ *b*	870,310	956,714	97·84	97·48	97·16
O *a*	875,929	959,271	98·47	97·92	97·42
„ *b*	875,929	959,879	98·47	97·95	97·49
H^m	889,538	984,652	...	...	...
S	854,780	921,168	96·09	94·76	93·55
	Years 51 to 55.				
EIII *a*	1,393,110	1,397,946	92·06	89·40	86·91
„ *b*	1,393,110	1,429,930	92·06	90·43	88·90
EII *a*	1,421,967	1,449,547	93·96	91·98	90·12
„ *b*	1,421,967	1,468,417	93·96	92·59	91·29
EI *a*	1,419,891	1,447,527	93·82	91·85	89·99
„ *b*	1,419,891	1,468,703	93·82	92·53	91·31
D *a*	1,458,607	1,519,267	96·38	95·39	94·45
„ *b*	1,458,607	1,529,019	96·38	95·70	95·06
C *a*	1,488,618	1,571,630	98·36	98·03	97·71
„ *b*	1,488,618	1,575,022	98·36	98·14	97·92
O *a*	1,497,194	1,580,536	98·93	98·59	98·26
„ *b*	1,497,194	1,581,144	98·93	98·61	98·30
H^m	1,513,378	1,608,492	...	...	...
S	1,474,150	1,540,538	97·41	96·57	95·77

TABLE K.

Office.	Surplus. No New Business.	Surplus. Double New Business.	Percentage of H^M Surplus. No New Business.	Percentage of H^M Surplus. Ordinary New Business.	Percentage of H^M Surplus. Double New Business.
	Years 6 to 10—Expenses 29–25 per-cent.				
EIII *a*	− 18,667	103,689	−370·30	201·67	279·34
„ *b*	+ 1,362	91,734	+ 27·02	220·82	247·13
EII *a*	− 12,637	86,975	−250·68	176·32	234·31
„ *b*	− 802	79,910	− 15·91	187·64	215·28
EI *a*	− 12,557	86,999	−249·10	176·57	234·38
„ *b*	+ 703	79,083	+ 13·95	189·25	213·05
D *a*	− 3,453	63,079	− 68·50	141·43	169·94
„ *b*	+ 2,654	59,434	+ 52·65	147·26	160·12
C *a*	2,299	46,479	45·61	115·70	125·22
„ *b*	4,423	45,211	87·74	117·72	121·80
O *a*	1,935	45,185	38·39	113·19	121·73
„ *b*	2,316	45,558	45·94	113·55	122·73
H^m	5,041	37,119	...	...	...
S	− 1,247	59,557	− 24·74	138·31	160·45
	Years 26 to 30—Expenses 14·6 to 13 per-cent.				
EIII *a*	62,025	220,479	47·28	85·45	110·57
„ *b*	82,054	208,524	62·53	87·89	104·58
EII *a*	79,564	215,274	60·64	89·18	107·96
„ *b*	91,398	208,208	69·65	90·62	104·42
EI *a*	79,719	215,373	60·75	89·26	108·01
„ *b*	92,980	207,458	70·86	90·87	104·04
D *a*	105,069	207,699	80·07	94·60	104·16
„ *b*	111,176	204,054	84·72	95·35	102·34
C *a*	122,325	202,603	93·22	98·28	101·61
„ *b*	124,449	201,335	94·84	98·54	100·97
O *a*	122,257	202,204	93·17	98·14	101·41
„ *b*	122,637	201,977	93·46	98·19	101·30
H^m	131,219	199,395	...	...	...
S	108,443	205,345	82·64	94·91	102·98
	Years 51 to 55—Expenses 13 per-cent.				
EIII *a*	97,496	257,582	53·09	81·24	101·64
„ *b*	117,525	246,627	64·00	83·09	96·92
EII *a*	118,876	256,218	64·74	85·82	101·10
„ *b*	131,712	250,154	71·73	86·91	98·71
EI *a*	118,446	255,732	64·50	85·61	100·91
„ *b*	131,706	247,816	71·73	86·83	97·78
D *a*	148,679	252,941	80·97	91·89	99·81
„ *b*	154,785	249,295	84·29	92·45	98·37
C *a*	170,095	252,005	92·63	96·58	99·44
„ *b*	172,220	250,738	93·79	96·77	98·94
O *a*	172,312	253,892	93·84	97·52	100·18
„ *b*	172,693	253,665	94·05	97·55	100·09
H^m	183,627	253,435	...	...	...
S	155,905	254,439	84·90	93·89	100·40

TABLE L.—*Showing the amount of Profit derived from New Business of £300,000 per annum.*

Office.	Years 6–10.		Years 26–30.		Years 51 to 55.	
	Surplus.	Per-cent of H^M Office.	Surplus.	Per-cent of H^M Office.	Surplus.	Per-cent of H^M Office.
EIII *a*	61,178	381·4	79,227	232·4	80,043	229·3
" *b*	45,186	281·7	63,235	185·5	64,051	183·5
EII *a*	49,806	310·5	67,855	199·1	68,671	196·7
" *b*	40,356	251·6	58,405	171·3	59,221	169·7
EI *a*	49,778	310·4	67,827	199·0	68,643	196·7
" *b*	39,190	244·3	57,239	167·9	58,055	166·3
D *a*	33,266	207·4	51,315	150·5	52,131	149·4
" *b*	28,390	177·0	46,439	136·2	47,255	135·4
C *a*	22,090	137·7	40,139	117·7	40,955	117·3
" *b*	20,394	127·2	38,443	112·8	39,259	112·5
O *a*	21,925	136·7	39,974	117·3	40,790	116·9
" *b*	21,621	134·8	39,670	116·4	40,486	116·0
H^m	16,039	...	34,088	...	34,904	...
S	30,402	189·6	48,451	142·1	49,267	141·2

In Table I we see that the effect of a falling off of new business will be to increase the percentages of reserve to H^m, the increase becoming less, however, with the advanced age of the office; whilst the issue of a larger amount of new assurances has a contrary effect; the limits for (EIII*a*) being 30·15 per-cent at the first period and 5·15 per-cent only at the last.

In Table K we see that if no new business be transacted, there is on a mixed valuation a deficiency for (EIII*a*), which is 370·30 per-cent of the (H^m) surplus; whilst double new business gives a surplus, 279·34 per-cent of the amount for office (H^m), the limits being 649·64 of the (H^m) values. These at the second period are reduced to 63·29 per-cent, and at the third to 48·55 per-cent. The new business produces therefore a contrary effect on the profits and reserves, since a falling off of new business increases the percentages of reserve and reduces those of profit, whilst an increase of new business reduces the percentages of reserve and largely increases those of profit.

From Table K we obtain Table L, which shows the amount of profit extracted from the new business in the three periods before mentioned. By Table K we see that for (EIII*a*), if no new business be done, we have a deficiency of 18,667; whilst for an ordinary amount we have a surplus of 42,511—which is increased to 103,689 if the amount of new assurances be doubled. It follows, then, that 300,000 of new business clears off the deficiency, and

moreover creates the surplus; so that the profit derived by (E_{III}*a*) from the new business is 61,178, and a further 300,000 of new business adds 61,178 to the ordinary surplus of 42,511—thus bringing the total to 103,689. If we take the (H^m) office, we find when no new business is done a surplus of 5,041—a normal amount gives a surplus of 21,080, so that the profit from the new business is 16,039, and double new business adds this last amount of profit to the preceding, bringing the total surplus to 37,119.

The amounts in Table K were obtained by direct calculation for both assumptions as to new business, and the results are easily checked. For the difference of profit, when no new business and a normal amount is done, is equal to the excess of profit over the amount in the latter case when a double amount of new business is transacted; and by means of Table L, and the reserves at the first quinquennium valuation in Table F, we have the means of ascertaining what would be the reserve and surplus for any other amount of new business, provided it be constant (or nearly so), during the quinquennium. It will also be observed that the excess of profit extracted by any office over (H^m) is equal to the difference in the amounts of reserve or profit at the first valuation in Table F; that a constant of 15,179, added to the amounts of profit there stated, gives the amount extracted at the valuation in the 10th year; that a further constant of 18,049 added to the preceding gives the amount of profit derived in the 30th year, and a further constant addition of 816 gives the amount for the 55th year. These constants represent the accumulated saving from reduction in expenditure. The amounts in Table L also represent what would have been the various surpluses in the first quinquennium in Table F, if the rates of expenditure assumed had been as stated in Table K. It may be further interesting to know, that if the expenses had been 10 per-cent only during the years 6 to 10, when no new business was done, a saving of no less than 34,150 would have been effected.

With reference to office (S), if we charge a part of the expenses—equal to, say, 50 per-cent of the first premium—to the new business, and the balance to the renewals, then the amounts in Table L no longer represent the amount of surplus derived from the new business, but some portion is due to the reduced rate of expenditure on the old business.

It is sometimes stated, when valuations are made otherwise than on the pure premium method, that the whole of the loading (understood to mean the difference between the office and the original net

premium) has been reserved for future expenses and profit; but how can this be so in the case of (EIII*a*), where the reserve at the first valuation is insufficient to pay the expected claims on its outstanding risks?—or even taking any other of the offices marked (*a*), where the future loading is already mortgaged to repay the negative values included in the valuation on those policies which will not be renewed? Taking credit for negative values is an anticipation of future profit, which is duly realized on those policies that are kept up, but lost on those that are not; and the profit expected from these having been spent, a further drain on the profits of the older policies is made to refund the amount lost. The above statement as to loading can never be correct where any amount of negative values, no matter how small, is included in a valuation. Moreover, the objection to these is much greater in practice when a policy is valued on the average in the middle of the year than it would be if it were valued when the premium falls due; for in the latter case, if the amount of the negative value be divided as profit, and the policy dropt, the loss would be small; whilst if the premium be paid, it would more than cover the risk for the year on the total of sum assured and negative value, such being the amount at risk. But in practice, if a policy for 1 be reckoned as an asset of m, no reserve is made for the unexpired risk; and on the policy becoming a claim the office incurs a loss of $(1+m)$. It is not therefore sufficient only to strike out negative values when they occur in a valuation, but a reserve to meet the current risk should also be made, or otherwise the true position of the office cannot be ascertained.

Mr. Sprague's method of valuation suggests a very simple way of meeting heavy expenditure in starting a company, or in acquiring new business, or a deficiency in the funds arising from any other cause. But if it be attempted to do this by valuing the net premiums of one table by the annuities of another, in order to avoid the anomaly of negative values, such net premiums should first be compared with the risk to be run, and should not be valued until the assured has attained a valuation age $(x+n)$, when

$$\mathrm{A}_{x+n}-\mathrm{P}'_x(\cdot 5+a_{x+n})=\text{or}>\frac{v^{\frac{1}{2}}q_{x+n}}{2},$$

(or say until $\mathrm{P}'_x=\varpi_{x+n}$), and up to that time at each investigation the same reserve should be made for the policy as for a term assurance.

NOTE.—Since this paper was completed, Mr. Sutton has suggested that the sum assured, instead of being constant for all ages at entry, increases with the age at entry. Assuming this to be so, it may be interesting to know what effect it would have on the values given in Table F; and in the following Table M I have given at four different periods the percentages of the various reserves to the H^m, taking the policies issued at ages 20 and 25 for £80·42, at 30 and 35 £98·47, 40 and 45 £118·61, and 50 and upwards for £130. As this however increases the annual new business from £300,000 to £311,623, I only give the percentages.

TABLE M.

Office.	Increasing. (1)	Uniform. (2)	Difference. (1)−(2)	Increasing. (1)	Uniform. (2)	Difference. (1)−(2)
	AGE OF OFFICE.					
	5 Years.			25 Years.		
EIII	11·58	5·08	6·50	82·05	80·97	1·08
EII	34·73	29·00	5·73	86·67	85·69	·98
EI	36·00	29·06	6·94	86·80	85·58	1·22
D	70·43	63·78	6·65	93·30	92·12	1·18
C	92·62	87·28	5·34	97·89	96·90	·99
O	88·99	87·62	1·37	97·75	97·51	·24
S	70·11	69·80	·31	93·64	93·66	− ·02
	50 Years.			70 Years.		
EIII	89·39	89·11	·28	89·79	89·59	·20
EII	92·07	91·76	·31	92·36	92·12	·24
EI	92·07	91·63	·44	92·36	91·99	·37
D	95·79	95·27	·52	95·94	95·47	·47
C	98·49	97·99	·50	98·53	98·06	·47
O	98·65	98·55	·10	98·70	98·61	·09
S	96·35	96·47	− ·12	96·49	96·63	− ·14

It will be seen that the difference, at first considerable, is much reduced at the end of the 25th year, and subsequently becomes comparatively unimportant.

As the "mixed" offices suffer principally from negative values at the younger ages, the reduction in the amounts assured is beneficial to them; whilst the increase at the older ages, where the Carlisle and Davies's Equitable mixed reserves are in excess of the H^m net values, improves the position of these two considerably. The

contrary occurs with office S; for the difference in the reserves by the net and Mr. Sprague's methods increases with the age at entry; and although there is an improvement in the early valuations in consequence of the heavier reserve for policies issued at the advanced ages, yet, by the time the larger proportion of assurances issued at the younger ages comes into play, the position of the office is slightly impaired by the suggested alteration.

Actuarial Notes.

MR. JAMES SORLEY, of the Life Association of Scotland, sends us the following Notes :—

I.

Results of an unsuccessful attempt to graduate a Mortality Table by Makeham's Method: $l_x = \kappa g^{q^x} \div a^x$.

The second column in the following table gives the unadjusted numbers living at intervals of ten years of age, according to observation: the remaining columns give the numbers living according to five attempted graduations :—

Age x.	Unadjusted.	Adjusted by means of Constants found from				
		Ages 30, 50, 70, 90. (1)	Ages 40, 60, 80, 100. (2)	Mean of (1) and (2). (3)	Ages 20, 40, 60, 80. (4)	Mean of (1) and (4). (5)
30	100,000	100,000	100,309	99,696	100,168	99,664
40	91,315	92,505	91,315	90,573	91,315	90,691
50	83,030	83,030	81,622	78,808	81,678	79,165
60	68,981	69,598	68,981	61,191	68,981	61,976
70	49,953	49,953	49,087	35,184	48,986	36,492
80	20,613	25,214	20,613	9,159	20,613	10,195
90	5,726	5,726	1,717	295	1,786	405
100	0	213	0	0	0	0

The following are the logarithms of the various constants :—

	(1)	(2)	(3)	(4)	(5)
Log κ	5·0786 740	5·114 0720	5·096 3730	5·111 0341	5·094 8541
Log a	·0023 512	·003 6958	·003 0235	·003 6094	·002 9803
Log g	− ·0006 970	− ·000 0643	− ·000 3807	− ·000 0736	− ·000 3853
Log q	·0355 756	·048 6767	·042 1261	·047 9901	·041 7828

After adjustments (1) and (2) had been constructed and found unsatisfactory, a note of their deviations from the unadjusted numbers living was made as follows:—

Age.	Deviation of (1).	Deviation of (2).
30	0	+ 309
40	+1 190	0
50	0	−1 408
60	+ 617	0
70	0	− 866
80	+4 601	0
90	0	−4 009
100	+ 213	0

The consideration of these differences led to the conclusion that a nearer approximation to the truth would be found by taking something like a mean of the two tables. In order to attain this object, therefore, without sacrificing the property of "uniform seniority", a mean of the constants was taken, and (3) constructed therefrom. In justification of this course may be quoted the authority of Mr. Woolhouse, who, in graduating the H^{MF} table by Gompertz's formula, adopted a similar process with entire success. Having first constructed two adjusted life lines from independently ascertained constants, he next made a note of their differences from the ungraduated numbers. Then, noticing from these differences that a new table giving the living at each age $x = \frac{3}{8} l_x$ of first curve $+ \frac{5}{8} l_x$ of second curve, would be a better representation of the facts, he simply took a set of constants in the same proportions respectively of the old sets, and from it produced a table standing to the two others in almost exactly the relation desired. (See *Journal*, vol. xv, p. 405.)

Although such a close agreement was hardly expected in the present case, still it occasioned considerable surprise to find curve (3) falling so wide of the mark, as from the table it will be seen to do. The remaining curves proving equally unsuccessful, made it apparent that the formula would not apply to the case in hand.

It may be added further, that after a very successful graduation by Mr. Woolhouse's new method, a final attempt to use Gompertz's method was made, by determining a set of constants from the adjusted numbers living at 30, 50, 70, and 90, and from them constructing a curve. This curve, however, also failed; indeed it did not differ greatly from curve (1) above.

The preceding suggests the enquiry—What should be the test of a satisfactory graduation? It appears to me that if the adjusted probability of dying in a year will, on being multiplied into the number exposed to risk, reproduce the actual deaths, this is the best evidence that the adjusted table expresses the essential teaching of the data, and is far more satisfactory than the agreement of annuity values before and after graduation, which is the test adopted by Mr. Sutton. In truth, when the facts are somewhat irregular, especially when the numbers exposed to risk differ greatly and suddenly, the adjusted table probably *ought not* to give annuity values agreeing with the values before adjustment. On the other hand, a little consideration will show, that by whatever formula the graduation may be effected, if the test now advocated be insisted upon, we insure that due value is given to the *weight*, as well as to the indication of the observation at each age.

II.

On the Valuation Reserve necessary for Diseased Lives; and for Female Lives subjected to an extra premium.

Having in view the fact that one class of lives may experience throughout a shorter expectation of life than another, while the reserve required for assurances on the former class may be not only not greater but even less than for assurances on the latter, it appears interesting to enquire whether it is necessary on the whole to make an increased reserve for lives classified as diseased and charged an increased rate of premium.

The Institute's Experience does not readily afford us the means of settling this point, as no pecuniary results have been deduced from its investigations as to diseased lives. In Mr. Meikle's *Observations on the Mortality of Assured Lives*, however, there is given (page 51) a table of annual premiums, at 3 per-cent interest, deduced separately from the total healthy lives and the total diseased lives embraced in the Scotch Offices' Experience. By means of the annuity values derived from these premiums, I have constructed the following table, to exhibit the effect on the reserve of the different laws of mortality prevailing in the two classes of lives. The relation chosen as the basis of comparison (distinguishing the functions relating to diseased lives by an accent) may be expressed thus ${}_nV'_x > {}_nV_x$ if $\frac{a'_x}{a_x} > \frac{a'_{x+1}}{a_{x+1}}$ for all values of x.

Or, in other words, the value of a policy according to the experience of diseased lives, is uniformly greater than the value of a policy according to the experience of healthy lives, if the ratio of the annuity-due by the former experience to the annuity-due by the latter at the same age, continually diminishes as the age increases. On referring to the table it will be found that this is actually the case, with the exception of a slight retrogression, probably arising from accidental causes, at ages 60 and 65.

Age x.	a'_x	a_x	$\frac{a'_x}{a_x}$
20	22·595	23·005	·9822
25	21·225	22·005	·9646
30	20·100	20·905	·9615
35	18·917	19·680	·9612
40	17·460	18·313	·9534
45	15·895	16·788	·9468
50	14·198	15·108	·9398
55	12·479	13·292	·9388
60	10·745	11·406	·9420
65	8·980	9·532	·9421
70	7·194	7·757	·9274
75	5·424	6·157	·8810
80	3·726	4·752	·7841
85	2·128	3·701	·5750
90	0		

Since, then, it is seen to be necessary to make a higher reserve for diseased lives taken as a whole (although perhaps it might not prove to be required for all classes of diseases were they to be investigated separately), the question suggests itself—What is the best way in which this may be done? It has hitherto been a common practice to value all assurances accepted at an extra premium as if they were effected at a higher age, corresponding to the premium charged. Unfortunately we cannot at present determine from existing data, without a good deal of preliminary calculation, whether this is a sufficiently near approximation to the truth—at least as regards diseased lives.

The same practice is, I believe, adopted by a good many companies in valuing policies on female lives, which up to about age 50 are commonly charged an extra premium. The following table, however, constructed in the same manner as above, shows that such a mode of valuation unnecessarily swells an office's reserves:—

Table of the Ratio $a_x^{(H^F)} \div a_x^{(H^M)}$, *at 3 per-cent.*

Age (x).	Ratio.	Age (x).	Ratio.
20	·9682	60	1·0573
25	·9678	65	1·0534
30	·9843	70	1·0528
35	·9999	75	1·0660
40	1·0148	80	1·1252
45	1·0323	85	1·0627
50	1·0463	90	1·2926
55	1·0548	95	2·0161

We learn that the true reserve for a policy on a healthy female life, instead of being greater, is generally less than the reserve for a policy on a healthy male life of the same age. We may therefore infer that in a valuation by the H^M table, we are at least perfectly safe in neglecting all female extras, and valuing the lives at their real ages. A partial set-off against any excess in the reserve thus made will be found in the heavier liability for current risk up to age 47. But after that age the probability of dying is not greater, but less than that of the H^M table; and one cannot help thinking that, if female lives are not to be separated altogether from male lives and valued by themselves, it would be more correct to use the H^{MF} table, and thus give an assurance company at its valuation the benefit of the superior vitality of those females on its books who have survived the child-bearing period.

MR. DAVID J. A. SAMOT, Actuary of the National Life Insurance Company of Rotterdam, sends us the following:—

I. Formulas for the Values of Endowment Policies that have been in force a given number of years.

(a) When the premiums are not returnable:—

The value of an endowment policy, without return of premium, after it has been *one* year in force, is, when the premium is paid yearly—

$$\frac{l_x \cdot P_{\overset{1}{x\overline{n}|}} \cdot (1+i)}{l_{x+1}},$$

or, dividing both numerator and denominator by $(1+i)^{x+1}$,

$$= \frac{D_x \cdot P_{\overset{1}{x\overline{n}|}}}{D_{x+1}}.$$

The value after *two* years is: $\dfrac{D_x.P_{\overline{x n}|}^{\;1}+D_{x+1}.P_{\overline{x n}|}^{\;1}}{D_{x+2}}$

$$=\frac{P_{\overline{x n}|}^{\;1}(D_x+D_{x+1})}{D_{x+2}}.$$

For the value after t years, we thus have by analogy:

$$\frac{P_{\overline{x n}|}^{\;1}(D_x+D_{x+1}+D_{x+2}+\ldots\ldots+D_{x+t-1})}{D_{x+t}}$$

$$=\frac{N_{x-1}-N_{x+t-1}}{D_{x+t}}.P_{\overline{x n}|}^{\;1}.$$

But $$P_{\overline{x t}|}^{\;1}=\frac{\dfrac{D_{x+t}}{D_x}}{1+|_{t-1}a_x}=\frac{\dfrac{D_{x+t}}{D_x}}{\dfrac{N_{x-1}-N_{x+t-1}}{D_x}}=\frac{D_{x+t}}{N_{x-1}-N_{x+t-1}};$$

hence $$\frac{N_{x-1}-N_{x+t-1}}{D_{x+t}}.P_{\overline{x n}|}^{\;1}=\frac{1}{P_{\overline{x t}|}^{\;1}}.P_{\overline{x n}|}^{\;1}=\frac{P_{\overline{x n}|}^{\;1}}{P_{\overline{x t}|}^{\;1}}.$$

The value of an endowment policy on x, payable in n years, after it has been t years in force, is thus equal to the annual premium for that endowment, divided by the annual premium for an endowment of 1 on x, payable in t years.

(b) When the premiums are returnable:—

Let P denote the premium that is to be returned for every paid annual premium ϖ, if x might die before attaining the age $x+n$, then the value of the policy after *one* year will be:

$$\frac{l_x.\varpi(1+i)-d_x.P}{l_{x+1}};$$

or, dividing both numerator and denominator by $(1+i)^{x+1}$,

$$=\frac{D_x.\varpi-C_x.P}{D_{x+1}}.$$

The value after *two* years will be:

$$\frac{\{l_x.\varpi.(1+i)-d_x.P+l_{x+1}.\varpi\}(1+i)-2d_{x+1}.P}{l_{x+2}};$$

or, dividing both numerator and denominator by $(1+i)^{x+2}$,

$$=\frac{D_x.\varpi-C_x.P+D_{x+1}.\varpi-2C_{x+1}.P}{D_{x+2}}$$

$$=\frac{(D_x+D_{x+1})\varpi-(C_x+2C_{x+1})P}{D_{x+2}}.$$

Hence the value after t years will be:

$$\frac{(D_x+D_{x+1}+D_{x+2}+\ldots\ldots+D_{x+t-1})\varpi-(C_x+2C_{x+1}+3C_{x+2}+\ldots\ldots+tC_{x+t-1})P}{D_{x+t}}$$

or, representing the series $C_x+2C_{x+1}+\ldots\ldots+tC_{x+t-1}$ by the symbol $|_t R_x$,

$$=\frac{(N_{x-1}-N_{x+t-1})\varpi-|_t R_x . P}{D_{x+t}}.$$

As the expression $N_{x-1}-N_{x+t-1}$, as well as $|_t R_x$, have been used respectively for calculating the premiums P and ϖ, the values of policies, according to this formula, may be speedily computed, principally by making use of a table of Gauss's logarithms.

The following is an example of the calculation:—

To find the value of an endowment policy of 1, on a person of the age 11, payable in 20 years, after it has been 15 years in force; the annual premium paid being ·03156.., and the premium to be returned ·02998.., according to the H^M Table, 4 per-cent. For this case, the value required

$$=\frac{(N_{11-1}-N_{11+15-1})\times{\cdot}03156-|_t R_x\times{\cdot}02998}{D_{11+15}}$$

$$=\frac{(N_{10}-N_{25})\times{\cdot}03156-|_t R_x\times{\cdot}02998}{D_{26}}$$

$\log(N_{10}-N_{25})$	$=5{\cdot}8629131$	$\log |_t R_x$	$=4{\cdot}4393872$
„ ·03156	$=8{\cdot}4991370$	„ ·02998	$=8{\cdot}4768943$
	$4{\cdot}3620501$		$2{\cdot}9162815$

If we put $4{\cdot}3620501=\log a$
and $2{\cdot}9162815=\log b$, we find
$B=(\log a-\log b)=1{\cdot}4457686$
$A=1{\cdot}4299228$
$\log b=2{\cdot}9162815$
+
$4{\cdot}3462043=\log(a-b)$

According to the seven-figure table of Gauss's *Logarithms*, published by Prof. Wittstein.

Thus we have for the logarithm of the value required $\log(a-b)-\log D_{26}$

$$=4{\cdot}3462043-4{\cdot}5230129=9{\cdot}8231914-10;$$

or, for the value itself, ·66557.

If $P=\varpi'_{\overline{x n|}}\,{}^{1}$, or the premium is to be *wholly* returned, our formula becomes:

$$\frac{(N_{x-1}-N_{x+t-1})\varpi'^{\,1}_{\overline{xn}|}-|_t R_x . \varpi'^{\,1}_{\overline{xn}|}}{D_{x+t}}$$

$$=\frac{[(N_{x-1}-N_{x+t-1})-|_t R_x]\varpi'^{\,1}_{\overline{xn}|}}{D_{x+t}}=\frac{(N_{x-1}-N_{x+t-1}-|_t R_x)\varpi'^{\,1}_{\overline{xn}|}}{D_{x+t}}.$$

But $\dfrac{D_{x+t}}{N_{x-1}-N_{x+t-1}-|_t R_x}=\varpi'^{\,1}_{\overline{xt}|}$, and thus:

$$\frac{N_{x-1}-N_{x+t-1}-|_t R_x}{D_{x+t}}\times\varpi'^{\,1}_{\overline{xn}|}=\frac{1}{\varpi'^{\,1}_{\overline{xt}|}}\times\varpi'^{\,1}_{\overline{xn}|}=\frac{\varpi'^{\,1}_{\overline{xn}|}}{\varpi'^{\,1}_{\overline{xt}|}}.$$

We find thus for the value of a policy, whereon the whole premium paid is to be returned, a formula analogous with that for a policy without return of premium.

When $\varpi^{\,1}_{\overline{xn}|}=P^{\,1}_{\overline{xn}|}+p$, we find

$$\frac{(N_{x-1}-N_{x+t-1})\varpi^{\,1}_{\overline{xn}|}-|_t R_x . P^{\,1}_{\overline{xn}|}}{D_{x+t}}$$

$$=\frac{\varpi^{\,1}_{\overline{xn}|}}{\varpi^{\,1}_{\overline{xt}|}}+p\times\frac{|_t R_x}{D_{x+t}}.$$

As p is always small, compared with $\varpi^{\,1}_{\overline{xn}|}$ and $\varpi^{\,1}_{\overline{xt}|}$, we have *approximately:*

$$\frac{(N_{x-1}-N_{x+t-1})\varpi^{\,1}_{\overline{xn}|}-|_t R_x . P^{\,1}_{\overline{xn}|}}{D_{x+t}}=\frac{\varpi^{\,1}_{\overline{xn}|}}{\varpi^{\,1}_{\overline{xt}|}}.$$

II. Method of interpolating the Values of Premiums for Endowments and Endowment Assurances, when these are given only for certain intervals of age.

The most common practical mode of interpolating premiums, which are not given in tables, that only contain the premiums at intervals, say of five years, consists in considering those premiums as increasing or decreasing between the calculated premiums with equal first differences. According to that mode, the interpolation for finding the annual premium for an endowment assurance of 100, from the table in Hardy's book, p. 113, for $x=55$ and $x+n=64$, would be:

Given premium for	55-60 =	18·45
„ „	55-65 =	9·10
	5)	9·35
		1·87
		9·10
		+
		10·97 = premium required.

But as the true premium for this case is only 10·10, the interpolated premium is by ·87 too great.

Looking at the large errors which this mode of interpolation gives in some calculations, I have devised a new one, which produces remarkably accurate results, and which may in practice be used, when, in tables of annual premiums for endowments and endowment assurances, and other such-like tables, the numbers are only given at certain intervals of age.

A few examples will, no doubt, suffice to give an insight into the process, and illustrate at the same time the difference between the old and new methods of interpolation.

For the above-given case, the new method of calculation is:

$$n=9$$

$$\begin{array}{rl}
18{\cdot}45\times\ 5=92{\cdot}25 & \\
u\times\ 6= & 92{\cdot}00(=92{\cdot}25-{\cdot}25)\\
z\times\ 7= & 91{\cdot}75(=92{\cdot}00-{\cdot}25)\\
y\times\ 8= & 91{\cdot}50(=91{\cdot}75-{\cdot}25)\\
x\times\ 9= & 91{\cdot}25(=91{\cdot}50-{\cdot}25)\\
9{\cdot}10\times 10=91{\cdot}00 & \\
\hline
5)\ 1{\cdot}25 & \\
{\cdot}25 & \dfrac{91{\cdot}25}{9}=\underline{10{\cdot}14=\text{premium required.}}
\end{array}$$

The true premium being 10·10, the difference is $=+{\cdot}04$.

For $x=55$, $x+n=61$, thus $n=6$, we find, from the same calculation, by the old method 16·58, and by the new one $\dfrac{92{\cdot}00}{6}=15{\cdot}33$.

As the true premium is 15·28, the errors are respectively:

$$+1{\cdot}30 \text{ and } +{\cdot}05.$$

For $x=42$, and $x+n=47$, we have:

Old Method.	New Method.
42-45 = 30·37 } Hardy's book, p. 112.	30·87 × 3 = 92·61
42-50 = 10·77 }	10·77 × 8 = 86·16
5)20·10	6·45 × 3 = 19·35 ÷ 5
4·02 × 3 = 12·06	3·87
10·77 +	86·16 +
22·83	5)90·03
	18·01

The true premium being 17·96, the errors are

22·83	18·01
17·96	17·96
+ 4·87	+ ·05

For $x=30$, $x+n=52$; and $x=25$, $x+n=48$; we get in the same way the errors:

	Old Method.	New Method.
	3·27	3·22
	3·21	3·21
	+ ·06	+ ·01
and	3·01	2·97
	2·96	2·96
	+ ·05	+ ·01

As this new mode of interpolation thus gives only a very little error on the side of safety, we may, I believe, conclude that it can advantageously be employed whenever we cannot easily, or quickly, compute the real premium.

DAVID J. A. SAMOT.

Rotterdam, 21 *April* 1877.

Life Insurance and Suicide. By J. W. EASTWOOD, M.D.Edin., M.R.C.P.Lond., *Dinsdale Park, Darlington.*

[Abridged from the *British Medical Journal*, 17th February 1877.]

IN looking over the rules of a few insurance societies, I could not avoid observing the want of unanimity which prevailed with respect to the manner in which they deal with suicides whose lives have been insured. This induced me to take up the subject, and obtain all the information I could from the different offices in England and Scotland. My information is gained from eighty-one out of ninety-two offices, and from the actuaries or secretaries of several of these I have received valuable assistance and useful facts. The ways are so various in which the different companies treat this subject, that it is not easy to classify them briefly. The general rule is, that a life policy is forfeited by suicide, whether the assured has been of unsound mind or not; and the following rule occurs

so frequently, that it may be quoted to represent a very large proportion, which I cannot accurately ascertain, of all the insurance companies.

" Assurances in this company made by persons on their own lives who shall die by their own act, whether sane or insane, or by duelling, or by the hands of justice, shall become void so far as respects such persons, but shall remain in force so far as any other person or persons shall then have acquired a *bonâ fide* interest therein by assignment or by legal or equitable lien; the extent of such interest to be proved to the satisfaction of the directors."

Some offices act literally upon this rule, whilst others, retaining the rule, modify it according to individual cases, so that their practice is more liberal than the rule itself warrants. Some of the companies return a portion or the whole of the premiums paid, others pay part, or the whole of the sum assured, the directors judging according to circumstances; some refuse to pay anything, if the suicide be shown to be sane; others, again, make no difference between the sane and the insane. Occasionally, the directors weigh the evidence as to *felo de se* and judge accordingly. A few are more liberal and pay in full, as though the assured had died a natural death; and a considerable proportion pay the policy, if a certain time have elapsed after the insurance has been effected. The following table shows the numbers, as far as I can obtain them accurately.

5	always pay without any conditions, and these are the Mutual, New York, Northern, Positive, and Standard.
2	pay after 6 months' insurance.
8	" 12 or 13 months' insurance.
4	" 3 years' insurance.
1	" 4 years' "
12	" 5 years' "
1	" 7 years' "
48	pay according to the discretion of the directors.
81	Total.

In the last class, the payments are more or less liberal, and the directors do that which seems right to them, and act according to circumstances. There is much uncertainty in this practice, for there appears to be no distinct principle involved which is allowed to guide them in their mode of dealing with the insured. In those companies in which the policy becomes a positive one after the lapse of a certain time, if the assured commit suicide before that time the directors are willing, in some instances, to take the case into consideration and to return part or the whole of the

premiums, or make some allowance. Very few of them, however, look upon the matter as a question of disease, or consider that there is great difficulty in ascertaining that the person who has committed suicide is really of sound mind. Coroners' inquests are not to be relied upon for such a purpose, as it is now comparatively rare to return the verdict as one of *felo de se*. The expression, "Died by their own hands", has been suggested to form part of the rule on this subject; but this term is objectionable, as it would include accidental deaths, including a recent case where an outward application was taken by mistake for medicine. There is, however, a decided tendency amongst insurance offices to regard the act of suicide as the result of disease. Without absolutely saying that, in every case it ought to be so regarded, it is very certain that there are few exceptions; for, even where the act is the result of a sudden reverse of fortune or domestic calamity, the shock to the nervous system is so great as to prevent the individual from recovering from the effects and thus bearing his misfortunes. Such cases as these are as much to be pitied as any other, and it is sad to see that assurance companies act on a principle which increases the distress of a family, instead of being the means of producing some measure of comfort. The result of my inquiries has fully satisfied me that insurance societies have made too great a bugbear of suicide, and have probably rather injured their business than otherwise by making such penal rules as are generally adopted, although their practice has usually been more lenient. From the prospectus of the Mutual, I quote the following passage.

"Suicides are abhorrent to the English mind, and in this country at least it may be safely held that suicide is always the result of a diseased mind. The new rule, freeing policies from the risk of being vitiated by suicide, came into force six years ago; the experience of the Society as to suicide over five periods of six years each is as follows.

1844-1849 ...	Percentage of suicide to claims	1·7
1850-1855 ...	" "	1·0
1856-1861 ...	" "	2·3
1862-1867 ...	" "	1·9
1868-1873 ...	" "	0·9

"These figures are somewhat remarkable, and although it would be absurd to suppose that the new rule prevented suicide, yet it is clear that it has not stimulated it as a practice. The experience of the Mutual thus shows that the risk of an Englishman taking away his own life, in order that his family may reap the benefit

of his assurance money, has been hitherto overestimated, and that it is, in fact, so small as to be inappreciable."

On the other side, one manager writes to me and says that he has "long had an impression that the usual condition of forfeiture may in some cases operate as a protection to the assured, presenting a consideration that may influence a very morbid mind to abstain from an act which it is otherwise disposed to commit." This opinion is not supported by any facts, but it is not at all unreasonable. Of the two companies which act upon the principle of only waiting six months before declaring the policy positive, or nearly so, the Scottish Provident Institution has some important observations in its report, which I shall quote. It says: "According to a strict view, suicide should be a cause of forfeiture only where it has been committed with the intention of defrauding the assurers. But the extreme difficulty of obtaining proof that such was the object has rendered it necessary to adopt a broader rule, which has varied in the regulations of different assurance societies. In some, there is a forfeiture, if the party assured die by his own hand, although the act would not amount to *felo de se;* while, in others, it was provided that he must be held *felo de se,* and, therefore, of sound mind. The former rule is adverse to every dictate of justice and sound feeling; for, if the suicide be insane, although his hand be the instrument, his malady is the true cause of death as much as if he died through cholera or fever."

There is another point on which there seems to have been a general agreement amongst the directors of insurance companies; that is, the necessity of taking some precaution that a person should not insure his life with the deliberate intention of relieving himself from the cares and troubles of this life, whilst, at the same time, his concern for those he leaves behind him is such as to impel him to insure his life. It is difficult to obtain instances sufficient to prove anything certain on this subject, as the only evidence at my disposal is given in the prospectus of the Mutual Society, quoted above, and these facts show that a more liberal arrangement does not increase the number of suicides amongst the assured. As there are only five offices which make their policy certain without any conditions, it is impossible to give many facts; but it appears to me that the occurrence of such a businesslike and deliberate design of suicide must be very unusual, and would imply an amount of "method in madness" which is extremely rare, or which does not exist at all. In the opinion of the directors of the Caledonian, which is the second company that requires only a delay of six

months before accepting the policy as a positive one, "the probability is, that the act of self-destruction, would, in such circumstances, follow closely the taking out of the policy, and the knowledge that no payment would be made, if death occurred in that way within six months, would naturally deter a person having an intention of the kind from assuring with this company."

There is still another subject on which directors have been generally unanimous, according to the reports which I have received. They appear to have been more influenced by a commercial spirit in framing their rules than by a concern for the welfare of the family of the person assured. In more than one-half of the reports—and the proportion is really greater—it is stated that the policy shall remain in force, under any circumstances whatever, where any other person shall have acquired a *bonâ fide* interest in it by assignment or by legal or equitable lien.

I have not been able to ascertain the annual number of suicides and particulars respecting them. From one society, however, I have been favoured with such details, and the number of deaths of this class was twenty-seven during thirty-six years. In very few of the cases could real *felo de se* be proved; for, in some instances, the persons were found drowned, others were obviously insane, whilst in some the cause of death appears to have been accidental.

The following are Dr. Eastwood's principal practical conclusions:—

The idea of a person's assuring his life, and then deliberately committing suicide, is of such rare occurrence, as only to justify very moderate precautions.

The fact of a person having insured his life prevents him from committing suicide is a very questionable statement, and has not been proved by any known facts.

The present restrictions are severe and unreasonable towards those who are almost always of unsound mind, and ought to be done away with or materially modified.

In order to prevent a person from insuring his life and immediately committing suicide, my opinion is that, in every case, the full amount of the policy should be paid after thirteen months; that is, when two annual premiums have been paid. This would be a sufficient protection to the offices, and, if death took place, it would not appear so great a loss to the family, to whom the premiums might be returned. After thirteen months, the policy

should be a positive one, and the question of soundness or unsoundness of mind should not be raised at all.

In the preceding paper, Dr. Eastwood has touched upon one point in connection with his subject, which has not generally received the attention it deserves. He points out that the expression "died by their own hands" is objectionable, as it would include accidental deaths; and he no doubt intends to imply that a man cannot be said to commit suicide unless he intentionally takes away his own life. This is, we believe, a sound distinction. It was brought into some prominence in the Belfast trial in 1868—*Thomas Sinclair's Executor* v. *The Scottish Widows' Fund.* The company there alleged that the deceased committed suicide, the facts being that he insured his life on 19 September 1866, and died on 2 January 1867, in consequence of a fall from his bedroom window, he being then in an unsound state of mind. Mr. Justice Morris in his charge to the jury stated that, if the deceased committed suicide, the office would not be liable. He added however that, if he were not bound by precedent, he would consider the more logical and proper construction to give to that, would be that the office would not be liable if "the assured took away his "life while he was a responsible human being. But it has been "decided otherwise, and we must take the law as we find it. It is "now settled law that, if he took away his own life intentionally, "whether he was at the time of sound mind or not, whether he was "responsible or irresponsible, the company are not bound to pay." This being the law, he proceeded to point out that it was necessary to consider what was the intention of the deceased. It is "one of "the most difficult things in the world to dive into the minds of "sane men"; it is consequently held in many cases that persons of sound mind must be judged by their acts and not by their intentions—in short, that a man of sound mind must be held to intend the natural consequences of his acts. The difficulty of arriving at the intention of a madman must, of course, be much greater. However difficult the task, it was the duty of the jury to undertake it, and to ascertain whether Mr. Sinclair deliberately intended to destroy his own life. He added, "The question now to consider "in this case is this—Is the mode or means by which Mr. Sinclair "met his death, capable of any explanation except that he intended "to take away his life?" In the event, the jury decided that Mr. Sinclair did not intend to kill himself, but had some other

object in view when he got out of his bedroom window, such as escaping from the restraint in which he was kept, and that, consequently, he did not commit suicide. The company accordingly had to pay the claim.—Ed. *J. I. A.*

Report of the Committee of the British Association (consisting of W. Spottiswoode, F.R.S., Professor Stokes, F.R.S., Professor Cayley, F.R.S., Professor Clifford, F.R.S., and J. W. L. Glaisher, F.R.S.), *appointed to report on Mathematical Notation and Printing, with the view of leading Mathematicians to prefer in optional cases such forms as are more easily put into type, and of promoting Uniformity of Notation.*

WITH a view to the questions referred to them for consideration, your Committee have made inquiries into the nature and processes of mathematical printing, and the difficulties attendant thereon; and it appears to them that a statement of the results of these inquiries, will form the best introduction to the suggestions which they have to make.

The process of "composition" of ordinary matter, consists in arranging types uniform in height and depth (or "body" as it is termed) in simple straight lines. The complications peculiar to mathematical matter are mainly of two kinds.

First, figures or letters, generally of a smaller size than those to which they are appended, have to be set as indices or suffixes; and consequently, except when the expressions are of such frequent occurrence as to make it worth while to have them cast upon type of the various bodies with which they are used, it becomes necessary to fit these smaller types in their proper positions by special methods. This process, which is called "justification", consists either in filling up the difference between the bodies of the larger and smaller types, with suitable pieces of metal, if such exist; or in cutting away a portion of the larger, so as to admit the insertion of the smaller types.

The second difficulty arises from the use of lines or "rules", which occur between the numerator and denominator of fractions, and (in one mode of writing) over expressions contained under radical signs. In whatever part of a line such a rule is used, it is necessary to fill up, or compensate, the thickness of it throughout the entire line. When no letters or mathematical signs occur on a line with the rule, the process is comparatively simple; but when,

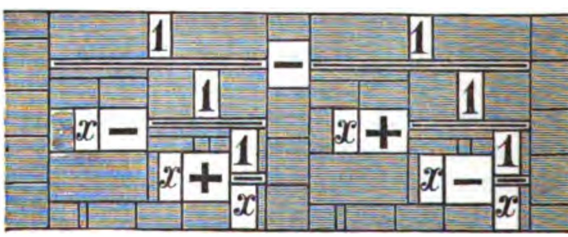
for example, a comma or sign of equality follows a fraction, or a + or — is prefixed to it, the middle of these types must be made to range with the rule itself, and the thickness of the rule must be divided, and half of it placed above and half below the type.

The complications above described may arise in combination, or may be repeated more than once in a single expression; and in proportion as the pieces to be "justified" become smaller and more numerous, so do the difficulties of the workman, the time occupied on the work, and the chances of subsequent dislocation of parts augment.

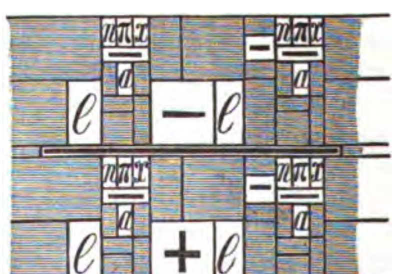
The cost of "composing" mathematical matter may in general be estimated at three times that of ordinary or plain matter.

With a view of illustrating these remarks, we have taken as an example an expression of not unfrequent occurrence in mathematics, but of considerable difficulty to the printer; and have marked out in compartments the different types of which it has to be composed. The shaded parts represent the "justification" spoken of.

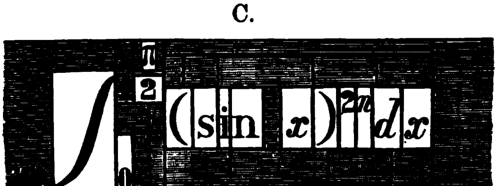
A.



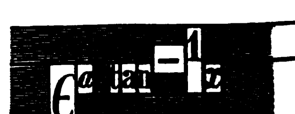
B.



C.



D.



There are many expressions occurring in mathematics which are capable of being written in more than one way; and of these some present much greater difficulties to the printer than others. This being so, your committee are of opinion that, instead of making any specific recommendations, the most useful course they can take will be to append a table of equivalent forms, specifying those which do and those which do not involve justification, and also a list of mathematical signs which may fairly be expected to be found, in the usual sizes, ready to hand among a printer's materials. It will, of course, be understood that neither one nor other of these is even nearly exhaustive. But the specimens here given form the principal elements from which others are formed; and from the explanations given in the earlier part of the report the intelligent reader will be able to discriminate in most cases between forms difficult and forms easy to be printed.

In recommending in this qualified way some forms of notation in preference to others, your committee wish it to be distinctly understood that they are speaking from the printing, and not from the scientific point of view; and they are quite aware that, even if some of the easier forms should be adopted in some cases, they may still not be of universal application, and that there may be passages, memoirs, or even whole treatises, in which they would be inadmissible.

Your committee are unwilling to close this report without alluding to the advantages which may incidentally accrue to mathematical science, by even a partial adoption of the modifications here suggested. Anything which tends towards uniformity in notation, may be said to tend towards a common language in mathematics; and whatever contributes to cheapening the production of mathematical books, must ultimately assist in disseminating a knowledge of the science of which they treat.

MATHEMATICAL SIGNS NOT INVOLVING "JUSTIFICATION".

$\times \quad - \quad + \quad = \quad \surd \quad \pm \quad :: \quad \therefore \quad \because \quad : \quad \} \quad < \quad > \quad \div$

$\Big(\quad \Big[\quad \Big\} \quad \int \quad \sqrt{}$

$a \quad a^1 \quad a_1 \quad a^2 \quad a_2$ &c., $a^{\frac{1}{2}} \quad a_{\frac{1}{2}}$

EQUIVALENT FORMS.

Involving justification.	Not involving justification.
$\frac{x}{a}$	$x \div a$ or $x : a$
$\sqrt{x}$	$\surd x$ or $x^{\frac{1}{2}}$
$\sqrt[3]{x}$	$\sqrt[3]{}x$ or $x^{\frac{1}{3}}$
$\sqrt{x-y}$	$\surd(x-y)$ or $(x-y)^{\frac{1}{2}}$
$\sqrt{-1}$	i
$x.\overline{x+a}$	$x(x+a)$
$\epsilon^{\frac{n\pi x}{a}}$	$\exp\,(n\pi x)(a)$
$\tan^{-1}x$	arc tan x
$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$	$x:y:z=l:m:n$

HOME AND FOREIGN INTELLIGENCE.

CAPE OF GOOD HOPE MUTUAL LIFE ASSURANCE SOCIETY.

Special Investigation Report by Mr. T. B. Sprague to the Directors.

I have made a careful investigation into the affairs of the Society, and have the satisfaction of reporting that, as far as I have the means of judging, I have found it not only in a thoroughly sound, but in a highly prosperous condition. Three of the most important points to be considered in judging of the position of a life insurance company are (1) the rate of expenditure at which it is managed, (2) the rate of interest realized upon the investments, and (3) the rate of mortality prevailing among the lives insured. On all these points I find the experience of the Society to be very satisfactory.

1. I find from the printed accounts that the premiums and expenses for the last seven years have been as shown in the following table:—

Year ending 31st May.	Premiums, &c.	Total Expenses.	Percentage of Expenses on Premiums.
	£	£	
1868	35,257	3,004	8·52
1869	36,739	3,337	9·08
1870	37,854	3,410	9·01
1871	39,260	3,209	8·17
1872	41,390	3,848	9·30
1873	44,945	4,078	9·07
1874	48,272	3,984	8·25
Total ...	283,717	24,870	8·77

I consider the rate of expenditure here shown to be extremely moderate.

2. Next as regards the interest on the investments. The amount of the Society's funds and the interest received in the last seven years is shown in the following table:—

Year ending 31st May.	Amount of Funds in the middle of the Year.	Interest.	Deductions for Bad Debts and Depreciation.	Net Interest.	Percentage of Net Interest to Funds.
	£	£	£	£	
1868	260,630	13,662	418	13,244	5·08
1869	284,980	14,745	798	13,947	4·86
1870	303,149	15,774	4,221	11,553	3·81
1871	324,313	17,363	2,301	15,062	4·64
1872	349,785	19,254	1,484	17,770	5·08
1873	377,135	19,513	533	18,980	5·03
1874	405,379	19,815	1,597	18,218	4·49
Total ...	2,305,371	120,126	11,352*	108,774	4·71

The general result is that, after allowing for the losses on the investments, the Society has during the last seven years realized an average rate of interest of about £4. 14*s*. per-cent per annum upon the whole of its funds. This is a substantially higher rate than most of the life insurance companies in the United Kingdom are able to obtain. I notice that considerable losses have occurred from time to time from bad debts.* I assume, however, that any losses of this kind that may occur in the future can be met without inconvenience by an appropriation of a portion of the interest, and that there is no reason to apprehend a loss of such a magnitude as to imperil the stability of the Society.

3. As regards the mortality, I have not the means of making a strict investigation into the rate of mortality that has prevailed among the members of the Society since it was established. I have, however, made such comparisons as the materials furnished to me allowed of. I find that the number of members on 31st May 1874, was 2,855, and that, assuming the rate of mortality among these to be the same as is found to prevail in this country, thirty-eight or thirty-nine members might be expected to die in the course of a year. The number of deaths in the last office year of the Society, 1873–4, was forty-four, being considerably in excess of the expectation; but in the previous year the number was only thirty;—the mean of these being thirty-seven. I therefore conclude that the mortality among the members is very much the same as is found to prevail in this

* Note by the Secretary.—The losses on Investments have amounted to £6,182, of which £1,375 arose out of a mortgage bond on property at Prince Albert, which was found to be a forgery. The balance (£5,229) is made up of amounts written off the Society's premises in Darling Street. The losses on Investments since the creation of the Society in 1845 to the year 1867 inclusive, amounted to £2,135, including £1,572 lost on the sale of the Society's premises in St. George's Street.

country. This conclusion is strengthened by a comparison I have made between the amounts of the claims paid and the premiums received in the Society and in a few British Life Insurance Companies which were established about the same date. The result of this comparison is seen in the following table:—

Established	Name of Company.		Premiums received.	Claims paid.	Percentage of Claims to Premiums.
			£	£	
1838	City of Glasgow .	5 yrs. ending 20 Jan. 1874	557,885	353,790	63·42
1840	Reliance . . .	5 „ 31 Dec. 1872	304,581	149,726	49·16
1843	Star.	5 „ 31 Dec. 1873	729,045	436,387	59·85
1843	Scottish National	4 „ 15 May 1872	259,898	160,041	61·58
1844	Equity and Law.	5 „ 31 Dec. 1874	524,967	293,076	55·81
1844	Great Britain .	5 „ 31 Dec. 1873	302,860	186,011	61·41
1845	Royal	5 „ 31 Dec. 1869	974,564	437,245	44·85
1845	Sovereign . . .	6 „ 31 Dec. 1873	447,764	279,735	62·47
1846	London and Provincial Law .	5 „ 31 Dec. 1870	349,906	155,102	44·33
1847	British Empire .	3 „ 31 Dec. 1872	250,497	147,367	58·83
Total .	10 Companies .		4,701,967	2,598,480	55·26
1845	Cape Mutual . .	7 yrs. ending 31 May 1874	283,718	178,976	63·09

4. At first sight these results appear to indicate that the mortality among the members of the Society must have been greater than that among the lives insured in most of the British offices founded about the same time; but I do not think this has been the case. The claims paid, as given in the above table, include not only the original sums assured, but also the bonus additions to the policies; and these bonus additions have been on the average very much larger in the Society than in the British offices. I believe therefore that if I had the means of determining in each case the amount of the original sums assured and that of the bonus additions, it would be found that the claims in the Society have not on the average at all exceeded those of the other offices. I have ascertained the amount of the bonus additions in two cases—the *Royal* and the *Equity and Law*—and give the figures, together with those of your own Society, in the subjoined table:—

Office.	Sums Assured.	Bonus Additions.	Percentage of	
			Bonus Additions to Sums Assured.	Sums Assured to Premiums.
	£	£		
Royal	416,881	20,364	4·9	42·77
Equity and Law . .	251,655	41,421	16·6	47·94
Cape	133,950	45,026	33·6	47·22

5. The above results show that the Society has in it all the elements of success. These, however, by themselves, are not sufficient to secure the permanent prosperity of a life insurance company; for a company possessing them all may be brought into difficulties by declaring too large bonuses, and treating as divisible profit sums that should be reserved to meet the liability under its policies.

6. One of the questions submitted to me is, whether the bonus of $2\frac{1}{4}$ per-cent per annum on the sums assured and former bonuses, declared at the last division of profits, was justified by the position of the Society. The calculations leading to this rate of bonus are based on the Northampton Mortality Table, with interest at 5 per-cent, being the table and the rate of interest which are prescribed in Article 38 of the Deed of Agreement of the Society, and which have always hitherto been used by it for the purpose. In order to answer the above question, I have carefully checked Mr. Elliott's calculations, and am able to report that the bonus of $2\frac{1}{4}$ per-cent (on the sum assured and former bonuses) was the proper rate to declare according to the regulations of your Society;—that is to say, it is the rate which results from the use of the Northampton 5 per-cent table in the valuation, reserving one-third of the surplus.

7. Another question on which my opinion has been requested, and one which is of still greater importance, is whether I would recommend any change in the system which has been hitherto adopted of valuing the Assets and Liabilities of the Society; or in other words, whether the Northampton Table with interest at 5 per-cent is a suitable one to use in the valuations, or whether it would be desirable to substitute for it some other table. I find that the late Mr. Todd, in his Report dated 7th March 1871, after speaking of the defects of the Northampton Table, states that it has been discarded by the Scottish Widows' Fund and the Scottish Equitable Society, which were both originally based upon it and for many years made all their calculations by means of it, and clearly indicates his own opinion that the Northampton Table is not by itself a safe guide in the future divisions of profit. In this opinion I entirely concur; and after full consideration, I am prepared to go still further, and to recommend that the use of the Northampton Table be altogether abandoned henceforth. Mr. Todd recommended that the results of the Northampton Valuation should not be adopted and acted upon unreservedly, but that they should be tested, and if necessary modified, by comparison with the results of a valuation made by the Carlisle Mortality Table with $3\frac{1}{2}$ per-cent interest. Such a test valuation was accordingly made by Mr. Elliott, and this also I have carefully examined. Comparing the results with those of the valuation made by Mr. Todd on the same basis as at 31st May 1868, and bearing in mind that the results obtained by the use of the Carlisle Table are undoubtedly much more trustworthy than those obtained by the use of the Northampton Table, I find that the continued use of the Northampton Table is now leading the Society in the direction of an anticipation of future profits; and I am of opinion that a continued adherence to the use of that table would be attended with danger to the future prosperity and stability of the Society—in other words, that the Northampton Table makes an insufficient reserve for the future

liabilities, and thus swells the apparent surplus, and exhibits as divisible surplus a sum which it is more correct to reserve to meet the liability under the policies.

8. For the purpose of illustrating this tendency of the Northampton Table I have prepared the statements appended to this Report. They show the progress under various suppositions of a fund formed by the yearly payments of 863 members, who are supposed to insure their lives for £100 each, being all of the same age, namely 35 next birthday at the outset, and all paying the same premium of £2. 17*s*. 6*d*. It is assumed that the mortality among these members is that found to prevail among insured lives in Great Britain, as shown by the Institute H^M Table, that the fund formed by the premiums of the members is accumulated at the constant rate of interest of 5 per-cent per annum, and that the expenses of management amount throughout to 10 per-cent of the premiums,—these being suppositions which correspond fairly well with the experience of the Society during recent years. On these suppositions the premium of £2. 17*s*. 6*d*. is considerably more than sufficient to provide for payment of the sums assured; and if we suppose that, as each member dies, his representatives receive simply the £100, there will be at the end of sixty years, when all the members have died, a surplus of £210,582. The progress of the fund under this supposition is shown in Table 1. If, instead of allowing the surplus to accumulate, the liability in respect of the insurances remaining in force is estimated every three years, and the surplus applied to declare a bonus among the members, the progress of the fund will differ materially according to the basis on which the liability is estimated. I have calculated that liability on two different basises: (1) according to the principles hitherto in use in the Society, namely, by the Northampton 5 per-cent Table, dividing two-thirds of the apparent surplus and reserving the other third; and (2) according to the method which I recommend for future adoption, namely, by the Institute Mortality Table, with interest at 4 per-cent; and the progress of the fund in these two cases is shown in Table 2.

9. Table 3 shows the estimated liability at each division of profits as found by the Northampton Table, the surplus divided, the total bonus added to the policies, and the bonus on each policy, also the rate of bonus and the rate on the sum assured and former bonuses. Table 4 gives the same particulars as regards the Institute Table.

10. It will be seen from a comparison of Tables 3 and 4 that the estimated liability under the policies remaining in force is throughout greatly less according to the Northampton 5 per-cent Table than according to the Institute 4 per-cent. There can be no doubt, in fact, that, taken by itself, the reserve made by the Northampton 5 per-cent Table would be dangerously small; but, in order to make a fair comparison, we must add to the estimated liability according to the former table the one-third of the apparent surplus which is reserved at each division of profits. At the first division at the end of three years there are 839 members remaining alive out of the original 863, making the total assurances in force £83,900. The liability under these policies, according to the Northampton 5 per-cent Table, is £2,398, and according to the Institute 4 per-cent, £3,036. The amount of the funds being £4,841, the former estimate

leads to a surplus of £2,442, and the latter to one of £1,805; but one-third of the former surplus, £814, being reserved, makes the total reserve when the Northampton Table is employed £3,212, which is greater than the reserve made by the Institute 4 per-cent valuation; and consequently the surplus divided is smaller. We see then that in the very early years of a society the effect of using the Northampton Table, reserving so large a proportion as one-third of the apparent surplus, is to reduce the amount of divisible profit. At the expiration of the second term of three years the effect produced by the two tables is reversed. There are then 814 members remaining alive, and the estimated liability under the insurances of £81,400, increased by the bonuses, is £6,529 according to the Northampton Table, and £7,995 according to the Institute. The apparent surplus in the case of the former is £3,474, one-third of which is £1,158, and adding this to the estimated liability, we get a total reserve of £7,687, so that we see that the reserve according to the Northampton Table, even including the one-third of the apparent surplus, is now less than that according to the Institute Table. The surplus divided will therefore be greater in the former case, being £2,315 against £1,989. At the third and fourth divisions of profit the surplus brought out for divisions by the use of the Northampton Table will be £2,759 and £2,970, against £2,094 and £2,262. Comparing the figures in column 6 of Table 3, with those of column 4 of Table 4, we see that the surplus divided when the Northampton Table is used, continues greater than the other, until the eighth division of profit inclusive; and the effect of dividing these larger sums is to increase greatly the reversionary bonuses, the total additions to each £100 policy being £62. 16*s*. according to the Northampton Table, and £52. 16*s*. according to the Institute Table. But, as I have before mentioned, the effect of using the Northampton Table is to treat as divisible profit sums which ought to be reserved to meet the future liabilities; and if we look a little further down Table 3, we see the ultimate effect of this premature distribution of profits. From the eighth division of profits the surplus shown by the Northampton Valuation to be available for division becomes rapidly less, until, when we arrive at the fourteenth division of profits, when the fund has been forty-two years in existence, and the 207 surviving members have attained the age of 77, we find that the Northampton Valuation, instead of showing a surplus, exhibits a deficiency. On the contrary, the use of the Institute Table is to cause a steady increasing bonus throughout the whole of the existence of the fund.

11. It must not of course be supposed that the state of affairs exhibited in these tables will ever be found to actually prevail in practice. The exact kind of progress here shown could only be exhibited by a life insurance company, if no new members joined it; and when, as is usually the case, a succession of new members join the Society each year, the effect of the use of the Northampton Table will be that the bonus will increase for a certain time and then diminish to a point depending on the amount of the new business. This law of progression is clearly seen in the rates of bonus actually declared in your Society up to the present time, which have been as shown in the following table:—

Year.	Rate of Bonus.
1850	2¼ per-cent per annum.
1853	2¼ " "
1856	3 " "
1859	3 " "
1862	3¼ " "
1865	2⅝ " "
1868	2½ " "
1871	2¼ " "
1874	2¼ " "

12. On a careful review of the whole circumstances of the Society, I strongly advise the Society at once to abandon the use of the Northampton Table. With very few exceptions the many British insurance companies which were founded upon it have seen reason, one after another, to abandon the use of it, and to adopt some other table that more correctly represents the actual mortality prevailing among their members. I am satisfied that, sooner or later, it will be found necessary for the Society to make a similar change; and I believe it will be the wisest course to do so while it is in a sound and prosperous state, rather than wait until the evil effects of an adherence to an erroneous table become more apparent.

13. The table which I recommend for adoption in future is that known as the Institute of Actuaries H^M Table, and the rate of interest 4 per-cent. For the purpose of being able to advise with greater confidence upon this important point, I have made three valuations of the liabilities of the Society by the Institute Table, (1) with interest at 4½ per-cent, (2) with interest at 4 per-cent, valuing the ordinary net premiums, and (3) with interest at 4 per-cent, the net premium values being increased so as to correspond to an age at entry one year greater than the actual age. I am of opinion that 4½ per-cent is too high a rate to use in the valuations, as it is not prudent for a life insurance company, which enters into contracts the greater number of which may be expected to last for 30 or 40 years, to stake its prosperity on the assumption that it will permanently be able to realize that rate of interest upon the whole of its assets, after making allowance for all losses on investments, and for unemployed capital, such as bank balances and premiums in course of collection. Considering all the circumstances of the Society, I do not recommend the adoption of the third of the above-mentioned methods. It will therefore be only necessary to state the results of the second method. The following is the balance sheet resulting from the use of the Institute 4 per-cent table:—

Liabilities.

Present Value of £1,991,751. 7*s*. (Sums Assured and Additions)	£828,850
Proportion of Net Premiums for Risk not run . . .	14,177
Surplus	33,003
	£876,030

Assets.

Realized Funds	£416,648
Present Value of Net Premiums (£30,897·798 per annum)	459,382
	£876,030

14. The surplus here shown was sufficient to admit of a bonus at the rate of £1. 8*s.* 9*d.* per-cent per annum, instead of the bonus of £2. 5*s.*, which resulted from the use of the Northampton Table. This is consistent with what I have already stated, namely, that too large a bonus has been divided among the members in past years, and shows that the change which I recommend to a more prudent valuation basis can only be carried out by a temporary reduction of the rate of bonus below that which the members have been accustomed to. I am nevertheless of opinion that the change should not be delayed, and that, although it may cause some temporary disappointment to the members, this will soon pass away. When once the finances of the Society have been placed upon a thoroughly sound footing, the rate of bonus will, I believe, no longer exhibit the constant tendency to reduction which it has shown since the division in the year 1862. The respective interests of existing members, and of new entrants will be most equitably adjusted by this arrangement, and the Society may look forward to a long future of unchequered prosperity.

TABLE 1.—*Showing the Progress of a Fund formed by the Contributions of* 863 *Members of the same age* 35, *insured for* £100 *each, on the supposition that no profits are divided.*

Age.	Number of Members surviving.	Number dying in the year.	Premiums payable at the beginning of the year.	Claims.	Expenses.	Funds at the beginning of the year.	Funds at the end of the year.	Age at the end of the year.
				Payable during the year.				
			£	£	£	£	£	
35	863	8	2,481	800	248	2,481	1,557	36
36	855	8	2,458	800	246	4,015	3,170	37
37	847	8	2,432	800	244	5,605	4,841	38
38	839	8	2,412	800	241	7,254	6,575	39
39	831	8	2,389	800	239	8,964	8,374	40
40	823	9	2,366	900	237	10,740	10,140	41
41	814	8	2,340	800	234	12,480	12,070	42
42	806	9	2,317	900	232	14,387	13,975	43
43	797	9	2,291	900	229	16,266	15,950	44
44	788	9	2,265	900	227	18,216	18,000	45
45	779	9	2,240	900	224	20,239	20,127	46
46	770	10	2,214	1,000	221	22,341	22,237	47
47	760	11	2,185	1,000	219	24,422	24,324	48
48	749	10	2,153	1,000	215	26,477	26,586	49
49	739	12	2,125	1,200	212	28,711	28,734	50
50	727	11	2,090	1,100	209	30,825	31,057	51
51	716	12	2,058	1,200	206	33,115	33,365	52
52	704	13	2,024	1,300	202	35,389	35,656	53
53	691	12	1,987	1,200	199	37,643	38,126	54
54	679	14	1,952	1,400	195	40,078	40,487	55
55	665	14	1,912	1,400	191	42,399	42,928	56
56	651	14	1,172	1,400	187	44,800	45,453	57
57	637	16	1,831	1,600	183	47,284	47,865	58
58	621	16	1,785	1,600	179	49,651	50,354	59
59	605	16	1,739	1,600	174	52,094	52,924	60
60	589	18	1,693	1,800	169	54,618	55,380	61
61	571	18	1,642	1,800	164	57,021	57,908	62
62	553	19	1,590	1,900	159	59,498	60,414	63
63	534	20	1,535	2,000	154	61,949	62,893	64
64	514	21	1,478	2,100	148	64,370	65,341	65

TABLE 1.—*continued.*

Age.	Number of Members surviving.	Number dying in the year.	Premiums payable at the beginning of the year.	Claims. Payable during the year.	Expenses. Payable during the year.	Funds at the beginning of the year.	Funds at the end of the year.	Age at the end of the year.
			£	£	£	£	£	
65	493	21	1,417	2,100	142	66,758	67,854	66
66	472	22	1,357	2,200	136	69,211	70,336	67
67	450	23	1,294	2,300	129	71,630	72,782	68
68	427	23	1,228	2,300	123	74,010	75,287	69
69	404	23	1,161	2,300	116	76,449	77,855	70
70	381	23	1,095	2,300	110	78,951	80,488	71
71	358	25	1,029	2,500	103	81,517	82,990	72
72	333	25	957	2,500	96	83,948	85,549	73
73	308	25	885	2,500	89	86,434	88,167	74
74	283	26	814	2,600	81	88,981	90,749	75
75	257	25	739	2,500	74	91,488	93,488	76
76	232	25	667	2,500	67	94,155	96,296	77
77	207	24	595	2,400	60	96,891	99,275	78
78	183	22	525	2,200	53	99,801	102,538	79
79	161	22	463	2,200	46	103,000	105,905	80
80	139	20	400	2,000	40	106,304	109,579	81
81	119	19	342	1,900	34	109,921	113,484	82
82	100	17	287	1,700	29	113,771	117,731	83
83	83	15	239	1,500	24	117,969	122,344	84
84	68	14	195	1,400	20	122,539	127,246	85
85	54	11	155	1,100	16	127,401	232,655	86
86	43	10	124	1,000	12	132,779	138,406	87
87	33	7	95	700	9	138,501	144,717	88
88	26	6	75	600	7	144,792	151,424	89
89	20	5	57	500	6	151,482	158,550	90
90	15	4	43	400	4	158,593	166,119	91
91	11	4	32	400	3	166,150	174,055	92
92	7	2	20	200	2	274,075	182,577	93
93	5	2	14	200	1	182,591	191,520	94
94	3	2	9	200	1	191,529	200,904	95
95	1	1	3	100	...	200,906	210,852	...

TABLE 2.—*Showing the Progress of the Fund on the suppositions that the Surplus is ascertained every three years, (1) by the Northampton 5 per-cent Table, reserving one-third of the Surplus, and (2) by the Institute 4 per-cent Table, dividing the whole of the Surplus.*

Age.	Number surviving.	Sum Assured.	Northampton 5 per-cent.			Institute 4 per-cent.		
At beginning of year.			Bonus.	Claims (sums assrd. and bonuses).	Fund at end of year.	Bonus.	Claims (sums assrd. and bonuses).	Fund at end of year.
		£	£	£	£	£	£	£
35	863	86,300	...	800	1,557	...	800	1,557
36	855	85,500	...	800	3,170	...	800	3,170
37	847	84,700	...	800	4,841	...	800	4,841
38	839	83,900	4,339	842	6,533	4,979	847	6,528
39	831	83,100	4,297	842	8,287	4,932	847	8,277
40	823	82,300	4,255	947	10,002	4,885	954	9,984
41	814	81,400	10,062	898	11,827	9,958	898	11,806
42	806	80,600	9,964	1,011	13,608	9,855	1,010	13,589

TABLE 2.—*continued.*

Age.	Number surviving.	Sum Assured.	Northampton 5 per-cent.			Institute 4 per-cent.		
At beginning of year.			Bonus.	Claims (sums assrd. and bonuses).	Fund at end of year.	Bonus.	Claims (sums assrd. and bonuses).	Fund at end of year.
		£	£	£	£	£	£	£
43	797	79,700	9,853	1,011	15,454	9,715	1,010	15,435
44	788	78,800	16,370	1,087	17,292	14,658	1,067	17,292
45	779	77,900	16,183	1,087	19,198	14,491	1,067	19,218
46	770	77,000	15,996	1,208	21,054	14,324	1,186	21,097
47	760	76,000	22,556	1,427	22,755	19,193	1,378	22,850
48	749	74,900	22,229	1,297	24,641	18,915	1,253	24,785
49	739	73,900	21,932	1,556	26,335	18,662	1,504	26,540
50	727	72,700	27,780	1,520	28,117	23,079	1,449	28,403
51	716	71,600	27,360	1,658	29,821	22,730	1,580	30,199
52	704	70,400	26,902	1,797	31,438	22,350	1,712	31,920
53	691	69,100	32,732	1,769	33,128	26,633	1,662	33,741
54	679	67,900	32,163	2,064	34,575	26,171	1,939	35,344
55	665	66,500	31,499	2,063	36,057	25,632	1,939	36,989
56	651	65,100	36,322	2,181	37,457	29,698	2,038	38,579
57	637	63,700	35,541	2,493	38,576	29,060	2,329	39,918
58	621	62,100	34,648	2,493	39,707	28,331	2,330	41,279
59	605	60,500	38,533	2,619	40,725	31,950	2,445	42,550
60	589	58,900	37,514	2,947	41,423	31,105	2,750	43,536
61	571	57,100	36,367	2,947	42,107	30,155	2,750	44,523
62	553	55,300	39,113	3,243	42,480	33,415	3,048	45,212
63	534	53,400	37,770	3,414	42,648	32,267	3,208	45,723
64	514	51,400	36,356	3,585	42,599	31,059	3,368	46,045
65	493	49,300	37,498	3,698	42,377	33,657	3,534	46,159
66	472	47,200	35,900	3,874	41,911	32,223	3,703	46,053
67	450	45,000	34,226	4,050	41,186	30,720	3,871	45,714
68	427	42,700	34,003	4,133	40,279	32,657	4,059	45,107
69	404	40,400	32,200	4,133	39,264	30,898	4,059	44,407
70	381	38,100	30,367	4,133	38,134	29,139	4,059	43,608
71	358	35,800	29,444	4,555	36,463	30,531	4,633	42,133
72	333	33,300	27,389	4,555	34,640	28,398	4,633	40,516
73	308	30,800	25,334	4,555	32,657	26,265	4,633	38,750
74	283	28,300	23,534	4,763	30,301	26,759	5,060	36,401
75	257	25,700	21,371	4,580	27,938	24,299	4,865	34,058
76	232	23,200	19,291	4,580	25,388	21,934	4,865	31,529
77	207	20,700				21,674	4,913	28,757
78	183	18,300				19,161	4,503	26,191
79	161	16,100				16,858	4,504	23,437
80	139	13,900				16,111	4,318	20,671
81	119	11,900				13,793	4,102	17,928
82	100	10,000				11,591	3,670	15,427
83	83	8,300				10,692	3,432	12,993
84	68	6,800				8,760	3,203	10,624
85	54	5,400				6,957	2,517	8,785
86	43	4,300				6,193	2,440	6,902
87	33	3,300				4,753	1,708	5,630
88	26	2,600				3,745	1,464	4,518
89	20	2,000				3,225	1,306	3,492
90	15	1,500				2,419	1,045	2,663
91	11	1,100				1,774	1,045	1,782
92	7	700				1,281	566	1,324
93	5	500				915	566	838
94	3	300				549	566	322
95	1	100				243	343	...

The progress of this fund is not traced further, as the valuation made at this point shows a deficiency, the funds remaining being insufficient to provide for payment of the sums assured and bonuses.

TABLE 3.—*Profits calculated by the Northampton 5 per-cent Table.*

Age.	Division of Profits.	Estimated Liability.	One-third of the Surplus.	Total Reserve.	Surplus divided.	Reversionary Bonus added to Policies.	Reversionary Bonus added to each Policy.	Rate of Bonus per-cent per annum. On Original Sum Assured.	Rate of Bonus per-cent per annum. On Total Sum Assd. and former Bonuses.
		£	£	£	£	£	£		
38	1	2,398	814	3,212	1,629	4,239	5·200	1·73	1·73
41	2	6,529	1,158	7,687	2,315	5,854	7·191	2.40	2·28
44	3	11,315	1,380	12,695	2,759	6,628	8·412	2·80	2·50
47	4	16,599	1,485	18,084	2,970	6,768	8·904	2·97	2·46
50	5	22,022	1,438	23,460	2,875	6,204	8·535	2·84	1·19
53	6	26,805	1,544	28,349	3,089	6,327	9·156	3·05	2·21
56	7	31,823	1,411	33,234	2,823	5,486	8·427	2·81	1·91
59	8	35,815	1,298	37,113	2,594	4,778	7·900	2·63	1·69
62	9	38,761	1,115	39,876	2,231	3,893	7·038	2·35	1·43
65	10	40,211	796	41,007	1,592	2,627	5·328	1·78	1·04
68	11	39,689	499	40,288	998	1,557	3·645	1·21	0·69
71	12	37,208	309	37,517	617	910	2·541	0·85	0·47
74	13	32,384	91	32,475	182	255	0·900	0·30	0·16
77	14	25,496	Deficiency of 108						

TABLE 4.—*Profits calculated by the Institute 4 per-cent Table.*

Age.	Division of Profits.	Estimated Liability.	Surplus divided.	Reversionary Bonus added to Policies.	Reversionary Bonus added to each Policy.	Rate of Bonus per-cent per annum. On Original Sums Assured.	Rate of Bonus per-cent per annum. On Total Sums Assrd. and former Bonuses.
		£	£	£	£		
38	1	3,036	1,805	4,979	5·900	1·96	1·96
41	2	7,975	1,989	5,122	6·292	2·10	1·98
44	3	13,341	2,094	5,023	6·374	2·12	1·89
47	4	18,835	2,262	5,055	6·651	2·22	1·87
50	5	24,277	2,263	4,721	6·494	2·16	1·73
53	6	29,510	2,410	4,695	7·794	2·26	1·72
56	7	34,462	2,527	4,605	7·074	2·36	1·70
59	8	38,733	2,546	4,349	7·188	2·40	1·65
62	9	41,903	2,620	4,210	7·613	2·54	1·66
65	10	43,498	2,547	3,866	7·842	2·61	1·63
68	11	43,276	2,438	3,508	8·215	2·74	1·63
71	11	41,303	2,305	3,151	8·801	2·93	1·66
74	13	36,739	2,011	2,627	9·283	3·09	1·67
77	14	29,855	1,674	2,105	10·170	3·39	1·74
80	15	22,156	1,281	1,557	11·201	3·73	1·82
83	16	14,520	907	1,071	12·903	4·30	1·99
86	17	8,220	565	653	15·186	5·06	2·21
89	18	4,213	305	344	17·200	5·73	2·35
92	19	1,643	139	152	21·714	7·27	2·77
95	20	265	57	60	60·000	20·00	6·06

We are in a position to state that the changes in the method of valuation recommended in the preceding report, were adopted by the members of the Society, and that the valuation as at 31 May 1877 was made accordingly. The result was that a bonus was declared at the rate of 2 per-cent per annum on the sums assured and former bonuses, whereas according to the old method of valuation the rate of bonus would have been 2½ per-cent per annum. The following is the amended rule referring to the method of valuation.

> " That the liability of the society under its contracts depending " on life contingencies shall be calculated, according to " the generally recognized rules, by what is known as a " net premium valuation on the basis of the Mortality " Table commonly called the Institute of Actuaries' " Experience H^M Table, with interest at the rate of 4 per- " cent per annum, or at such lower rate as the directors " shall from time to time determine. Whenever the " amount of the property of the society shall exceed that " of the liability so calculated, the surplus shall be con- " sidered as profit."

THE MUTUAL ASSURANCE SOCIETY OF VICTORIA, LIMITED.

First Quinquennial Investigation, Report from 1*st January* 1871 *to* 31*st December* 1875.

We have received a copy of this Report from the Actuary, Mr. Robert M'Caskie, of Melbourne, and think the following extracts from it will interest our readers :—

In entering into the investigation of the Society's affairs, the first consideration was the tables to be used and the method of valuation to be followed; and after mature deliberation, I determined to take the Actuaries' H^M Table with 4 per-cent interest, published under the auspices of the Institute of Actuaries, as the basis of valuation, and to follow a method proposed by Mr. Thos. B. Sprague, the editor of the " Journal of the Institute of Actuaries" and one of its Vice-Presidents, published in vol. xv. of the Journal of the Institute, and which is approved of by the leading members of the profession.

This method may be briefly described as recognizing the fact that the expenses attending a life policy are not uniform during the whole years of its existence, but, on the contrary, that a very large expenditure has to be incurred the first year, consisting of increased commission to agents for introducing the business, medical fees, printing and advertising, &c., and in after years the expenses are confined to the renewal commission allowed to agents and the ordinary expenses of the office. It is not therefore necessary to reserve the whole of the loading added to the net premium to form the office premium, seeing that such a course would only be required were the expenses uniform during the whole years of the existence of the policy. A part of the loading, equal to the increased expenditure during the first year of the existence

of the policy, has therefore been valued, and this course is allowed by the highest authorities to be perfectly legitimate. But while following this method, care must be taken that in the results brought out no negative values appear, and that if the Society is under an obligation, express or implied, to allow a surrender value of a certain magnitude for its policies, it is essential that it should reserve at least this amount to meet its liabilities under its policies.

The Society has, in my opinion, entered into an implied contract with its members to return a third of the premiums received after three annual premiums have been made for surrender, and in the method of valuation adopted this has been steadily kept in view, and the guaranteed amount of surrender reserved.

The amount necessary to be reserved to meet the surrender value requires that a lower net premium should be valued than would be the case if no guaranteed amount of surrender had been promised, and to that extent this system of valuation approaches nearer to a net premium method. I may also mention that by the method of valuation followed, no negative values appear, but every policy included in the valuation has a liability attached to it.

Increase of Business.

The amount of new assurances during each year of the quinquennial period was as shown in the following *vidimus* of new business transacted:—

Year.	Policies issued.	Amount Assured.	First Premiums thereon.
		£ s. d.	£ s. d.
1871	246	79,618 11 11	7,868 17 7*
1872	157	53,245 9 9	1,079 13 1
1873	202	68,500 0 0	1,111 14 0
1874	518	147,743 0 0	2,780 11 11
1875	644	176,000 0 0	3,165 0 0
		£525,102 1 8	16,005 16 7

* This amount includes £6,700 received as single premiums on Foundation Policies.

Expenses of Management.

For the reasons stated in the introduction, the expenses of management as compared with old-established offices appear to be heavy, but this is more apparent than real. In an old-established office the amount of new business bears a much smaller proportion to the whole amount of the existing assurances than in a young office, so that while an old-established office may be able to show a small percentage of expenses on its premium income, a close examination of the accounts will reveal the fact that the expenses incurred in procuring new business are as heavy as those in a comparatively young office.

In a paper read by Mr. Deuchar, Actuary to the Caledonian Life Assurance Company, before the Actuarial Society of Edinburgh,

1st April 1874, and published in the "Assurance Magazine", October 1874, he shows that the expenses of the Scottish offices (which are generally allowed to be well managed, and transact a business similar to that of this Society) incurred in acquiring new business, amount to 80 per-cent of the first year's premium, clearly demonstrating the fact that the expenses are exceptionally heavy during the first year of assurance, and amount to a small percentage in after years. While, therefore, it is undeniable that the expenses during the five years under review have been heavy, yet when it is borne in mind that the new assurances bear so large a proportion to the whole existing business, it will be seen that the expenses incurred have been no heavier than in the Scottish Offices, many of which are of old standing, the dates of their establishment ranging from 1815 to 1867, and nearly all of them having a most extensive business.

Increase of Resources.

The net funds and revenue have increased during the five years as follows:—

Year.		Funds.	Annual Revenue.
		£ s. d.	£ s. d.
1871	Commencement . . .	Nil.	...
1871	31st December, end of 1st year	4,650 9 8	8,727 6 6*
1872	" " 2nd "	6,102 4 10	3,967 16 5
1873	" " 3rd "	7,969 3 0	5,883 4 5
1874	" " 4th "	13,330 5 1	9,950 4 10
1875	" " 5th "	22,254 14 8	15,518 14 8

* This amount included £6,700 received as single premiums on Foundation Policies.

Mortality Experience.

During the five years 18 deaths of members of the Society have occurred, entailing claims to the amount of £5,754. 10*s*. 3*d*. This is considerably under the expectation as shown by the Actuaries' H^M table; by that table the expected number of deaths was 31 and the expected claims £9,075. This result is no doubt in a large measure due to the careful and recent selection of lives.

Investigation of the Society's Affairs.

The result cf the valuation shows that the total funds as per Valuation Balance Sheet, amount to £22,254. 14*s*. 8*d*.; the net liability of the Society under its assurance, endowment and annuity contracts amounts to £21,915. 14*s*. 3*d*. the difference is £339. 0*s*. 5*d*., being the surplus profit in the quinquennium.

This result will perhaps be a disappointment to the members, but it should be borne in mind that the accounts and returns now required to be furnished to the Government prove that it is almost impossible for an assurance society to divide a profit on its first five years' business. The fact that many offices, now well established, have done so, and divided more profit in their first five years than in any sub-

sequent five years, only proves that in the absence of the returns now required by the Insurance Companies' Act their true position could not be detected.

The following further particulars are extracted from the schedules.

The whole-term assurances of one year's duration and upwards were valued in classes, according to the ages attained by the lives assured on the 31st December 1875.

The ages attained were taken as the office ages at entry plus the number of years and quarters the policies have been in force.

In valuing the policies, fractional parts of a quarter have been treated as whole quarters, and the proportion of premium for which each policy was renewed beyond 31st December 1875, has been taken into account as a liability.

The liability of the Society is ascertained by taking the difference between the present value of the sums assured and the present value of the future premiums after deducting the loading.

The net premiums valued were those as exhibited by the "Institute of Actuaries' H^M Table of Mortality" with 4 per-cent interest, with a small addition equal to ·7955 per-cent on the net premium for age 20, and increasing to 3·3018 per-cent on the net premium for age 60, such increase being considered equal to the initial expenses incurred during the first year of assurance which do not recur in after years. Each premium was assumed to be just due.

[Mr. M'Caskie writes to us. To find what net premium to value for the endowment assurances, I first ascertained the net single premium for the assurance for the life at the age at which the assurance was effected and accepted and added to it £1. 10*s*. per £100 assured for initial expenses. I then divided the sum of these by the corresponding immediate temporary annuity, the quotient giving the net premium which I valued thus: the single premium for an endowment assurance for age 20 payable at 45, is 41·855, to which add 1·5 giving 43·355, which divided by 15·118 is 2·868.]

The whole-term assurances under one year's duration and all assurances other than whole-term assurances, the children's endowments, and the annuities, were valued separately.

All assurances "loaded up" and charged at rates corresponding with those charged at an increased age, have been treated as if the increased ages were the actual ages of the assured.

The percentage of the gross premium income reserved as a provision for future expenses and profits is equal to 22·21 per-cent on the premium income.

CONSOLIDATED REVENUE ACCOUNT.

	£	*s.*	*d.*
Amount of Funds on 1st January 1871	Nil.		
Premiums (no Reassurance Premiums)	40,496	3	10
Interest (no dividends)	3,164	15	10
Consideration for Annuities granted	3,500	0	0
Fees and Fines	254	15	0
Forfeited Deposits	84	7	2
Deposits	47	5	0
	£47,547	6	10

	£	s.	d.
Claims under Policies	5,754	10	3
Surrenders	208	1	3
Annuities	237	0	0
Commission	4,943	16	3
Expenses	13,983	19	1
Bad Debts	165	5	4
Amount of Funds on 31st December 1875	22,254	14	8
	£47,547	6	10

The total amounts assured, premiums received annually, and the total premiums received on all policies other than for the whole term of life with uniform annual premiums, as at 31st December 1875, are shown in the following table:—

PARTICIPATING POLICIES.

Class of Assurance.	No. of Policies.	Amount Assured.			Annual Premium.					Total Premiums received.			Reserve.
					Ordinary.			Extra.					
		£	s.	d.	£	s.	d.	£	s.	£	s.	d.	£
[W]hole-Life Policies—													
1. Limited Number of Premiums	8	3,684	10	10	162	13	3	...		803	10	4	675
2. Paid-up Policies	64	18,393	16	3	...			...		6,400	0	0	8,176
[E]ndowment Assurance	240	61,100	0	0	3,031	13	6	5	0	3,804	10	4	2,261
[As]surance on Joint Lives	176	47,400	0	0	2,159	6	11	5	10	3,221	4	5	982
[C]hildren's Endowments	27	3,450	0	0	248	14	10	...		315	3	3	309
	515	134,028	7	1	5,602	8	6	10	10	14,541	18	4	12,403

The life assurance fund was invested—

		£	s.	d.	
On the 31st December 1871	at	3	11	0	per-cent.
" " 1872		7	4	0	"
" " 1873		6	14	5	"
" " 1874		6	12	9	"
" " 1875		8	2	7	"

The practice of the Society has been to allow a surrender value of one-third of the premiums paid; but only after three annual premiums have been paid on the policy.

Policies effected on lives believed to be under the average expectation and "loaded up" or accepted at rates corresponding with those charged at an increased age, are treated both for surrender value and in the classification of age and liability valuation as if the increased ages were the actual ages of the assured.

The Society does not transact business at other than European rates; but in the case of persons travelling between the 25th parallels of north and south latitude an extra premium of £1 per-cent is charged. This extra premium is remitted when persons whose lives are assured come to reside south of the 25th parallel of south latitude.

Single Lives, with Profits.

Premiums payable during Whole Term of Life to secure £100.

Age next Birthday.	Annually. £ s. d.	Age next Birthday.	Annually. £ s. d.	Age next Birthday.	Annually. £ s. d.	Age next Birthday.	Annually. £ s. d.
20	1 16 2	31	2 8 5	41	3 6 10	51	4 18 4
21	1 17 1	32	2 9 11	42	3 9 8	52	5 2 6
22	1 18 0	33	2 11 5	43	3 11 10	53	5 7 0
23	1 19 0	34	2 13 0	44	3 14 6	54	5 11 8
24	2 0 0	35	2 14 9	45	3 17 4	55	5 16 8
25	2 1 0	36	2 16 6	46	4 0 4	56	6 2 1
26	2 2 1	37	2 18 4	47	4 3 6	57	6 7 10
27	2 3 3	38	3 0 4	48	4 6 11	58	6 13 11
28	2 4 6	39	3 2 5	49	4 10 6	59	7 0 6
29	2 5 9	40	3 4 7	50	4 14 3	60	7 7 6
30	2 7 1						

NEW ZEALAND GOVERNMENT INSURANCE DEPARTMENT.

First Quinquennial Investigation.

ANNUAL REPORT, 1875—76.

To Policyholders.

The Commissioner requests the co-operation of the policyholders in pointing out to their friends the benefits of life insurance, and the very favourable terms on which these may be obtained in this department.

The advantages offered by this department, in comparison with those of private offices, are,—

Government Security for the payment of every policy.

A much larger Original Insurance, generally from £1,150 to £1,250 for the same yearly premium as would elsewhere be charged for £1000, with the right to share in whatever surplus may arise; all profits being divisible by Act among policyholders only.

Copies of the reports and other pamphlets will be forwarded on application.

Extracts from Annual Report of the Government Insurance Commissioner.

New Business.

1,986 proposals were received during the year for £717,341, of which 501 were declined or not completed, leaving 1,485 policies issued for £504,509, the particulars of which are shown in the following summary:—

Class.	No. of Policies.	Sum Assured. £ s. d.	Single Premium. £ s. d.	Annual Premium. £ s. d.
Insurance . . .	1,215	477,120 0 0	279 6 9	16,224 1 1
Endowment . .	27	3,950 0 0	...	193 8 9
Annuity . . .	6	£218. 7*s.* 5*d.* per an.	2,356 4 5	...
Industrial . . .	237	23,439 1 0	...	848 0 8
Total . .	1,485	504,509 1 0	2,635 11 2	17,265 10 6
				2,635 11 2
Total New Premiums				£19,901 1 8

Claims.

The claims that fell in during the year were 41 in number, and amounted to £21,550, caused by the death of 37 persons. The number of losses is but slightly larger than those for the previous year, but the amount is increased by upwards of £9,000. This is accounted for by the fact that 3 of the claims amounted to more than one-third of the whole sum that was claimed. On the other hand, 2 annuitants have died, by which annuities amounting to £180 have ceased to be payable.

Policies Discontinued.

A summary of the policies discontinued during the year is given in the following table:—

How discontinued.	Assurance Branch.			Endowment Branch.			Annuity Branch.			Industrial Branch.		
	No. of Policies.	Sum Assured.	Annual Premiums.	No. of Policies.	Sum Assured.	Annual Premiums.	No. of Policies.	Amount of Annuity per annum.	Annual Premiums.	No. of Policies.	Sum Assured.	Annual Premiums.
		£	£ s. d.		£	£ s. d.		£	£ s. d.		£ s.	£ s. d.
y Death .	41	21,550	683 1 0	...	...	...	2	180	...	..	...	...
, Expiry .	1	100	1 10 3	...	...	...	...	...	...	..	...	...
, Surrender	25	7,000	252 3 3	2	200	14 4 0	...	...	...	...	...	...
, Change .	4	2,700	64 19 10	..	...	...	...	...	...	..	...	...
, Lapse .	27	7,010	199 11 7	4	450	23 18 3	...	...	...	28	2,778 10	92 14 8
ot taken .	..	...	...	..	...	...	..	...	...	10	982 17	45 14 4
Totals .	98	38,360	1,201 5 11	6	650	38 2 3	2	180	...	38	3,761 7	138 9 0

The number of the policies issued since the establishment of the department is 7,413, assuring £2,602,285, of which 1,007 have been discontinued, leaving 6,406 existing policies, assuring £2,303,662.

Accounts.

The Revenue Account and Balance Sheet have been prepared in accordance with the 41st section of "The Government Insurance and Annuities Act, 1874", and are attached to this Report. In the ordinary branch, notwithstanding the expenses attending the Quinquennial Valuation (which are not properly chargeable to the year's expenses), the ratio of the expenses of management (including commission) to the premium income, has been reduced to 16·3 per-cent, as against 37·4 per-cent in the year ending 30th June 1872, clearly showing that the expenditure incurred on new business has been a good investment, although no credit whatever was taken for such investment in the recent Quinquennial Valuation.

Industrial Branch.

The new business in this branch was discontinued at the end of the financial year, although it was fast increasing; the Government

having considered that there were circumstances connected with the business that rendered its continuance undesirable, and the report of the English Actuaries shows that this discontinuance was in accordance with their views.

General Remarks.

Messrs. Bailey and Pattison having pointed out that the rates of premium charged on older ages were too low, the tables are now being revised on the basis of the "Mortality Experience of the Institute of Actuaries".

A valuation is also being made by the Department, which is intended to be done annually, in addition to the Quinquennial Investigation prescribed by law. The whole of the operations will thus be kept under review, while errors (if such should creep in) will be detected, and the stability of the office placed beyond doubt. These arrangements will give additional confidence to the public, and doubtless will tend still further to increase the business of the Department.

Revenue Account *of the* Ordinary Branch *from* 1*st July* 1875 *to* 30*th June* 1876.

	£	s.	d.
Amount of Funds on 1st July 1875	109,967	12	1
Renewal Premiums	61,136	13	6
New Premiums	8,414	2	1
Single Premiums	346	1	9
Consideration for Annuities granted	2,356	4	5
Interest	6,669	6	11
Fines	Nil.		
Miscellaneous	11	19	0
	£188,901	19	9

	£	s.	d.
Claims	21,550	0	0
Surrenders	820	4	7
Annuities	1,384	17	0
Commission, New	1,507	4	2
Commission, Renewal	1,408	5	7
Travelling Allowance, Agents'	1,266	4	3
Medical Fees	1,794	0	0
Travelling Expenses	299	6	6
Advertising	276	7	3
Salaries, Head Office	2,500	0	0
Extra Clerical Assistance	284	10	0
Salaries, Travelling Agents'	938	1	2
Printing	205	4	6
Rent	115	8	0
Cost of Actuarial Valuation	705	11	8
Miscellaneous	76	7	10
Written off Furniture	52	19	9
Amount of Funds on 30th June 1876	153,717	7	6
	£188,901	19	9

BALANCE SHEET *of the* ORDINARY BRANCH, 30*th June* 1876.

LIABILITIES.

	£	s.	d.
Total Funds, as per Revenue Account	153,717	7	6
Claims announced but not paid	2,800	0	0
Annuities due and unpaid	122	8	8
Commission, New	660	4	11
Commission, Renewal	36	14	3
Travelling Expenses	59	5	7
Medical Fees	585	10	0
Advertising	37	16	4
Agents' Salaries	6	1	0
Stationery	119	8	1
Rent	35	8	0
Messrs. Pattison & Bailey's Fees	500	0	0
Miscellaneous	13	1	5
	£158,643	5	9

ASSETS.

	£	s.	d.
Loans upon Policies	584	1	3
Investments (Treasury Bills)	147,400	18	6
Office Furniture, Head Office and Agencies	476	0	0
Agents' Balances	55	17	4
Outstanding Premiums	5,817	2	4
Outstanding Interest	42	10	10
Accrued Interest	1,227	14	1
Advance to Industrial Branch	478	4	3
Cash	2,560	17	2
	£158,643	5	9

REPORT BY THE ACTUARIES (MR. A. H. BAILEY AND MR. W. P. PATTISON).

ORDINARY BRANCH.

The money transactions of the Ordinary Branch, during the five years of its existence, may be thus summarized:—

	£ s. d.	£ s. d.
INCOME FROM		
Premiums	146,150 5 0	
Annuity Purchase Money	16,689 12 7	
Interest	8,424 2 2	
Fines and Fees	81 5 11	
		171,345 5 8
OUTGO FOR		
Claims	22,750 0 0	
Surrenders	293 7 11	
Annuities	2,950 3 4	
Commission	7,061 3 5	
Expenses	28,322 18 11	
		61,377 13 7
FUND AT 30TH JUNE 1875	...	£109,967 12 1

The Department was liable at the same date for 4,958 Assurances for £1,833,819, 21 Immediate Annuities of £1,249, and 10 Annuities of other descriptions of £935. The premiums payable under these several contracts amounting to £57,510 per annum.

In order to determine the sufficiency of the fund to meet these liabilities, we had to consider upon what basis the valuation should be made. No sufficient experience has yet been acquired of the mortality among assured lives in New Zealand or in any of the Australian Colonies; but, having regard to the climate and character of the people, there is reason to believe that it will not differ materially from the corresponding mortality in England. We consider, therefore, that the most suitable table of mortality for the present investigation is that known as the Institute of Actuaries H^M, and we have adopted it accordingly for the assurance risks. This table exhibits the mortality among 130,243 male lives, assured at ordinary rates of premium, by twenty of the principal companies in Great Britain, between the years 1797 and 1863. The number of years of life passed through was 1,283,034; the number of deaths 20,521; and from the care and pains bestowed upon all the details of the work, as well as from the reliance to be placed upon the accuracy of the facts, there can be little doubt that this table is the most trustworthy exponent of the mortality of assured lives that has yet been published. The annuities have been valued by the Carlisle Table, the H^M Table not being suitable for this class of liabilities, many annuities being granted upon female lives.

As regards the rate of interest, the 36th and 37th sections of "The Government Insurance and Annuities Act, 1874", direct that all moneys received under the Act shall be paid into the public account, and may be invested in certain specified securities. For any uninvested part of such moneys, interest at the highest Bank rate allowed at the time for other trust funds shall be credited to the insurance account.

From the last report of the Commissioner, it appears that, with the exception of a small working balance, the whole fund is invested in Treasury Bills, bearing interest at the rate of £5. 2*s*. 6*d*. per-cent per annum. This may, perhaps, be a somewhat higher rate than will be always obtainable in future, but we think that there can be little doubt that for some time to come the rate of interest will be materially higher in New Zealand than in England; and we consider, therefore, that, for the purpose of the present investigation, we shall be quite justified in assuming for our calculations 4½ per-cent as the rate of interest, and have done so accordingly. This rate is employed in the calculations of the Insurance Department of New York, and of several other of the United States of America, and, in our judgment, is the best suited for the Insurance Department of New Zealand.

To make provision for future expenses, profits, and contingencies, the value of what is called the "loading" has been computed and reserved: that is, a special calculation has been made for each policy of the bare premium which would be sufficient for the risk according to the table of mortality and rate of interest assumed for the valuation, and the difference between the premiums actually payable and the premiums thus computed, has been capitalized and reserved.

Provision is thus made, among other things, for the heavier mortality expected among those lives on which extra premiums have

been charged for health or occupation. The additional annual premiums form in this manner a special fund available to meet the increased annual mortality.

The summary and valuation of the Ordinary Branch made on these principles will be found in the annexed Schedule. The general result is as follows:—

Fund	£109,967
Estimated Liability	97,776
Surplus . .	£12,191

Having regard to the rates of premium charged, we think that this result is satisfactory; but we wish to add that, in our opinion, this surplus would not be properly divisible. As no division is to be made until another five years have elapsed, the question does not now arise. But when that time shall come, we think that a further reserve will have to be made for the future bonuses on those policies on which a limited number of premiums only is payable, and which constitute about 11 per-cent of the number. If the loading on this class of assurances had been distributed equally over the whole term of life, and a corresponding reserve made on the present occasion, we believe, from some calculations we have made, that the surplus would have been materially reduced.

The rates of premium are undoubtedly very low, especially at the older ages. They are below the prevailing rates in this country, and are lower than are generally charged by the principal assurance societies in the Australian Colonies.

No considerable surplus is, we think, likely under any circumstances to accrue from such premiums. Their sufficiency will depend mainly upon the cost at which the business will in future be conducted, and the rate of interest at which the funds will be invested.

During the five years now under review, commission and expenses have absorbed more than 24 per-cent of the premiums; but by the last report it appears that the rate of expense for the year ending 30th June 1875, was 19·2 per-cent, and there is reason to believe that it will continue to decline. If, however, the rate of interest should fall materially in future, we think that the rates must then undergo a general revision; and under any circumstances we think that the premiums at the older ages, are too low and ought to be raised.

Industrial Branch.

The money transactions of this branch show a deficiency of £501. 3*s*. 7*d*. There have been granted 76 policies for assurances of £8,105, paying in premiums £5. 5*s*. 7*d*. weekly. A valuation of these risks, made on the same principle as for the ordinary branch, results in an estimated liability of £53, so that the apparent deficiency is thereby increased to £554. This of itself is unimportant, as the business has been started so recently, and the amount is small; but it seems to us to be worthy of serious consideration whether the Department should grant such assurances at all. The intention apparently was to undertake risks like those of Friendly Societies. The object of the death

payments in these societies is not family provision or business requirements, but almost invariably to provide a small sum of money (not exceeding £10) for funeral expenses; but the assurances of this branch of the Department belong to an entirely different class, the average amount being £107. It is surely unnecessary to incur the trouble and expense of a separate organization for such business as this. It is not easy to understand that persons in a position to effect these assurances can have any real difficulty in paying premiums quarterly; and it is surely to their interest to avoid the increased expense which a system of weekly collections must occasion.

FOURTH SCHEDULE.

On the present occasion no distribution of profits has to be made.

The table of mortality used in the valuation of policies was the Institute of Actuaries Life Tables (H^M), deduced from the mortality experience of life assurance companies. Fourteen policies involving more than one life were valued by the Carlisle Table.

The annuities were valued by the Carlisle Table.

The rate of interest assumed in the valuation of all the policies and annuities was 4½ per-cent.

The difference between the premiums payable under the policies and the computed net premium by H^M, 4½ per-cent, has been in each case reserved. The amount so reserved on the ordinary whole-life policies by an uniform premium amounted to 21½ per-cent on the premiums payable.

The time during which a policy must be in force in order to entitle it to share in the profits.

We are not aware that the New Zealand Insurance Department have yet determined this point.

Summary *and* Valuation (see p. 381).

FIFTH SCHEDULE.

The average rate of interest at which the funds of the Department were invested at the close of each financial year:—

30 June 1871	6	per-cent.
„ 1872	6	per-cent.
„ 1873	5	per-cent.
„ 1874	5	per-cent.
„ 1875	5·08	per-cent.

The table of mortality used for calculating the surrender values of policies is that known as the "Seventeen Offices' Experience". The rate of interest assumed in every case is 4 per-cent. A deduction of 20 per-cent from the value thus calculated determines the surrender value allowed. In the case of children's Endowments, the return of all premiums paid, without interest, is allowed as a surrender value.

In the case of policies on which extra premiums have been charged

ORDINARY BRANCH.

DESCRIPTION OF TRANSACTIONS.	PARTICULARS OF THE POLICIES FOR VALUATION.				VALUATION. I.—By the H^M Institute Table.—Interest at 4½ per-cent. II.—By the Carlisle Table.—Interest at 4½ per-cent.			
	Number of Policies.	Sums Assured.	Office Yearly Premiums.	Net Yearly Premiums.	Sums Assured.	Office Yearly Premiums.	Net Yearly Premiums.	Net Liability.
I.—ASSURANCES.	£	£	£	£	£	£	£	£
Whole term of life, subject to Premiums throughout life	3,217	1,198,043	33,015·0	25,935·5	411,554·1	471,763·1	369,112·1	42,442·0
Ditto, by a limited number of Premiums	547	280,523	10,842·1	8,705·7	88,301·1	89,330·3	71,589·3	16,711·8
Ditto, by single Premium	7	853	...	...	277·9	...	...	277·9
Endowment Assurances	1,064	323,750	12,569·8	10·458·2	143,123·8	148,852·8	123,369·0	19,754·8
Endowments, returnable Premiums	75	8,450	511·3	...	...	...	...	1,505·4
Ditto, ditto, by single Premium	12	1,200	...	...	...	...	...	850·3
Ditto, non-returnable Premiums	5	1,100	58·9	55·9	616·7	476·8	454·0	162·7
Assurances for terms of years	17	14,800	252·1	...	...	...	...	126·1
Assurances on Joint Lives	13	4,600	175·7	148·7	2,072·6	2,123·5	1,797·0	275·6
Survivorship Assurances	1	500	22·0	15·1	99·5	111·1	76·3	23·2
Extra Premiums	...	...	63·5	...	...	...	...	32
Total Assurances	4,958	1,833,819	57,510·4	...	646,045·7	712,657·6	566,397·7	82,161·8
II.—ANNUITIES.		Per Annum.						
Immediate	21	1,249·1	...	...	...	...	...	12,193·7
Survivorship	2	335	9·2	7·6	1,143·8	125·2	104·0	1,039·8
Deferred	5	300	94·2	89·2	1,747·7	744·1	703·7	1,044·0
For a fixed term of years	3	300	...	...	...	...	...	1,336·2
Total of the Results	4,989	...	57,613·8	...	...	...	...	97,775·5

INDUSTRIAL BRANCH.

DESCRIPTION OF TRANSACTIONS.	PARTICULARS OF THE POLICIES FOR VALUATION.				VALUATION. By the H^M Institute Table.—Interest at 4½ per-cent.			
	No. of Policies.	Sums Assured.	Office Yearly Premiums.	Net Yearly Premiums.	Sums Assured.	Office Yearly Premiums.	Net Yearly Premiums.	Net Liability.
		£	£	£	£	£	£	£
Industrial Assurances	76	8,105	274·5	161·2	2,479·2	4,179·3	2,426·4	52·9

for impaired health or other causes, the surrender value is calculated as if the premium for the true age at entry had been charged.

Policies on lives believed to have an expectation of life less than the average are issued at increased premiums, corresponding with those charged at higher ages. In all calculations, whether for purposes of valuation or surrender, they are dealt with at the true ages, as if no increase in the premiums had been made.

The Published Tables of Premiums.

For the Assurance of Lives resident in any part of the World, except within 25 degrees of the Equator.

Premiums to secure £100, payable at Death.										Endowment Assurance.—Annual Premiums to secure £100, to be paid as indicated, or at Death, if prior.									
Age next Birthday.	Single Premium.			Annual Premiums to cease after 20 years.			Annual Premium, Whole Life.			At Age 65.			At Age 55.			At Age 45.			Age next Birthday.
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
20	28	1	8	2	1	6	1	10	4	1	13	2	2	2	0	3	1	3	20
21	28	12	8	2	2	5	1	11	1	1	14	3	2	3	8	3	4	7	21
22	29	4	4	2	3	6	1	11	11	1	15	5	2	5	5	3	8	4	22
23	29	16	6	2	4	7	1	12	10	1	16	7	2	7	5	3	12	4	23
24	30	9	2	2	5	7	1	13	10	1	17	11	2	9	5	3	16	10	24
25	31	2	4	2	6	8	1	14	11	1	19	3	2	11	9	4	1	9	25
26	31	15	10	2	7	9	1	16	0	2	0	7	2	14	3	4	7	3	26
27	32	9	10	2	9	0	1	17	1	2	2	3	2	16	10	4	13	4	27
28	33	7	0	2	10	4	1	18	2	2	3	11	2	19	8	5	0	4	28
29	33	17	2	2	11	5	1	19	4	2	5	6	3	2	6	5	7	11	29
30	35	9	6	2	12	5	2	0	6	2	7	1	3	5	8	5	16	5	30
31	35	1	11	2	13	5	2	1	8	2	8	9	3	9	0	6	6	2	31
32	35	14	10	2	14	6	2	2	10	2	10	8	3	12	6	6	17	5	32
33	36	8	6	2	15	10	2	4	0	2	12	6	3	16	8	7	10	8	33
34	37	3	0	2	17	0	2	5	2	2	14	7	4	1	0	8	6	5	34
35	37	18	0	2	18	2	2	6	6	2	16	11	4	6	0	9	5	5	35
36	38	13	4	2	19	8	2	8	0	2	19	5	4	11	6		...		36
37	39	9	10	3	1	0	2	9	6	3	2	1	4	17	7		...		37
38	40	6	3	3	2	7	2	11	2	3	4	11	5	4	5		...		38
39	41	3	2	3	4	0	2	12	11	3	8	2	5	12	1		...		39
40	41	19	11	3	5	7	2	14	8	3	11	3	6	0	9		...		40
41	42	16	0	3	7	0	2	16	5	3	14	7	6	10	8		...		41
42	43	12	0	3	8	5	2	18	2	3	18	3	7	1	9		...		42
43	44	8	0	3	10	0	2	19	11	4	2	1	7	14	9		...		43
44	45	4	10	3	11	6	3	1	10	4	6	6	8	10	3		...		44
45	46	1	11	3	13	0	3	3	10	4	11	2	9	8	9		...		45
46	47	0	1	3	14	9	3	6	0	4	16	5		...			...		46
47	47	19	4	3	16	10	3	8	5	5	2	5		...			..		47
48	48	19	11	3	19	2	3	11	0	5	9	3		..			...		48
49	50	2	5	4	1	8	3	14	0	5	17	2		...			...		49
50	51	6	6	4	4	0	3	17	4	6	6	2		...			...		50
51	52	12	1	4	6	9	4	1	1	6	16	9		...			...		51
52	53	18	2	4	10	5	4	5	0	7	8	10		...			...		52
53	55	4	8	4	14	5	4	9	3	8	2	11		...			...		53
54	56	11	7	4	18	3	4	13	6	8	19	7		...			...		54
55	57	19	3	5	2	3	4	18	0	9	19	8		...			...		55

Annuities and Endowments.

IMMEDIATE ANNUITIES.—Table showing the Sum to be paid for an Immediate Life Annuity of £1, payable by Half-Yearly Instalments, according to the Age and Sex of the Person upon whose Life the Annuity is to depend.

Age last Birthday.	Males.			Females.			Age last Birthday.	Males.			Females.		
	£	s.	d.	£	s.	d.		£	s.	d.	£	s.	d.
20	18	3	0	18	4	7	46	13	8	3	13	9	0
21	18	0	5	18	2	2	47	13	3	3	13	14	3
22	17	17	10	17	19	9	48	12	18	2	13	9	3
23	17	15	2	17	17	4	49	12	12	11	13	4	1
24	17	12	5	17	14	10	50	12	7	7	12	18	10
25	17	9	7	17	12	3	51	12	2	2	12	13	4
26	17	6	8	17	9	8	52	11	16	9	12	7	7
27	17	3	8	17	7	0	53	11	11	3	12	1	8
28	17	0	7	17	4	3	54	11	5	7	11	15	7
29	16	17	5	17	1	6	55	10	19	11	11	9	8
30	16	14	2	16	18	7	56	10	14	2	11	3	7
31	16	10	10	16	15	8	57	10	8	3	10	17	7
32	16	7	5	16	12	8	58	10	2	4	10	11	5
33	16	3	10	16	9	6	59	9	16	4	10	5	4
34	16	0	2	16	6	4	60	9	10	3	9	19	1
35	15	16	5	16	3	0	61	9	4	2	9	12	10
36	15	12	7	15	19	7	62	8	18	0	9	6	7
37	15	8	8	15	16	1	63	8	11	10	9	0	4
38	15	4	7	15	12	6	64	8	5	8	8	14	0
39	15	0	5	15	8	9	65	7	19	7	8	7	9
40	14	16	2	15	4	10	66	7	13	6	8	1	6
41	14	11	9	15	0	11	67	7	7	5	7	15	4
42	14	7	3	14	16	10	68	7	1	6	7	9	3
43	14	2	8	14	12	7	69	6	15	8	7	3	2
44	13	18	0	14	8	2	70	6	9	11	6	17	3
45	13	13	2	14	3	8							

ENDOWMENTS FOR CHILDREN, WITH RETURN OF PREMIUMS.—Rates of Premium to secure Endowments of £100. The Premiums to be returned without Interest if Death occur previous to the specified Age.

To be paid at the Age of 14.

Age next Birthday.	Single Premium.			Annual Premium.		
	£	s.	d.	£	s.	d.
1	59	5	6	6	3	7
2	62	7	3	6	17	0
3	65	7	5	7	12	10
4	68	6	8	8	11	10
5	71	6	6	9	15	1
6	74	7	3	11	4	2
7	77	9	7	13	1	8
8	80	13	11	15	11	9
9	84	0	4	19	1	10

To be paid at the Age of 21.

Age next Birthday.	Single Premium.			Annual Premium.		
	£	s.	d.	£	s.	d.
1	43	19	9	3	8	3
2	46	6	7	3	13	9
3	48	12	5	3	19	6
4	50	17	8	4	6	3
5	53	3	6	4	13	10
6	55	10	1	5	2	5
7	57	18	0	5	12	3
8	60	7	6	6	3	7
9	62	18	10	6	17	3
10	65	12	2	7	12	8
11	68	7	6	8	11	8
12	71	5	1	9	14	11
13	74	4	11	11	4	1
14	77	7	0	13	1	6

Table showing the Minimum Surrender Value of Policies for £100.

Age at Entry.	By Equal Annual Premiums for the Whole Term of Life.									For the Whole Term of Life. Premiums limited to 10 Annual Payments.									Endowment Assurances payable attaining the Age of 55, or at previous Death.							
	MINIMUM SURRENDER VALUES AT THE END OF																								
	5 Years.			10 Years.			15 Years.			5 Years.			10 Years.			15 Years.			5 Years.			10 Years.			15 Year
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.
20	2	13	3	5	16	0	9	9	8	10	18	1	24	9	10	27	4	11	5	9	8	12	3	11	20	8
25	3	4	11	7	1	1	11	10	7	12	2	7	27	5	0	30	9	8	7	4	2	16	1	2	27	0
30	3	19	5	8	12	8	14	2	3	13	11	8	30	9	8	34	5	9	9	14	7	21	15	4	36	15
35	4	18	2	10	13	6	17	2	11	15	4	4	34	5	9	38	11	1	13	14	1	30	16	3	52	5
40	6	2	10	13	0	9	20	9	1	17	3	7	38	11	1	43	2	11	20	13	0	46	11	6	100	0
45	7	9	4	15	10	1	23	18	5	19	4	10	43	2	11	47	19	1	34	18	10	100	0	0		
50	8	17	3	18	2	11	27	10	6	21	5	5	47	19	1	52	16	3	100	0	0					
55	10	8	10	20	19	9	31	2	7	23	4	1	52	16	4	57	9	9								

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ASSURANCE MAGAZINE.

On the carrying out of Reversionary Transactions by Life Assurance Companies. By JAMES R. MACFADYEN, *of the Legal and General Life Assurance Society.*

[Read before the Institute 26 November 1877.]

SOME years ago I had the honour of reading before this Institute a paper which, though mainly devoted to a study of surrender values, treated such surrender values as purchases by life offices of reversions. On that occasion I stated that sound British companies could well afford to repurchase their own acceptances at 6 or even at 5 per-cent. Subsequent experience has done much to confirm me in the opinion then expressed, but I did not then, nor do I now assert that offices should on such terms purchase other reversions than their own policies; and I trust it will not be thought wasting your time, if I preface this paper by enumerating a few considerations which seem to indicate that the present market price of reversions gives, comparatively, a return none too high to an investor. Such market rate I take to be 5 per-cent in the case of an investor who does not stand, or, what is the same thing, arrives at his price as if he did not intend to stand, any risk of loss, whether through longevity of life tenant or death of reversioner. Purchasers who,

though willing to bear the annuity risk, insist on opening a policy, and thus transfer, or apparently transfer, the assurance risk, look, I take it, for at least 6 per-cent on their investment. If the extreme yield of other investments suitable to life offices be assumed to be 4½ or 5 per-cent, reversions are more profitable by at most 1 or 1½ per-cent. This greater yield of reversionary transactions does not seem to be excessive when their peculiar nature is taken into consideration. Money is locked up in them for a very lengthened period. The amount to which the advances may run up is unknown, and thus the individual return is beyond the power of prediction. It is not uncommon that a *full* investigation of title or value is impracticable. The vendor may inflict on the purchaser, if such purchaser assure himself, a heavy loss from death through a breach of contract.* There may be difficulties of identification of parties. Altogether, the causes which induce a prudent investor in reversions to look nearly as much to the standing of the introducer of the business, as to what appears on the face of the transaction, combine to indicate that the excess interest rate obtained on reversions is none too high for the extra risk involved. I therefore have based the following investigations on the ordinary market price of the day, and have in my analysis tried to obtain results which on the average of transactions will return at least such market price. Nevertheless, however much we may deprecate the cutting down of the profit on reversions, it is to be feared the tendency nowadays is all in that direction.

The ordinary formulæ for the value of reversions keep back the cost of an annuity to provide the interest, and, if necessary, the premiums, till the benefit falls into possession. A private purchaser, if he wish to make himself absolutely secure, actually purchases this annuity; but when the buyer is an assurance company, this annuity is now, so far as I know, never taken out†; even although all offices, I believe, are careful, if assurances form part of the transaction, to set them up in whole or part. Whether these assurances be opened within the office itself, or elsewhere, is not now the question. In the following analysis the assurance

* Even when the policy is opened with third parties, this loss may arise unless all risks whatever be covered in the policy from the first. The ordinary clause protecting an innocent holder of a policy from loss, provided that on hearing of the transgression of conditions, he informed the office, and paid such extra premium as the Directors might consider reasonable, would be of little use in the case, when, say, the life assured volunteered for the War in the East, and got killed before the policyholder knew of his breach of contract.

† With reference to this point, we beg leave to refer our readers to Mr. Sprague's statement, vol. xvii, p. 248.—ED. *J.I.A.*

company as a buyer of reversions, and the assurance company as a seller of them, are treated as absolutely distinct bodies. Such profit or loss as may be realised on a reversion, is taken as simply the profit or loss arising from the deviation from the average of the annuity charged at average price to the seller. Now even of assurance policies it can hardly be said that the general run of companies have a sufficient number to save them from inconvenient mortality fluctuations. But the class of business in which the liability is greatest at the beginning of the transaction, is large as compared with that in which the liability is a maximum at the close. If then, companies have to endure severe fluctuations on their assurance trading, still more intense fluctuations may be looked for in their annuity trading. And thus the profit realised by an office on its purchase of reversions is of a very changeful and wavering character. If this be the case with the *purchaser* of reversions, how much more strongly is it so with their *seller!* Each individual vendor of a reversion is in the position of an assurance company with only one policy on its books, and which is accordingly liable to suffer ruinous loss, or it may be, to realise extravagant gain on the transaction. Any expedient then, which can be devised to lessen the excessive risk run by both parties to the contract in the sale of a reversion, is a gain to both, though it is especially so to the vendor. The main object then, I have set before myself in this paper, is to seek a method of dealing with reversions, which, while in no way lessening the *average* profit on such transactions, will sacrifice part of the excessive gain realised by a company when the life tenant dies early, in consideration of obtaining the diminution of the excessive loss endured when the life tenant dies late. It has been said that the method of dealing with reversions here analysed, "sacrifices" some part of the profit resulting from the reversion falling in early. The method adopted consequently makes no attempt to debit the seller with the cost of assurances to provide the profit relinquished. Such a principle of action would not only be contrary to the central idea of the system, but would break down in practice. For the heavy loadings on the extra assurances needful, would render the method unworkable in most cases.

Absolute Reversions.

First, to apply our analysis to absolute reversions. Say an absolute charge payable after the death of a person aged (x) is granted in the following manner. The amount of the advance to

the seller is the present value of an absolute reversion of 1. In consideration of this advance, there is granted a charge, the amount of which, if the person aged (x) die within the next n years, is to be $1:b$, but if the death occur after the n years, the amount of the charge is to be $\left(\frac{1}{b}+z\right)$. It is required to find the value of z. The symbol b of course represents any given number greater than 1, that may be wished. The following will be at once seen to be the equation of value:

$$\frac{1}{b}\left\{1-d(1+a_x)\right\}+z\frac{D_{x+n}}{D_x}\left\{1-d(1+a_{x+n})\right\}=1-d(1+a_x) \quad . \quad (1)$$

whence we have

$$z=\frac{\left(1-\frac{1}{b}\right)\left\{1-d(1+a_x)\right\}}{\frac{D_{x+n}}{D_x}\left\{1-d(1+a_{x+n})\right\}}.$$

In practice, the rates employed for the quantities within the brackets, would be those that would be used in the purchase of the reversion in the ordinary way; and for the $\frac{D_{x+n}}{D_x}$ would be employed the rate of interest sought on the transaction, and such table of mortality as would not bring out more favourable results to the seller than that employed in the other parts of the expression.

The problem may, however, be solved from another point of view. The ultimate possible charge, less the temporary assurance to secure it, is equivalent to the equalised charge. That is,

$$\left\{\frac{1}{b}+z\right\}\left\{1-d(1+a_x)\right\}-z\,{}_{\overline{n}|}A_x=1-d\,(1+a_x) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

whence

$$z=\frac{\left\{1-\frac{1}{b}\right\}\left\{1-d(1+a_x)\right\}}{1-d(1+a_x)-{}_{\overline{n}|}A_x}.$$

Here, in practice, the temporary assurance would be calculated by the company's selling rates for such assurances; the remaining parts of the expression being as in the former method of calculation.

These two expressions for z are of course identical when homogeneous. When otherwise, it is only possible to say which gives the more favourable results, after a comparison of the actual rates

employed. The latter formula, since it manages to extract assurances from absolute reversions, is likely to be the more popular in practice with a life company. Either formula can be employed when the advance is fixed, and the charge is the quantity to be determined. The latter expression is the one which most offices would use when purchasing a reversion of a fixed amount. Instead of the company paying the full cash value of such reversion, it offers the alternative of giving somewhat less than that cash value, compensating the seller by allowing him to retain for a term of years an interest in a portion of the reversion. The preceding analysis shows that this is equivalent to allowing the seller to purchase with the portion of the cash value of his reversion kept back from him, a temporary assurance for n years on the life of the person after whom the reversion falls into possession. Though this is the effect of the analysis, it would be objectionable to actually hand over to the vendor, as part payment for his reversion, a policy of assurance. By doing this, the company would be exchanging a security of its own, and of which it thoroughly knew the value, for a security concerning which, however good in fact, it could not have the same certain knowledge. And it is possible to imagine cases in which the reversion bought was not received, while the company was still legally liable under its policy. For these reasons, then, it seems objectionable to hand over to the seller the assurance policy as part payment of his reversion. All that is suggested is, that the temporary assurance policy be actually passed into the office books, and that the premium account draw on the reversion account, for the amount of the premium under such policy. If this be done, the reversion in all valuations will be held to be of the full amount it would otherwise only attain after n years. From the nature of the case, the assurance must be without medical examination. Of course such examination can be dispensed with by the company purchasing; for, if the proportion of the whole reversion which is to be payable in the event of the life aged (x) dying within n years be properly adjusted, the office will gain more by the death of (x) within the n years, than it will lose under the term policy. But equally of course, since this profit, gained by the office on the reversion, is not shared with other companies, no part of the risk under the term policy can be reassured. This inability to re-assure is, however, a difficulty more apparent than real. So far as the actual profit and loss is concerned, an office might open the temporary policy for any sum, however much it might nominally

exceed the maximum line on a single risk. The only objection, if objection it can be called, that I can see to its doing so is, that a company might not relish the possible appearance in its accounts for the year of an enormous increase in the amount of "claims under policies", while the still more enormous increase in the "profit on reversions fallen in" showed no connection with such claim amount. Even if this be considered an objection, the cases are very few in which a transaction is so large as to preclude the adjustment of the stages of the charge before and after n years, so that their difference shall not exceed the maximum amount held by a company under a policy. In such few cases this scheme of treating reversionary transactions does not fail, but the book entry method suggested may be thought objectionable to carry out. If in such few cases it be so considered, the transaction can be completed without the actual opening of the policy.

In consequence of the reversioner retaining under this scheme for a term of years an interest in the reversion, one of the gravest objections to the purchase of reversions will, for such term, be without force. So long as the seller has a balance to receive, he will take care that no time is lost in apprising the company of the death of the life tenant, and aiding it to prove the death. Even after the seller of a reversion has lost the interest he held during a term of years, the office purchasing will be in no worse a position with regard to supervision of the life tenant than under a reversion bought in the ordinary way.

The method of treating reversions, now under analysis, requires in practice that the stages of the charge to be given be so adjusted that, even if the reversion falls into possession just before the end of the n years, the company shall neither lose part of its capital, nor fail of a reasonable return upon it. I say "in practice," because in theory it might be held that the loss in one case would be balanced by the profit on another. An arrangement of terms, however, which compelled the office to lose, should death occur within the n years, would be obviously objectionable in practice, to say nothing of the danger of selection it would open up in cases where the life tenant was uninsurable. The proportion of the reversion to be received if it come into possession within the n years, must therefore not fall below the accumulated amount of the office's outlay at the end of n years at the minimum rate of interest required on the transaction. In adjusting the stages there are obvious general principles. The older the life tenant, the smaller

n should be, and the less should be the difference between the stages of the reversion. For simplicity sake, I have employed only two stages, but more than two can be introduced if desired. The multiplication of stages, however, will soon reach its limit, from the necessarily high terms which, with many stages, must latterly be required. My own idea is that in practice it is better to have only two stages, with n of sufficient length to be attractive to a seller.

This method of treating reversions peculiarly suits the case when the life in possession is reputedly in bad health. Nothing is more common in bargaining about reversions than to be told that such is the case. In some instances possibly the expectant heir's wish is father to his thoughts and statements on the subject. But none the less there are cases where the statement is true; and even if the seller is mistaken on the point, he feels it a grievance that offices buying reversions make no allowance for what he believes a special feature. The method under analysis exactly meets such instances. By it the office allows the reversioner to back his opinion that the life tenant is in bad health, by granting him a temporary assurance at ordinary rates on the said tenant's life. This, of course, it can well afford to do. As, if the n years be outlived, the Society in the individual case is better off than in a purchase on the ordinary principle—while, if they be not outlived, the minimum rate of profit insisted on will *at least* be realised. And the worse the life in possession is, the more profit above that minimum rate will be received. Though it is no part of the profit under consideration, it may be pointed out that, if the life tenant be really an invalid, the society will as an assurance company, reap a benefit through the special assurances, hereafter referred to as being required in connection with reversionary annuities, being fixed at a too high rate of premium.

This part of the subject may be concluded by a comparative table showing the fluctuation in interest realised on a reversion, according as the ordinary or the special mode of purchase has been adopted. The selling price of an absolute reversion to £1000, net, after a life aged 60 may be taken as £470. This is its value, whether the calculation be based on a 6 per-cent homogeneous Carlisle valuation, or on Mr. Jellicoe's terms. If, instead of handing the vendor £470 a company give him only £400, it will have kept back £70 of his money. If we take the office single premium for a 7 years' assurance on a life aged 60 as 24 per-cent, this £70 of present cash is equivalent to a 7 years' assurance of £300. By

the special method then, the vendor will only get in cash £400, but in the event of the death of the life tenant within 7 years, the office will only receive £700. If the death occur after 7 years, the office will receive the whole £1000. Assuming payment of the charge to be received at the end of 1, 2, 3, &c. years from now, the following table will show the rate of interest per annum realised over the whole period of the transaction. It has been arrived at as follows:—

For the ordinary mode of purchasing the reversion, the equation is

$$470(1+i)^t=1000, \qquad \text{whence } \lambda(1+i)=\frac{1}{t}\lambda\frac{1000}{470}.$$

For the special mode, if $t<n+1$,

$$400(1+i)^t=700, \qquad \text{whence } \lambda(1+i)=\frac{1}{t}\lambda\frac{700}{400}.$$

For the special mode, if $t>n$,

$$400(1+i)^t=1000, \qquad \text{whence } \lambda(1+i)=\frac{1}{t}\lambda\frac{1000}{400}.$$

If Reversion be paid at the end of	THE RETURN DURING THE CURRENCY OF THE TRANSACTION IS BY	
	Ordinary Method.	Special Method.
1 Year	113·0 per-cent.	75·0 per-cent.
2 „	46·0 „	32·3 „
3 „	28·6 „	20·5 „
4 „	20·8 „	15·0 „
5 „	16·3 „	11·9 „
6 „	13·4 „	9·7 „
7 „	11·4 „	8·3 „
8 „	9·9 „	12·1 „
10 „	7·8 „	9·6 „
15 „	5·2 „	6·3 „
20 „	3·9 „	4·7 „
25 „	3·0 „	3·7 „
30 „	2·5 „	3·1 „
35 „	2·2 „	2·7 „
40 „	1·9 „	2·3 „

From the foregoing table it will be seen, that even if death occur in the 7 years during which the office is only to receive a portion of the reversion, there will at the worst be realised 8·3 per-cent per annum on the investment. If the death of the life tenant occur in extreme old age, the preceding table shows that, by the special system, the company will be able to bear about a

quinquennium more of longevity than by the ordinary system, and yet realise as good a return as by that usual method. It would be an interesting enquiry to see how closely the point, at which, by the ordinary system, the expected interest rate was realised, coincided with the average age at death by the Carlisle Table. However it is outside the range of my subject at present, and so I have not followed it up.

Reversionary Annuities.

The greater part of what has been already said when treating of absolute reversions, is equally applicable to reversionary annuities, or contingent reversionary charges. To prevent repetition, therefore, attention will now only be drawn to the special points raised in considering these two latter classes of business.

Acting on the same principle as before, the formula required to find z, the additional reversionary annuity beyond $\frac{1}{b}$ to be paid during the lifetime of (y), after the death of (x), provided such death be deferred n years, will, when the consideration money is the present value of a reversionary annuity of 1, be found from the equation—

$$\frac{1}{b}\left\{\frac{1}{d+\mathrm{P}_y}-(1+a_{xy})\right\}+z\frac{l_{x+n}l_{y+n}v^n}{l_x\ \ l_y}\left\{\frac{1}{d+\mathrm{P}_{y+n}}-(1+a_{x+n,\,y+n})\right\}$$
$$=\frac{1}{d+\mathrm{P}_y}-(1+a_{xy})\quad.\ .\ .\ .\ .\quad(3)$$

whence

$$z=\frac{\left\{1-\frac{1}{b}\right\}\left\{\frac{1}{d+\mathrm{P}_y}-(1+a_{xy})\right\}}{\frac{l_{x+n}l_{y+n}v^n}{l_x\ \ l_y}\left\{\frac{1}{d+\mathrm{P}_{y+n}}-(1+a_{x+n,\,y+n})\right\}}.$$

If, as in the case of the absolute reversion, we analyse an alternative formula, in which, from the first, credit is given to the seller for the survivorship annuity of z, (though it only vests after n years) we have, were the rates homogeneous, the equation to find z—

$$\left\{\frac{1}{b}+z\right\}\left\{a_y-a_{xy}\right\}-z\left\{a_y-\left({}_{|n}a_{xy}+\frac{l_{x+n}}{l_x}\,{}_{n|}a_y\right)\right\}=a_y-a_{xy}\quad.\ .\ .\ .\quad(4)$$

whence

$$z = \frac{\left\{1-\frac{1}{b}\right\}\left\{a_y - a_{xy}\right\}}{\frac{l_{x+n}}{l_x}\,{}_{n|}a_y - {}_{n|}a_{xy}}.$$

When we convert this equation into one of working rates, the quantities into which (x) and (y) both enter, would (in the denominator after ages ($x+n$), ($y+n$) only) be left homogeneous (say if Mr. Jellicoe's terms were adopted, Carlisle 3½ per-cent); and only the quantities in which (y) alone appeared would be made non-homogeneous. One form into which these non-homogeneous terms might be thrown, is that given them under equation (3), and thus we might have the value of z whether arrived at through equation (3) or equation (4) identical. And as there is not the same reason as in absolute reversions for employing alternative formulæ, this identical value is the one I shall confine myself to as the basis of the working formula.

I have used the words "as the basis of the working formula", because there are practical difficulties in the way of employing equation (3) as it stands. The formula presupposes that if the ordinary kind of assurance on the life of (y) is employed, the office will delay opening the policy to protect the additional annuity of s till the said annuity vests. If the transaction were so small that no reassurances were necessary, this could be done; but the greater number of cases require assurances larger than the office maximum; and it cannot be expected that an outside company, when the additional protection becomes needful, will undertake the risk without reference to the then state of health of the person to be assured. Possibly, between offices having confidence in each other, an agreement, entered into at the opening of the transaction, on the one hand to offer, on the other to undertake the assurance,—it might be at special rates,—on the vesting of the additional annuity, might answer the purpose. At least the risk thus undertaken seems to me not much greater than various now current in the ordinary course of reassurance business. However, it will be well to see whether we cannot eliminate the question of confidence, and treat the matter in a mathematical way.

At the time of entry on the deferred reversion, the amount of assurance requisite is $\frac{z}{d+P_{y+n}}$. Now it is obvious that on the *average* of transactions, it would be immaterial whether the

assurance were then taken out by single or annual premium. In either case its then cost would be

$$\frac{z}{d+P_{y+n}}A_{y+n},$$

and the then cost discounted to the present time is

$$\frac{l_{x+n}}{l_x}\cdot\frac{l_{y+n}v^n}{l_y}\frac{z}{d+P_{y+n}}A_{y+n}.$$

This then is the sum which, if paid at the present moment on *every* reversionary transaction undertaken, would, at the end of n years, after transferring the accumulated amounts paid on account of those who did not outlive these years, to the credit of those who did survive, exactly suffice to furnish a full assurance of $\frac{z}{d+P_{y+n}}$ on such survivors. Let us next consider the properties of an assurance deferred n years on a life aged (y), the amount of which, however, is not to be paid unless another life aged (x) survive the said n years. The annual premium for such an assurance would be, assuming such net premium to be called ϖ,

$$\varpi=\frac{\dfrac{l_{x+n}\,l_{y+n}\,v^n}{l_x\quad l_y}A_{y+n}}{1+a_{xy}+\dfrac{l_{x+n}}{l_x}\cdot\dfrac{l_{y+n}\,v^n}{l_y}(a_{y+n}-a_{y+n,\,x+n})}.$$

The single premium corresponding is—

$$\frac{l_{x+n}\,l_{y+n}\,v^n}{l_x\quad l_y}A_{y+n};$$

and if the amount of the assurance were $\frac{z}{d+P_{y+n}}$, the single premium would be—

$$\frac{z}{d+P_{y+n}}\frac{l_{x+n}\,l_{y+n}\,v^n}{l_x\quad l_y}A_{y+n}.$$

But this is the same expression as we previously arrived at; so, leaving aside for the moment the effect on our analysis of varying interest and loadings, it appears that the ordinary assurance of $\frac{z}{d+P_{y+n}}$ opened n years hence, on such transactions as might then require it, is equivalent to taking from the outset, in every instance,

an assurance of the full amount $\frac{z}{d+P_{y+n}}$, but of the special nature above referred to. It is then by this special assurance on the life of (y), the amount being only payable provided both (x) and (y) outlive n years, that I propose to meet the difficulty raised through the inability to secure a favourable medical examination n years hence. Its rationale is as follows:—The office, though in all cases paying premiums from the first, for the full amount of assurance that may ultimately be required, does so on a scale of rates which is below the ordinary scale; and these premiums are so calculated, that the sums thus saved on such assurances as outlive the n years, suffice on the average to pay for the premiums lost on those policies which do not survive the n years.

The present value of all future premiums, whether the assurance be taken on the special plan from the first, or on the ordinary plan, n years hence, is in each case

$$\frac{z}{d+P_{y+n}} \frac{l_{x+n}}{l_x} \frac{l_{y+n}}{l_y} v^n A_{y+n}.$$

This identity might, however, be disturbed if the elements constituting the expression were not the same in both. Taking first the loading. Since the above expression is one of factors merely, it matters nothing whether the percentage of loading be put on the single or the annual premium, or whether it be put on the A_{y+n} or $\frac{l_{x+n}}{l_x} \frac{l_{y+n}}{l_y} v^n A_{y+n}$. All that is requisite is that it be the same percentage in each case. And thus if it be considered that in the special office premium P, there ought to be a higher loading than in the ordinary office premium P_{y+n}, it will be necessary in expression (3) to substitute for the ordinary office rate at age $(y+n)$ an annual premium having the same percentage of addition to ϖ_{y+n} as the special pure rate ϖ has to the special office rate P. Next as to the effect of varying interest and mortality rates.

When the expression

$$\frac{z}{d+P_{y+n}} \frac{l_{x+n}}{l_x} \frac{l_{y+n}}{l_y} v^n A_{y+n}$$

is considered in reference to the premiums required under the special policy, the office is in the character of a seller of assurance, and will thus be anxious to raise the premium as much as possible.

This it can do through the medium of the interest, the mortality, or through both combined. It seems an unnecessary complication to put it on more than the interest alone, so I shall assume that the office will be willing to sell the special assurance on an H^{M} 3 per-cent basis. The above expression for the assurance sale will thus be

$$\frac{z}{d+\mathrm{P}_{y+n}} \overset{\mathrm{H^M}\ 3\%}{\frac{l_{x+n}}{l_x}\frac{l_{y+n}}{l_y}v^n} \mathrm{A}_{y+n}.$$

When, however, the question is the determination of z, the office is in the character of purchaser of a reversionary annuity, and will thus want, say, 5 per-cent for its money. The difference between the two interest rates, or

$$\frac{z}{d+\mathrm{P}_{y+n}} \overset{\mathrm{H^M}}{\frac{l_{x+n}}{l_x}\frac{l_{y+n}}{l_y}} \mathrm{A}_{y+n}(\overset{3\%}{v^n}-\overset{5\%}{v^n}),$$

is the loss on the assurance sale and is an additional debit in expression (3) to the seller of the annuity. One thing further ought to be pointed out with regard to the expression

$$\frac{z}{d+\mathrm{P}_{y+n}} \frac{l_{x+n}}{l_x}\frac{l_{y+n}}{l_y}v^n \mathrm{A}_{y+n}.$$

When this is employed for annuity-valuing purposes, the premium during the joint existence of (x) and (y) after n years is on Mr. Jellicoe's principle, taken as capitalised by the Carlisle $3\frac{1}{2}$ per-cent table, this being assumed to be the price at which an annuity could be bought in the market. No doubt this principle of arriving at office single premiums from office annual premiums combined with office annuity rates, is strictly logical and accurate. However, even although its neglect leads to indefensible anomalies, I am afraid in practice generally we are more likely to use homogeneous 3 per-cent rates. I therefore use this rate for A_{y+n} in the above expression for the difference of interest.

We are now in a position to evolve the working formula for the purchase of a reversionary annuity, carried out on the principle under examination. Reverting to equation (3) it will be seen that, after making allowance for the loss on the special assurance sale, the working formula is

$$\frac{1}{b}\left\{\frac{1}{\underset{5}{d}+P_y}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{xy}})\right\}+z\,\overset{\text{H}^{\text{M}}\ 50/_0}{\frac{l_{x+n},\,l_{y+n}\,v^n}{l_x\quad l_y}}\left\{\frac{1}{\underset{5}{d}+P_{y+n}}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{x+n,\,y+n}})\right\}$$

$$-\frac{z}{\underset{5}{d}+P_{y+n}}\,\overset{\text{H}^{\text{M}}}{\frac{l_{x+n}\,l_{y+n}}{l_x\quad l_y}}\,\overset{\text{Office }30/_0\quad 50/_0}{A_{y+n}(v^n-v^n)}=\frac{1}{\underset{5}{d}+P_y}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{xy}})\quad . \quad . \quad (5)$$

whence

$$z=\frac{\left\{1-\frac{1}{b}\right\}\left\{\frac{1}{\underset{5}{d}+P_y}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{xy}})\right\}}{\overset{\text{H}^{\text{M}}\ 50/_0}{\frac{l_{x+n}\,l_{y+n}\,v^n}{l_x\quad l_y}}\left\{\frac{1}{\underset{5}{d}+P_{y+n}}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{x+n,\,y+n}})\right\}-\overset{\text{H}^{\text{M}}}{\frac{l_{x+n}\,l_{y+n}}{l_x\quad l_y}}\,\frac{\overset{\text{Office }30/_0\quad 50/_0}{A_{y+n}(v^n-v^n)}}{\underset{5}{d}+P_{y+n}}}\,.$$

It may be well to give an example.

An advance of £1000 is made to a life of (25) on condition of his paying for life after the death of (55) a reversionary annuity. This annuity he has now the option of declaring whether it will be on the ordinary Jellicoe terms, or whether it will be on the equivalent terms when three-fourths only of the Jellicoe annuity is to be paid if the death of (55) occur within the next 10 years, but a higher annuity (z) is to be paid if (55) outlive these 10 years. What is the amount of this higher annuity?

Here, taking 1 as the annuity,

$$z=\frac{\frac{1}{4}\left\{\frac{1}{\underset{5}{d}+P_{25}}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{25\cdot55}})\right\}}{\overset{\text{H}^{\text{M}}\ 50/_0}{\frac{l_{35}l_{65}v^{10}}{l_{25}l_{55}}}\left\{\frac{1}{\underset{5}{d}+P_{35}}-(1+\overset{\text{Car. }3\frac{1}{2}}{a_{35\cdot65}})\right\}-\frac{\frac{l_{65}l_{35}}{l_{55}l_{25}}\overset{\text{Office }30/_0\quad 50/_0}{A_{35}(v^{10}-v^{10})}}{\underset{5}{d}+P_{35}}}\,.$$

$$\begin{aligned}&\overset{\text{H}^{\text{M}}\ 30/_0}{\varpi_{25}}=\cdot01625\\&15^{\circ}/_{\circ}=\underline{\ 002437\ }\\&P_{25}=\cdot018687\\&\underset{5}{d}\quad=\underline{\ 04762\ }\end{aligned}$$

$$06631\quad \kappa\lambda=17842\;.\,\dot{}\,.\;\frac{1}{\underset{5}{d}+P_{25}}=15\cdot081$$

$$1+\overset{\text{Car. }3\frac{1}{2}}{a_{25\cdot55}}=11\cdot782$$

$$3\cdot299\quad \kappa\lambda=48162\;\therefore\;£303\cdot12=$$

Ordinary Jellicoe terms per £1000.

$\overset{H^M\,3\%}{\varpi_{35} = 021928}$

$15\% = \underline{003289}$

$ 025217$

$\underset{5}{d} \quad = \underline{04762}$

$07284 \quad \kappa\lambda = 13763 \therefore \dfrac{1}{\underset{5}{d} + P_{35}} = 13{\cdot}729$

$1 + \overset{\text{Car. } 3\frac{1}{2}}{a_{35\cdot65}} = \underline{8{\cdot}869}$

$4{\cdot}860 \quad \lambda = 68664$

$\lambda l_{65} = 69282$

$\lambda l_{35} = 93592$

$\kappa\lambda l_{55} = 17709$

$\overset{H^M\,3\%}{A_{35} = 42950} \qquad \kappa\lambda l_{25} = 08123$

$15\% = \underline{06448} \qquad \lambda\ \overset{5\%}{v^{10}} = \underline{78811}$

$49898 \quad \lambda = 69867 \qquad 81181 \qquad 2{\cdot}0508$

$\overset{3\%}{v^{10}} = 74409$

$\overset{5\%}{v^{10}} = \underline{61391}$

$13018 \quad \lambda = 11454$

$\lambda \dfrac{l_{65} l_{35}}{l_{55} l_{25}} = 83706$

$\kappa\lambda(\underset{5}{d} + P_{35}) = \underline{13763}$

$78290 \quad . \quad . \quad . \quad . \quad . \quad . \quad \underline{{\cdot}6066}$

$1{\cdot}4437 =$ denominator.

$\text{Numerator} = \dfrac{3{\cdot}299}{4} = {\cdot}82475 \quad \lambda = 91632$

$\lambda\,\text{denom.} = \underline{15948}$

$75684 \qquad \therefore\ {\cdot}57127 = x.$

If, therefore, the reversionary annuity for 10 years instead of 1 be ·75, it ought thereafter to be ·75 + 57127, or 1·32127. And if for £1000 advance the reversionary annuity for 10 years instead of £303·12 be $\dfrac{3 \times 303{\cdot}12}{4}$ or £227·34, it ought thereafter to be 303·12 × 1·32127 or £400·5.

The formula for the annual premium under the special policy is

$$\varpi = \frac{\dfrac{l_{65} l_{35} \overset{H^M\,3\%}{v^{10}}}{l_{55} l_{25}} A_{35}}{1 + a_{25\cdot55} + \dfrac{l_{65} l_{35} v^{10}}{l_{55} l_{25}} (a_{35} - a_{35\cdot65})}.$$

With 15 per-cent loading this will be found to give for P a value of ·01427. The amount to be assured under the ordinary scale is $\frac{227\cdot34}{\underset{s}{d}+P_{25}}=£3428$ and the premium thereon is 64·068. The amount to be assured under the special scale is $\frac{400\cdot5-227\cdot34}{\underset{s}{d}+P_{25}}=£2377$ and the premium thereon is 33·934.

With these particulars we are in a position to analyse the preceding calculations.

The amount advanced by the office is—

1st. On account of the usual annuity,

$227\cdot34\times3\cdot299$	=	£750 = Cash paid vendor for this annuity.
$227\cdot34\times11\cdot782$	=	£2678 = Annuity-due to provide interest and premiums during joint lives.

2nd. On account of the special annuity,

$173\cdot17\times2\cdot0503$	= £355	
Deduct drawback on account of loss on special assurance, $173\cdot17\times\cdot6066$	= £105	
	——	£250 = Balance of cash paid vendor.
		£105 = Amount of loss on special assurance.
$173\cdot17\times(1+a_{35\cdot65})\frac{l_{65}l_{35}v^{10}}{l_{55}l_{25}}$ $=173\cdot17\times8\cdot869\times\cdot4219=$		£648 = Value of deferred annuity to provide premiums and interest after 10 years on the annuity then to vest.
		£4431 = Cash advanced.

If we wish to compare this cash advanced with the amount of the assurances effected, it must be remembered that the amount, though not the actual cost, of the special assurance, is that which will only be required 10 years hence. Its present discounted value is $£2377\times\frac{l_{65}l_{35}v^{10}}{l_{55}l_{25}}$, or £1003. Add to this the amount of ordinary assurance, £3428,

and we have £4431, being the amount of the cash advance as previously arrived at.

As in the case of absolute reversions, it may be well to conclude this part of the subject by a table showing the fluctuation in interest realized by the office, in the preceding example, according as the ordinary or the special system of carrying out the transaction is adopted. The return stated is that yielded till the capital is actually replaced by the falling in of the assurance. The table has been arrived at tentatively and approximately from the following formulæ:—

For the ordinary method of purchase—

$$1000(1+i)^t+\overset{\text{Premium}}{85\cdot424}\left\{\frac{(1+i)^{t+1}-1}{i}-1\right\}=\frac{303\cdot12}{d+P_{25}}.$$

For the special method of purchase—

If $t<n+1$,

$$(1000)(1+i)^t+\overset{\text{Ordinary and Special Premiums}}{(64\cdot068+33\cdot934)}\left\{\frac{(1+i)^{t+1}-1}{i}-1\right\}=\frac{227\cdot34}{d+P_{25}}.$$

If $t>n$,

$$(1000)(1+i)^t+\overset{\text{Ordinary and Special Premiums}}{(64\cdot068+33\cdot934)}\left\{\frac{(1+i)^{t+1}-1}{i}-1\right\}=\frac{227\cdot34}{d+P_{25}}+\frac{173\cdot17}{d+P}.$$

If First Payment of Annuity occur in	The Annual Return till the replacement of Capital is by	
	Ordinary Method.	Special Method.
1 Year	25·6 per-cent.	18·5 per-cent.
2 "	19·7 "	14·6 "
3 "	16·2 "	12·2 "
4 "	13·8 "	10·5 "
5 "	12·2 "	9·3 "
6 "	10·9 "	8·3 "
7 "	9·8 "	7·4 "
8 "	8·9 "	6·8 "
9 "	8·4 "	6·3 "
10 "	7·6 "	5·7 "
11 "	7·1 "	8·4 "
15 "	5·5 "	6·5 "
20 "	4·3 "	5·1 "
25 "	3·5 "	4·2 "
30 "	2·8 "	3·5 "
35 "	2·5 "	2·9 "
40 "	2·1 "	2·6 "
45 "	1·8 "	2·3 "

From the foregoing table it will be seen that even if only the smaller annuity under the special scheme come into possession,

the return will be *at worst* 5·7 per-cent on the transaction, and after the 10 years, the office is in a better position than by the ordinary system. In comparing the respective interest returns under the two schemes, it must, of course, be borne in mind that the probabilities of death at the various dates given are not the same.

CONTINGENT REVERSIONARY CHARGES.

Very little need be said on the subject of contingent reversionary charges, as they have virtually been included in our previous analysis. The equation corresponding to (5) will be—

$$\frac{1}{b}\left\{1-(\underset{s}{d}+\mathrm{P}^1_{yx})(1+\overset{\text{Car. }3\frac{1}{2}}{a_{xy}})\right\}+z\,\frac{\overset{\mathrm{H^M}\ 5\%}{l_{y+n}l_{x+n}v^n}}{l_y\ \ l_x}\left\{1-(\underset{s}{d}+\mathrm{P}^1_{y+n,x+n})(1+\overset{\text{Car. }3\frac{1}{2}}{a_{x+n,y+n}})\right\}$$

$$-z\,\frac{\overset{\mathrm{H^M}}{l_{y+n}l_{x+n}}}{l_y\ \ l_x}\,\overset{\text{Office}}{\mathrm{A}^1_{y+n,x+n}}\,(\overset{3\%}{v^n}-\overset{5\%}{v^n})=1-(\underset{s}{d}+\mathrm{P}^1_{yx})(1+\overset{\text{Car. }3\frac{1}{2}}{a_{xy}})\quad .\ .\ .\ (6)$$

whence—

$$z=\frac{\left\{1-\frac{1}{b}\right\}\left\{1-(\underset{s}{d}+\mathrm{P}^1_{yx})\,(1+\overset{\text{Car. }3\frac{1}{2}}{a_{xy}})\right\}}{\frac{\overset{\mathrm{H^M}\ 5\%}{l_{y+n}l_{x+n}v^n}}{l_y\ \ l_x}\left\{1-(\underset{s}{d}+\mathrm{P}^1_{y+n,x+n})\,(1+\overset{\text{Car. }3\frac{1}{2}}{a_{x+n,y+n}})\right\}-\frac{\overset{\mathrm{H^M}}{l_{y+n}l_{x+n}}}{l_y\ \ l_x}\,\overset{\text{Office}}{\mathrm{A}^1_{y+n\,x+n}}(\overset{3\%}{v^n}-\overset{5\%}{v^n})}$$

and the special assurance annual premium will be—

$${}_{n}\varpi^1_{yx}=\frac{\frac{\overset{\mathrm{H^M}\ 3\%}{l_{y+n}l_{x+n}v^n}}{l_y\ \ l_x}\,\mathrm{A}^1_{y+n,x+n}}{1+a_{xy}}.$$

THE METHOD OF ACCUMULATION.

The outcome of the system of treating reversions I have just been analysing, is logically the method of accumulation, for by it the return on a reversion purchased is equalised throughout the duration of the transaction. This method is not open to the objection that at first sight it might seem liable to. The loss, if the accumulation outruns the security, is not greater than under the system where a fixed amount of charge is bought. Indeed, it is probably less, for no prudent mortgagee lends the full value of a reversion. There are, however, very serious drawbacks to this system of carrying out reversionary transactions. When

reversions are bought outright, the loss in one case is compensated by the profit in another. But under the system of accumulation there is no such compensation; and it is for this reason mortgagees should never lend the full value of a reversion, unless they have security for the keeping down of the accumulation. Again, the amount of ultimate assurance that may be necessary is almost always enormous. This, in itself, would render it very difficult to place it all, even were it on the ordinary method of assuring. But it is practically impossible that it can be on the ordinary method, as no borrower's solicitor would allow the premium for such an amount of assurance to be charged against his client from the outset. The natural kind of assurance would be that which increased from year to year, exactly on all fours with the accumulation. But this, or indeed any very special kind of risk, it would be hopeless to try to get placed to the requisite amount. For these reasons the accumulation method is not one that can be generally recommended, though there are a few cases which may be considered exceptional. For instance, I can see no objection to employing it for an absolute reversion where the corpus is on any hypothesis of longevity amply sufficient to provide the accumulation; unless indeed the spoiling of the market for purchase be considered an objection.

Advances by way of Mortgage.

This method allows the borrower to keep down the accumulation on the advance, under penalty, in the case of default, of the advance being converted into a purchase on terms previously arranged. The selection is thus left entirely in the hands of the borrower, and it may safely be assumed that, so far as his means will allow, it will be exercised in the way that best suits his interest. If it were certain that the accumulation would be kept down for a term of years, and the amount of the charge to vest in the event of failure were calculated on the assumption of a purchase from the first, there would be no doubt that selection would be fully provided for. Of course I am assuming that the value of the reversion is an increasing one. And even with the knowledge that instead of a term of years, there is no security that even one payment of interest or premiums will be made, I consider, though of course it is only an opinion, that this natural increase in the value of the reversion outweighs selection. So, next to purchasing reversions outright, this method of dealing with them seems to me

the most eligible. The misfortune of it is, however, the case seldom arises, as it is rare to find a borrower on reversions who really wants to pay anything in the meantime.

Whatever be the form adopted for carrying out the purchase of a reversion, there is always one objection present. The seller has from the outset to pay for an assurance very much in excess of the then requirements of the purchaser. And this necessarily makes the terms which the vendor obtains very onerous to him. The system of increasing assurances, though more troublesome to the office, and losing to it the enormous profit it makes through the death of the vendor of a heavily reassured reversion, enables the terms to be lightened. Mr. Sprague, in the 17th volume of the *Journal of the Institute of Actuaries*, has analysed a case in which the requisite power of increase was arrived at by way of option of the purchaser. Another means that might be adopted is an assurance of two or three stages necessarily grown into at fixed dates if the policy so long endured. Whatever be the system adopted, it can be grafted on the scheme of carrying out reversionary transactions, to which this paper has been mainly devoted, quite as readily as on any more usual method.

This paper has already run to such length, that I must not further trespass on your patience. The subject, however, is far from being exhausted. Indeed, the variety of ways in which reversionary transactions can be carried out by life companies is practically endless. In accord with what has been said by Mr. Sprague, in the paper already referred to, I hold that with this endless variety before him, the actuary ought not to content himself with applying to the reversions proposed to be purchased merely these ordinary formulæ which he knows will keep his company safe, whatever effect they may have on the interests of the vendor. Each case ought rather to be dealt with by him from the view-point of its special features, and that method of treatment should be adopted, which, while guarding the office, and allowing it a legitimate profit, will best consult the interests of the seller.

DISCUSSION.

The PRESIDENT (Mr. J. Hill Williams)—As far as my experience goes—and it has not been inconsiderable—I am afraid the reversioner would not much like the plan of taking £400 instead of £470 for his reversion of £1000. The owners of absolute reversions almost always prefer to sell them right-out, and they will go wherever they can get the highest price; so that I do not think an office offering £400 instead of £470, the open market value, will have much chance of securing the reversion. With reference to leaving a small part of the present value in the hands of the reversioner, with the view of making it his interest to assist in ultimately proving the death of the tenant for life, that is no doubt desirable; but I believe that in practice it has not been found that there is any great difficulty about proving the death of the tenant for life where you have purchased reversions. Strangely enough, and contrary to expectation, the difficulty occurs more frequently in the case of the person purchasing a life interest. It seems to be much more difficult to prove a man alive than it is to prove him dead. It always happens that somebody has got an interest in showing when a man is dead—somebody expects something or other, and is ready to supply the information on being applied to. Another point that occurs to me is this: I do not know that a contingent reversioner has much to complain of, in the fact that the very nature of the property which he has to sell, requires for the protection of the buyer a larger insurance than is wanted at the moment. We should bear in mind that the contingent reversioner in that case has really nothing. You create a property for him out of nothing, and the least he can expect you to do is that you should make your purchase money perfectly secure. Considering reversions as investments by reversionary and insurance companies, the mere fact of the life tenant outliving the expected period of longevity, appears to me to be of no importance. We have never, as assurers, thought of complaining when a man dies after paying only one assurance premium; and, for the same reason, I do not see why we should complain when a reversioner lives beyond the expectation. It is all a question of average, and I do not see any great advantage in specially modifying or equating that average by the somewhat complicated process which has been recommended to us to-night. If there are a sufficient number of reversionary transactions entered into by reversionary or insurance companies, the principle of average will rule and establish itself in the long run, and secure to the buyers of the reversions a moderate and sufficient amount of profit, considering the nature of the security.

Mr. A. BADEN—Looking at the question practically, those who wish to raise money on reversions are broadly divided into two classes. The one class desire as much money as can be got for the reversions they have to sell, and the other class are in a position to afford to leave some margin, so that they may themselves take advantage of the gain which would ensue if the life tenant should die in the earlier years of the transaction. Now in practice the wants of both these classes are really met by insurance and reversionary societies. To

those who want as much money as possible, they offer the full value of their reversion; and, on the other hand, they say to those who can afford to leave a margin on such value, "We are willing to make an advance upon the security which you offer to us." This latter mode practically meets the very case which Mr. Macfadyen has written this very elaborate paper to provide for—namely, the case of those persons who believe that the life tenant is not in good health and is very likely to die in the earlier years of the transaction, so that consequently an inordinate profit is likely to ensue to the purchaser of the reversion, of which he thinks it desirable to take advantage if he can. Let us take Mr. Macfadyen's example of a reversion of £1000: the life tenant being sixty, he takes the value to be by the 6 per-cent homogeneous Carlisle Table £470; and he would give the person offering the reversion, £400 instead of the £470 which is its full value, and in consideration of the difference there would be a policy actually taken up or written on the books for £300, so that, if the life dropped within seven years of the transaction, that policy would go to the seller of the reversion. Now if he dealt with it according to the ordinary practical mode, and took an advance on his reversion, the mortgagee might afford to lend him £385, we will say, which is arrived at in this way. At the end of seven years the value of the reversion would be £558, and the present value of £558, at 6 per-cent, is £385 or thereabout. What would be the position of the reversioner supposing the life tenant died near the end of the seventh year? He would have had his £385, which, improved at 6 per-cent, would amount to £558, and he would then receive the balance, that is £442, out of the £1000. Now, according to Mr. Macfadyen's method, if the same thing occurred, and the life tenant died in the course of the seventh year, the reversioner would have received £400, which, at 6 per-cent interest, would have amounted to £601 about, and he would then get the insurance of £300—a total receipt of about £901, as against £1000 received by the other method. But supposing the life tenant died in the third year, what would be the effect? If he had an advance upon his reversion, that advance and the interest together would amount to £460, and he would receive net £540 at the end of three years, making up the total of £1000. Or, going by Mr. Macfadyen's method, he would have had, with interest, £476, and he would receive the £300, a total of £776, as against £1000. This, if he is in a position to calculate at all upon the eventual chance of his transactions, he is not likely to prefer. This is an objection which may be urged on the part of the seller or mortgagor of the reversion; but I think that a decided objection is to be taken on the part of the purchaser of the reversion, the office. Mr. Macfadyen says that if the life tenant dies within the time which he assigns for the temporary assurance, although there will be a loss on the life policy, there will be such a gain on the reversion, that the insurance loss will be a matter of no consequence. Now this is a piece of reasoning which I confess I am at a loss to understand. It appears to me that the whole *rationale* of buying reversions consists in the confidence you have that you will get a sufficient number for an average to be struck between great gains in the earlier period of the transaction and great losses in the latter

period. Now, what does it matter whether you call it a loss on the policy or a loss on the reversion? The loss is there notwithstanding, and you cannot make it other than a loss; and it comes to this, if you choose to say that you do not lose on your reversions, simply because you have paid the loss out of the policies, you have been selling short period policies to the public at a price at which no office can afford to sell them; and for this reason, that the class of persons who will choose that transaction, are those who honestly believe that the life tenant is really in a bad way and will not live to complete the term, and the consequence is that you must write these insurances in your own books as you cannot get them reassured. With reference to the concluding remark in Mr. Macfadyen's paper, that actuaries have it in their power to meet every possible case and to make the most elaborate calculations to satisfy the whim of every vendor, I doubt whether any actuary of experience will choose deliberately to court that kind of dealing. The great inconvenience and the endless trouble which would be occasioned in periodical valuations, by our dealing with every transaction as a special case, instead of being able to class them, is a considerable practical objection to any such mode of looking at the subject.

Mr. A. H. Bailey—The first part of the paper discusses what ought to be the rate of interest on reversions. Surely that is a matter of market price. The rate of interest upon reversions has fallen somewhat of late years, not from the philosophical reasons which Mr. Macfadyen assigns, but because they are more sought after than formerly. Private persons will not buy reversions, because of the inconvenience of having their money locked up. Several years ago insurance companies would not look at reversionary securities. To meet that difficulty, reversionary interest societies came into existence; but for some time past insurance companies have been extensive competitors for reversions, having found out that they are a suitable investment; and as a matter of course the rate of interest on that class of securities has fallen. But I do not for one moment suppose that the difference in the rates of interest on reversionary securities and on such other securities as are suitable to insurance societies, is as much as 1¼ per-cent. Then the seller really does not reason in the way that Mr. Macfadyen supposes. All he wants is money. He believes that any money given for his reversion is very much too little, and consequently he does not sell the reversion until reduced to the last extremity. And, generally, when reversions are sold, there are prior charges to pay off. The existence of these charges will somewhat interfere with Mr. Macfadyen's proposals. Loans on reversions are more easily effected than formerly; and there is no difficulty in borrowing a certain sum on a reversion, with the stipulation that, if the borrower do not pay interest, it shall be accumulated at compound interest. Care must be taken by the mortgagee to lend, not the value of the reversion, but such an amount as, accumulated at compound interest for the possible period of life which the tenant for life could attain, will be within the amount of the fund. The reversioner usually goes on borrowing as long as he can. The sale of the reversion is his last resource, and he then wants the utmost he can get for it.

While I am struck with the ingenuity with which Mr. Macfadyen has worked out the formulas, I must say I do not think they will seriously affect the purchase of reversions in practice.

Mr. G. W. BERRIDGE—My experience also has been that a man either wants the whole value of his reversion at once, in which case he sells it; or, if not the whole value, he simply gets a loan upon it, and trusts to the chapter of accidents to pay the interest. Perhaps a solicitor advances the interest; and finding these advances accumulating, the reversioner goes for a fresh advance, and this makes a little more business for the solicitor. If the man has no kind friend to pay the interest, the reversion goes to the hammer, and there is an end to the matter. Reversions submitted to auction generally fetch very good values. If a man is prudent, he can often make a very good arrangement for himself for a limited term of years. Supposing, in the case quoted by Mr. Baden, he would take an advance of something under the value, he could provide for the interest accumulating for a certain number of years. If in that time he did not provide the interest, the reversion should become the property of the office. I generally find that, if a man does not want to part with the whole reversion but only requires a certain sum less than the present value, he prefers to sell a part of his reversion outright, instead of allowing the advances to accumulate and result in a larger charge. That raises a point which is an objection to Mr. Macfadyen's scheme with regard to reversionary annuities. In the instance given, when you take an annuity for £200 for a certain term of years and £400 afterwards, if a further sum were required, the larger annuity would be taken as the amount of the prior charge. I had this point brought to my notice in a very marked case, when, as there was a very large margin of income, I was enabled to arrange terms which were more advantageous than they could be under the ordinary arrangement. The contract was signed, part of the money was had, the man then took the advice of his solicitor, who quashed the arrangement and reverted to the reversionary annuity, because, he said, the estate would be so encumbered when the charge began to operate. It also would bar him to a great extent from further advances, and thus stop the way to his own needs. There would be also very great difficulty in arranging the terms of a transaction under Mr. Macfadyen's scheme. Generally speaking, you are asked to quote terms at once, and you have often scanty information; and when you have quoted them, you are asked to have them varied, and you cannot go round to half-a-dozen offices to ask what they will do some special policy for, but have to trust to your own judgment as to what can be done. A whole-life policy also is generally done cheaper than any special form of insurance.

The PRESIDENT—I wish to ask a question of Mr. Macfadyen. When you say that it would be objectionable to hand over to the reversioner an actual policy of assurance (for a short term on the life of the tenant for life) as part payment of his reversion, has it occurred to you that, whether you hand him a policy or not, if you have contracted to grant him an assurance, he is surely entitled to have some deed or document to show his title to the assurance which you say belongs to him?

Mr. MACFADYEN—It is true, no doubt, that from an actuarial view-point, the seller of a contingent reversion has no real grievance on being compelled to pay for heavy protecting assurances. But no one outside the profession takes this view. All that is looked to is the small amount of cash given, and the heavy amount that in the future is demanded. The effect of the protecting assurances on the terms is often such, that I was not surprised at what I once heard—that actuarial terms are so exorbitant, that it is cheaper to go to Jew money-lenders for advances than to life offices. As to the question put to me whether the seller would be content if he did not actually receive the life policy, there seems some misapprehension. The life policy is only a book-entry, and the seller knows nothing of it. All that he knows in (say) the case in question, is, that if the reversion comes into possession within seven years, he will only pay £700 instead of £1000. With Mr. Baden's statement that the method of accumulation suits the borrower's interests better than the scheme in question, I quite agree. In fact, the method of accumulation suits the borrower so well, that the lender has not a chance. At best, his accumulated advance will be repaid. He stands, without compensation, the risk of the accumulation outrunning the security. Mr. Baden asks what is the position of the office in a transaction carried out on the principle in question, and seems to think that I have simply shifted the risk to the assurance policy. If he considers further, I think he will agree with me that this is not so. I have provided for an assurance loss on one transaction by an assurance gain on another. The fact that compensation is taken for the possible loss on the assurance policy, by keeping back the premium from the sum paid the seller, will, I am sure, on consideration, convince Mr. Baden that it is not a mere hocus-pocus between reversion and assurance, I have been dealing with; and that the only gain sacrificed, or rather, partly sacrificed, is the unearned one now made when the life tenant is below the average in health. Mr. Bailey thinks that it is not the reasons mentioned in my paper but competition, that has reduced the price of reversions. Quite so, but I did not give the objections to reversionary transactions as reasons why the price has been reduced, but as reasons why it ought not to be. Mr. Berridge thinks that any scheme which may ultimately cause the burden to the seller of a reversion to exceed the ordinary terms, will not succeed, as it may deprive the seller of his future income. No doubt a seller wants to pay in future as little as he can; and the objection tells against any form of reversionary transaction, and against the method of accumulation most of all. However, taking the example in the paper, since £100 per annum is the whole difference between the ordinary and the special form, it may be questioned whether a seller would think the possible loss of this so great a drawback as to prevent his entering on the transaction.

The Computation and Adjustment of Probabilities derived from Observation. *By* WILHELM LAZARUS, F.I.A., *Hamburg.**

I.

ALL our empirical knowledge depends not only upon the exactness, trustworthiness, and frequency of the observations on which it is founded, but quite as much upon the possibility of drawing general conclusions from the observations. The great variety of form in which the results of observation appear, even independently of the diversity of the nature and the method of observation, will easily explain why the range of the conclusions to be drawn from the observations, differs widely according to the field of the science to which they refer. While the natural philosopher must be satisfied with the mere knowledge of a fact in a field which has not previously been investigated, he goes in other realms of science far beyond this first step of observation; he obtains an explanation about the causes of the phenomenons and the connection of the forces and the laws which govern the events that come under his observation. Such explanations cannot in general be drawn directly; they require a number of intermediate steps in which the observed result has to be critically examined and analyzed into its single elements. The proceeding to be adopted for this purpose depends upon the nature and form of the observation, and it would be a mistake to suppose that we can follow the same method for all kinds of observation.

Among the many kinds of observation there are at all events two classes which may be distinguished as essentially different, namely, those in which certain magnitudes are measured, and those in which the frequency of an occurrence is counted. The theory of probability deals more with the first class than with the second. After it had been recognized that measurements, even when made with the greatest care, are never free from small errors; and that, on account of these errors, different measurements of the same quantity do not agree with each other; and that measurement of a number of quantities related to each other in a certain known manner, brings out results opposed to the well known theorems of Mathematics; it became necessary to find an answer to the question,

* I am greatly indebted to Mr. T. B. Sprague for the valuable assistance he has rendered me by carefully revising the language of this paper, and suggesting the suitable English phrases in many cases where I had followed the German form of expression too closely.—W. L.

how these different results might be corrected (or adjusted) so as to furnish a system of values which under the existing circumstances might be considered as nearer to the truth than any other. A strict examination of the nature of these errors led at last to the method of adjustment, known as the method of least squares, which is considered with good reason to be one of the greatest triumphs of science. It has been applied with complete success to all kinds of questions in which the adjustment of inconsistent measurements is involved, and its rules have been very extensively used in practice even by many persons who have mechanically performed the proper calculations without knowing much of the theory on which they are based. This being the case, we need not wonder that the theory of least squares has been applied to observations for which it is not at all suitable. Misled by a certain outward similarity, various persons have tried to adjust by the method of least squares, the irregularities found to exist in the records of the frequency of certain occurrences as ascertained by counting. These irregularities have been very improperly called errors of observation, but they must rather be considered as caused by the insufficiency of the number of observations, and a wholly different method must be pursued for the purpose of removing them.

II.

The probability of the happening of a certain event being known, we can easily calculate the probability that, in a given number of cases, it will happen with any assigned degree of frequency, say, for instance, in one tenth of the whole cases; and if the number of cases is very large, there is so great a probability that the degree of frequency will lie between certain very narrow limits, that in popular language a transgression of these limits may be termed impossible. The probability that the frequency of the event will be an assigned quantity, diminishes as the number of cases increases, but the probability that it lies within the above mentioned limits, increases; so that, if we retain the same probability, the limits of the degree of frequency approach each other as the number of cases increases. If the number of cases is large, we may with some confidence predict that the event will happen with a frequency lying within certain limits, while nothing can be safely predicted when the number of cases is small. If the probability of the happening of the event is slightly altered, the

limits just mentioned are correspondingly altered, but most of the numbers lying between them remain the same as before; and therefore, if the probability of the happening of the event is only slightly altered, there will be but little alteration in the probability that the frequency of the event will lie between the same limits as before.

This consideration indicates already that if it should be at all possible to infer the probability of an event from the observation of the frequency of its happening in a known number of cases, that probability can be fixed only approximately. We shall see further on that no such inference can be absolutely drawn, but that we can only assign a certain probability to each inference, and that the most we can do is to fix certain limits within which this probability of the event will lie.

Before proceeding further, it will be useful to call to memory a few well known theorems in the theory of probability, of which we shall have to make use.

If the probability of an event happening is $\frac{M}{M+N}=p$, the probabilty of its failing will be $\frac{N}{M+N}=1-p$; and if h trials are made, then the probability that the event will occur m times, and fail n times (where $h=m+n$) is $\frac{1.2\ldots\ldots\ldots h}{1.2\ldots\ldots m\cdot 1.2\ldots\ldots n}p^m(1-p)^n$, which, applying Stirling's formula, becomes

$$\frac{h^{h+\frac{1}{2}}p^m(1-p)^n}{\sqrt{2\pi}m^{m+\frac{1}{2}}n^{n+\frac{1}{2}}}.$$

This probability has a maximum value when $m=hp$, and $n=h(1-p)$, and is then $=\frac{1}{\sqrt{2\pi hp(1-p)}}$.

If $m=hp+x$, $n=h(1-p)-x$, where x therefore signifies the deviation from the most probable occurrence, the probability for this difference x, which may be designated by Ω, becomes

$$=\frac{h^{h+\frac{1}{2}}p^{hp+x}(1-p)^{h(1-p)-x}}{\sqrt{2\pi}(hp+x)^{hp+x+\frac{1}{2}}\{h(1-p)-x\}^{h(1-p)-x+\frac{1}{2}}}$$

$$=\frac{1}{\sqrt{2\pi h\left(p+\frac{x}{h}\right)\left(1-p-\frac{x}{h}\right)}}\left\{\left(1+\frac{x}{ph}\right)^{-(hp+x)}\left(1-\frac{x}{(1-p)h}\right)^{-\{h(1-p)-x\}}\right\},$$

and consequently approximately*

$$\Omega = \frac{1}{\sqrt{2\pi h p(1-p)}} \cdot e^{-\frac{x^2}{2hp(1-p)}}.$$

$$\text{If } \frac{1}{hp(1-p)} = \mathrm{H}^2, \; \Omega \text{ becomes } = \frac{\mathrm{H}}{\sqrt{2\pi}} e^{-\frac{1}{2}\mathrm{H}^2x^2}.$$

The extreme values of x are $-hp$ and $h(1-p)$, corresponding to the values $m=0$ and $m=h$, and the integral $\int x^2 e^{-\frac{1}{2}\mathrm{H}^2x^2} d_x$ can therefore only be taken between the limits $-hp$ and $h(1-p)$; but it is obvious that the value of $e^{-\frac{1}{2}\mathrm{H}^2x^2}$ is extremely small for all values of x which are not very small, so that the value of the integral will not be materially altered if we take it between the limits $-\infty$ and $+\infty$ instead of $-hp$ and $h(1-p)$.

In the same way as is done in the method of least squares, we will put

$$\int_{-\infty}^{+\infty} x^2 \frac{\mathrm{H}}{\sqrt{2\pi}} e^{-\frac{1}{2}\mathrm{H}^2x^2} dx = \mathrm{M}^2,$$

and call M the mean deviation from the most probable occurrence —a deviation for which we must be prepared, if from the *known* probability of an event we infer that it will in h trials occur the most probable number of times.

It follows by known theorems

$$\frac{\mathrm{H}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\mathrm{H}^2x^2} dx = \mathrm{M}^2 = \frac{1}{\mathrm{H}^2} = hp(1-p),$$

and consequently the mean deviation from the most probable occurrence becomes $\mathrm{M} = \pm\sqrt{hp(1-p)}$, supposing that h is a large number. The mean deviation absolutely increases with h, but its ratio to h diminishes as h increases.

* This is found by developing

$$l\left\{\left(1+\frac{x}{hp}\right)^{-(hp+x)}\left(1-\frac{x}{h(1-p)}\right)^{-\{h(1-p)-x\}}\right\} = \frac{-x^2}{2hp(1-p)} + \frac{x^3(1-2p)}{6h^2p^2(1-p)^2}\cdots\cdots$$

consequently

$$\left(1+\frac{x}{hp}\right)^{-(hp+x)}\left(1-\frac{x}{h(1-p)}\right)^{-\{h(1-p)-x\}} = e^{\frac{-x^2}{2hp(1-p)}+\frac{x^3}{6h^2p^2(1-p)^2}}\cdots\cdots$$

and

$$\frac{1}{\sqrt{\left(p+\frac{x}{h}\right)\left(1-p-\frac{x}{h}\right)}} = \left\{p(1-p)+x\frac{1-2p}{h}-\frac{x^2}{h^2}\right\}^{-\frac{1}{2}} = \left\{p(1-p)\right\}^{-\frac{1}{2}} \times$$

$$\left\{1-\frac{1-2p}{hp(1-p)}x-\frac{x^2}{h^2p(1-p)}\right\}^{-\frac{1}{2}} = \left\{p(1-p)\right\}^{-\frac{1}{2}}\left\{1+\tfrac{1}{2}\frac{1-2p}{hp(1-p)}x\cdots\cdots\right\}.$$

III.

Before we proceed to our problem, we have to prove the following fundamental proposition.

If an observed event (either single or compound) may have arisen from any one of several different causes, all of which are *à priori* equally probable, then the probability that the event is the consequence of a *certain specified* cause, is equal to a fraction; the numerator of which is equal to the probability of the event on the supposition that the specified cause existed; and the denominator of which is equal to the sum of the probabilities of the event when each of the different possible causes is supposed to exist.

To prove this we will take the illustration of drawing white and black balls from different urns. Suppose there are z urns, each of which contains the same number of balls, $M+N$, which may be either white or black; and that in the first urn there are M_1 black and N_1 white balls; in the second urn, M_2 black and N_2 white balls; in the third urn, M_3 black and N_3 white balls; and so on. Let the observed event in question be the drawing of m black and n white balls from some one of the urns in $m+n$ drawings, the drawn ball being replaced in the urn after each drawing; and let it be required to find the probability that the drawings were made out of a specified urn. The observed event may be considered as possibly arising from any one of z different causes, each urn representing a different cause; and as the number of balls is the same in each urn, the probability that the drawing will take place from this or that urn is *à priori* the same.

We will now find, according to the above proposition, the probability that the event observed has arisen from the sth urn, putting for brevity the probability of drawing

	a white ball	a black ball
from the 1st urn	$\frac{N_1}{M_1+N_1}=y_1$	$\frac{M_1}{M_1+N_1}=1-y_1$
„ 2nd „	$\frac{N_2}{M_2+N_2}=y_2$	$\frac{M_2}{M_2+N_2}=1-y_2$
„ 3rd „	$\frac{N_3}{M_3+N_3}=y_3$	$\frac{M_3}{M_3+N_3}=1-y_3$
	and so on.	and so on.

The numerator is to consist of the probability of the observed event,

computed as if there existed only that single cause, the urn s; and is

$$=\frac{1.2\,.\,.\,.\,.\,.\,.\,.\,.\,(m+n)}{1.2\,.\,.\,.\,m\,1.2\,.\,.\,.\,.\,n}y_s^n(1-y_s)^m.$$

The denominator is to consist of the sum of the probabilities computed from all existing causes, and is therefore

$$=\frac{1.2\,.\,.\,.\,.\,.\,(m+n)}{1.2\,.\,.\,m\,1.2\,.\,.\,.\,n}\Big\{y_1^n(1-y_1)^m+y_2^n(1-y_2)^m+\ldots$$
$$+y_s^n(1-y_s)^m+\ldots\quad+y_z^n(1-y_z)^m\Big\};$$

whence the probability we seek is

$$\frac{y_s^n(1-y_s)^m}{y_1^n(1-y_1)^m+y_2^n(1-y_2)^m+\ldots+y_s^n(1-y_s)^m+\ldots\;+y_z^n(1-y_z)^m}.$$

We can prove the correctness of this expression by comparing the number of favorable cases with that of all possible cases.

Disregarding for a moment the circumstance that different urns exist, let us consider the probability of the observed drawing from one single urn, say the rth. It contains M_r black and N_r white balls. The number of all possible cases for a combination of $m+n$ balls out of M_r+N_r, if the drawing is performed in the way we have described, is $(M_r+N_r)^{m+n}$; and if this binomial is developed, its terms express the number of cases in which $m+n$, $m+n-1$, $m+n-2$, ... 2, 1, 0, black balls can be combined with 0, 1, 2, ... $m+n-2$, $m+n-1$, $m+n$, white balls. The exponent of M_r corresponds with the number of the black and the exponent of N_r with the number of the white balls, and consequently $\frac{1.2\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,(m+n)}{1.2\,.\,.\,.\,.\,.\,.\,m\cdot1.2\,.\,.\,.\,.\,.\,n}M_r^m\,N_r^n$ is the number of the favorable cases.

The probability of drawing m black and n white balls from the rth urn would be *à priori*

$$=\frac{\dfrac{1.2\,.\,.\,.\,.\,(m+n)}{1.2\,.\,.\,.\,m\,1.2\,.\,.\,.\,n}M_r^mN_r^n}{\{M_r+N_r\}^{m+n}}.$$

It has been observed that m *black and* n *white balls have been drawn,* and we have to determine the probability that the drawing was made from the sth urn. It is obvious that the number of favorable cases is

$$\frac{1.2\,.\,.\,.\,.\,(m+n)}{1.2\,.\,.\,m\,1.2\,.\,.\,.\,n}M_s^mN_s^n,$$

and the number of all possible cases, *after* m *black and* n *white balls have been drawn,* consists of the sum of the favorable cases from all urns, and is consequently

$$\frac{1.2\ldots(m+n)}{1.2\ldots m\,1.2\ldots n}\left\{M_1^mN_1^n+M_2^mN_2^n+\ldots+M_s^mN_s^n+\ldots+M_z^mN_z^n\right\}.$$

The probability that the event observed arose from the sth urn is consequently

$$\frac{M_s^mN_s^n}{M_1^mN_1^n+M_2^mN_2^n+\ldots\ldots+M_s^mN_s^n+\ldots+M_z^mN_z^n}.$$

This coincides with the expression found according to our proposition, as will be immediately seen if we divide the numerator and the denominator of the last expression by $(M+N)^{m+n}$, and, remembering that $M_1+N_1=M_2+N_2=\ldots\ldots\ldots=M+N$, introduce again the notation $y_1, y_2, \ldots\ldots$ The expression then becomes, as before,

$$\frac{y_s^m(1-y_s)^n}{y_1^m(1-y_1)^n+y_2^m(1-y_2)^n+\ldots\ldots+y_s^m(1-y_s)^m+\ldots\ldots+y_z^m(1-y_z)^n}.$$

IV.

We will illustrate this theorem by an example. Suppose we have four urns, which may be designated by A, B, C, D, and we know each urn to contain the same number of balls. Suppose we know besides that among each five balls there are in A one, in B two, in C three, in D four white balls, the other balls being black. In eight drawings from one of the urns, replacing each ball as drawn, three white and 5 black balls have been drawn. What is the probability that the drawing was made from urn C? This probability is according to our proposition

$$\frac{\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^5}{\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^5+\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^5+\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^5+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)^5}=\frac{864}{3896}.$$

In the same manner, we could find the probability that the drawing was made from urn A, $\frac{1024}{3896}$; from B, $\frac{1944}{3896}$; from D, $\frac{64}{3896}$. The probabilities are very different for the different urns, and if we are to draw a conclusion from the observed event to the urn from which it has taken place, we certainly will decide in favor of that urn, the supposition of which gives the greatest probability to the event observed; in our example this is the urn B.

Without making any change in the results, we might have

designated the urns, instead of A, B, C, D, by the proportions in which the white or black balls are contained therein; and might have asked, what was the proportion of the white balls which gave rise to the event observed? But this question is the same with the question, what probability of drawing a white ball may be considered as the cause of the observed event? In the case which we have chosen for our example, we are bound to choose among the only four existing probabilities. But this restraint does not prevail in all cases.

V.

As we have seen in the example just given, each possible probability for the happening of a single event may be considered as a cause for the event observed; and it is obvious we need not introduce the different urns to represent the causes. One urn will be quite sufficient, if the proportion of the white and black balls therein is unknown; and the different possible proportions, each corresponding to a different probability, may be considered the causes of the event. If the proportion of the white and black balls, or, in other words, the probability of drawing a white ball, is unknown, and if no observation has been made, there exists no reason to decide in favor of a certain probability, so that *à priori* each value of it is equally probable.

Let z be the number of balls which the urn contains, then white and black balls may be combined in the following ways,

0	white	and	z	black	balls
1	"	"	$z-1$	"	"
2	"	"	$z-2$	"	"
	⋮			⋮	
$z-2$	"	"	2	"	"
$z-1$	"	"	1	"	"
z	"	"	0	"	"

Let the probabilities of drawing a white ball according to these $z+1$ hypothesises, be $y_0, y_1, y_2, \ldots\ldots, y_{z-2}, y_{z-1}, y_z$; the probability of drawing a black ball becomes $1-y_1, 1-y_2, \ldots\ldots$ $1-y_{z-1}, 1-y_z$. It is obvious that y_0 and $1-y_z$ are equal to 0.

If the unknown existing probability of drawing a white ball, which must be contained among the values $y_0, y_1, \ldots\ldots y_z$, be designated by y, and if the drawing of n white and m black balls in the manner described has been observed, the probability of the

hypothesis that y is the probability of drawing a white ball, is according to the proposition before proved,

$$\frac{y^n(1-y)^m}{y_1^n(1-y_1)^m+y_2^n(1-y_2)^m+\ldots \qquad +y_{z-1}^n(1-y_{z-1})^m}.$$

The terms $y_0^m(1-y_0)^n$ and $y_z^m(1-y_z)^n$ are here omitted in the denominator, as they are both zero. This expression contains only one variable y; and if we are at liberty to dispose of it, we will certainly choose it of such value that the probability of our hypothesis assumes a maximum value. In other words, among the different hypothesises which we may advance as the cause of the event observed, we will choose the most probable, and this is the same hypothesis the supposition of which would give a greater probability to the event observed (if no observation had been made) than any other admissible supposition. As the denominator of the fraction above is constant, the fraction becomes a maximum when the numerator $y^n(1-y)^m$ is a maximum. In solving the problem, we must not forget that with reference to our example, we are not quite at liberty respecting the value of y, but that we are obliged to choose one of the values $y_0, y_1, \ldots\ldots y_z$.

VI.

Hitherto we have confined ourselves to the case of drawing white and black balls, but this has been only for the sake of simplicity. Our conclusions will apply equally well in other cases, supposing the conditions expressed in our general proposition are fulfilled. We will now take an example of another kind.

If it has been observed, that out of $m+n$ persons of a certain age, m have died within a year, and n have survived the year, what is the best hypothesis, founded upon this observation, for the probability that a person of the age in question will live for a year? If there existed no observation, every possible value of the probability considered to be the cause of the observed event, would be equally probable. After the observation has been made, the probability of each hypothesis can be determined; and that hypothesis is called the best one which has the greatest probability. The problem implicitly contains the supposition that the probability of living a year is considered to be the same for every person of the given age, so that all individual peculiarities in this direction are disregarded. From this point of view there is no difficulty in conceiving the probability of surviving a certain period to be analogous to that of drawing a white ball; but as this probability may have any

value between 0 and 1, we must, if we wish to follow up the analogy with the urn and balls, suppose the number of balls in the urn to be infinite. In this case, the numerator of the fraction which expresses the probability of the hypothesis y, remains unaltered, $y^n(1-y)^m$; but the denominator will now consist of the sum of an infinite number of terms of the form $y^n(1-y)^m$, where y continuously changes from 0 to 1. If we multiply the denominator by dy, it becomes $\int_0^1 y^n(1-y)^m dy$. We must multiply the numerator also by dy, in order that the value of the fraction may not be altered; and thus we get the probability of the hypothesis that y is the probability of living a year $= \dfrac{y^n(1-y)^m dy}{\int_0^1 y^n(1-y)^m dy}$.
The value of the definite integral $\int_0^1 y^n(1-y)^m dy$, can be found by partial integration, and is

$$= \frac{1.2 \ldots\ldots\ldots\ldots (m-1)m}{(n+1)(n+2)\ldots\ldots(n+m-1)(n+m)} \cdot \frac{1}{n+m+1},$$

and therefore depends only upon n and m: let us for brevity put it equal to $1 \div \text{A}$. Then the probability in question becomes $\text{A}y^n(1-y)^m dy$. In consequence of the factor dy, this probability is infinitely small; and we therefore cannot assign any finite probability to the hypothesis of any assumed value of y. But we can find certain limits u and v, such that the probability that y will lie between them, $\text{A}\int_u^v y^n(1-y)^m dy$, is so great that values of y which lie beyond these limits may be considered inadmissible.

We may also determine y so that $\text{A}y^n(1-y)^m dy$ is a maximum, and consider this value of y as the best supposition. Obviously the limits u and v will span the narrowest interval, when they include those values of y which give the greatest values to $\text{A}y^n(1-y)^m dy$; and from this point of view, too, that value of y which makes the probability mentioned a maximum, is an important one. In this case $y^n(1-y)^m dy$ must be a maximum, or $d\{y^n(1-y)^m\}=0$, whence $\dfrac{m+n}{y(1-y)}\left\{y-\dfrac{n}{m+n}\right\}=0$. As the first factor of the term on the left side cannot be 0, the second factor must be put $=0$, so that $y=\dfrac{n}{m+n}$, $1-y=\dfrac{m}{m+n}$. Hence, starting with the assumed observation, the best hypothesis we can make is that the probability of living a year at the age in question is $\dfrac{n}{m+n}$, and the probability of dying within the year $\dfrac{m}{m+n}$.

VII.

If the event observed is a more complicated one; if, for instance, we have observed that

out of m_1+n_1 persons of the age t_1, m_1 died in the following year, n_1 survived it,
„ m_2+n_2 „ „ t_2, m_2 „ „ n_2 „
„ m_3+n_3 „ „ t_3, m_3 „ „ n_3 „
⋮
„ $m_\mu+n_\mu$ „ „ t_μ, m_μ „ „ n_μ „

we should find by a process similar to the preceding that the probability of the hypothesis that y_1 is the probability of living a year,

$$\text{at the age } t_1=\frac{y_1^{n_1}(1-y_1)^{m_1}dy}{\int_0^1 y^{n_1}(1-y)^{m_1}dy}=A_1y_1^{n_1}(1-y_1)^{m_1}dy, \text{ suppose}$$

$$\text{that } y_2 \text{ „ „ } t_2=\frac{y_2^{n_2}(1-y_2)^{m_2}dy}{\int_0^1 y^{n_2}(1-y)^{m_2}dy}=A_2y_2^{n_2}(1-y_2)^{m_2}dy \text{ „}$$

$$\text{„ } y_3 \text{ „ „ } t_3=\frac{y_3^{n_3}(1-y_3)^{m_3}dy}{\int_0^1 y^{n_3}(1-y)^{m_3}dy}=A_3y_3^{n_3}(1-y_3)^{m_3}dy \text{ „}$$

$$\text{„ } y_\mu \text{ „ „ } t_\mu=\frac{y_\mu^{n_\mu}(1-y_\mu)^{m_\mu}dy}{\int_0^1 y^{n_\mu}(1-y)^{m_\mu}dy}=A_\mu y_\mu^{n_\mu}(1-y_\mu)^{m_\mu}dy \text{ „}$$

The probability that all these suppositions, $y_1, y_2, \ldots\ldots y_\mu$, for the different ages, may be made simultaneously, is equal to the product of these probabilities, the single parts of the event observed being independent of each other; it is

$$=A_1A_2\ldots\ldots A_\mu y_1^{n_1}(1-y_1)^{m_1}y_2^{n_2}(1-y_2)^{m_2}\ldots\ldots y_\mu^{n_\mu}(1-y_\mu)^{m_\mu}(dy)^\mu.$$

If this is to be a maximum,

$$y_1^{n_1}(1-y_1)^{m_1}y_2^{n_2}(1-y_2)^{m_2}\ldots y_\mu^{n_\mu}(1-y_\mu)^{m_\mu}$$

must be a maximum, consequently the first derivative must be zero. Taking the logarithms and differentiating, we find

$$\left(\frac{n_1}{y_1}-\frac{m_1}{1-y_1}\right)dy_1+\left(\frac{n_2}{y_2}-\frac{m_2}{1-y_2}\right)dy_2+\ldots\ldots+\left(\frac{n_\mu}{y_\mu}-\frac{m_\mu}{1-y_\mu}\right)dy_\mu=0,$$

or

$$\frac{m_1+n_1}{y_1(1-y_1)}\left\{y_1-\frac{n_1}{m_1+n_1}\right\}dy_1+\frac{m_2+n_2}{y_2(1-y_2)}\left\{y_2-\frac{n_2}{m_2+n_2}\right\}dy_2+\ldots\ldots$$
$$+\frac{m_\mu+n_\mu}{y_\mu(1-y_\mu)}\left\{y_\mu-\frac{n_\mu}{m_\mu+n_\mu}\right\}dy_\mu=0,$$

which may be written thus—

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)dy\right\}=0.$$

VIII.

We have now to consider two cases; according as (1) the probabilities $y_1, y_2, \ldots\ldots y_\mu$, are independent of each other, or (2) they are connected with each other by certain relations. In the first case, $y_1, y_2, \ldots\ldots y_\mu$, being independent of each other, the above equation leads to the μ equations,

$$\frac{m_1+n_1}{y_1(1-y_1)}\left\{y_1-\frac{n_1}{m_1+n_1}\right\}dy_1, \text{ which gives } y_1=\frac{n_1}{m_1+n_1},\ 1-y_1=\frac{m_1}{m_1+n_1};$$

$$\frac{m_2+n_2}{y_2(1-y_2)}\left\{y_2-\frac{n_2}{m_2+n_2}\right\}dy_2, \quad ,, \quad ,, \quad y_2=\frac{n_2}{m_2+n_2},\ 1-y_2=\frac{m_2}{m_2+n_2};$$

and so on.

The supposition that $y_1, y_2, \ldots\ldots\ldots y_\mu$, are independent of each other, allows no further "adjustment" of the result. The problem is completely and definitely solved. Whatever may be the irregularities in the values of $y_1, y_2, \ldots\ldots\ldots y_\mu$, we must accept them; no objection can be made to them, and there is no reason to alter the values found. No "adjustment" is permissible unless we admit that the probabilities found are to satisfy certain conditions, either expressly proved or tacitly assumed, in addition to the observed facts. The problem of adjustment will then consist in solving the equation

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)dy\right\}=0$$

in such a way that $y_1, y_2, \ldots\ldots y_\mu$, satisfy the proved or assumed conditions.

IX.

The relations between the probabilities $y_1, y_2, \ldots\ldots\ldots y_\mu$, or the conditions they are to fulfil, may be introduced in very different ways. The most complete way is when the probability (as for instance the probability of living a year) is assumed to be a function of the age of known form. Assume, then, that for all the ages under observation $y=f(\alpha, \beta, \gamma \ldots\ldots\ldots t)$ where the form of f is known and $\alpha, \beta, \gamma \ldots\ldots$ are constants to be determined so as to satisfy the equation

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{n+m}\right)dy\right\}=0$$

where we must substitute $df(\alpha, \beta, \gamma \ldots t)$ in the place of dy. Since $\alpha, \beta, \gamma \ldots\ldots$ vary independently of each other, we have to

compute the partial differential quotients of $f(a, \beta, \gamma \ldots . t)$ with reference to $a, \beta, \gamma \ldots .$ and introduce them into the above equation. This leads to the equations

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)\frac{dy}{da}\right\}=0$$

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)\frac{dy}{d\beta}\right\}=0$$

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)\frac{dy}{d\gamma}\right\}=0$$

. . . .

We thus have as many equations as there are constants, and the constants are to be computed from these equations.

X.

The solution of the equations depends on the form of the function f, and will differ according as the values $a, \beta, \gamma \ldots .$ appear in a linear form or not. But even in the first case, where $f(a, \beta, \gamma, \ldots . t)$ may be put $=a+\beta t+\gamma t^2+\ldots \ldots$, the factor $\frac{m+n}{y(1-y)}$ contains the unknown quantities $a, \beta, \gamma, \ldots \ldots$ in the denominator, that is, in non-linear form; and this obliges us to adopt a method of solution by approximation.

Let A be an approximate value of y, and let

$A_1+E_1=y_1$, $A_2+E_2=y_2$, $A_3+E_3=y_3$, &c., ..., it follows that

$$\frac{m+n}{y(1-y)}=\frac{m+n}{(A+E)(1-A-E)}=\frac{m+n}{A(1-A)+E(1-2A)-E^2}$$

$$=\frac{m+n}{A(1-A)\left\{1+\frac{E(1-2A)-E^2}{A(1-A)}\right\}}=\frac{m+n}{A(1-A)}\cdot\frac{1}{1+E\left(\frac{1}{A}-\frac{1}{1-A}\right)-E^2\left(\frac{1}{A}+\frac{1}{1-A}\right)}.$$

If A is very nearly equal to y, E will be so small that E : A and E : (1 − A) may be neglected compared with the value 1, and we may take $\frac{m+n}{A(1-A)}$ approximately equal to $\frac{m+n}{y(1-y)}$.

The partial differential quotients of $y=a+\beta t+\gamma t^2+\ldots \ldots$ are $\frac{dy}{da}=1$, $\frac{dy}{d\beta}=t$, $\frac{dy}{d\gamma}=t^2$, &c., and substituting these values in the principal equations, we get

$$\Sigma\left\{\frac{m+n}{\mathrm{A}(1-\mathrm{A})}\left(a\ \ +\beta t\ +\gamma t^2+\ldots\ldots\qquad -\frac{n}{m+n}\right)\right\}=0$$

$$\Sigma\left\{\frac{m+n}{\mathrm{A}(1-\mathrm{A})}\left(at\ +\beta t^2+\gamma t^3+\ldots\ldots\qquad -\frac{nt}{m+n}\right)\right\}=0$$

$$\Sigma\left\{\frac{m+n}{\mathrm{A}(1-\mathrm{A})}\left(at^2+\beta t^3+\gamma t^4+\ldots\ldots\qquad -\frac{nt^2}{m+n}\right)\right\}=0,\ \ \text{\&c.}$$

If, for brevity, we put

$$\left\{\frac{m+n}{\mathrm{A}(1-\mathrm{A})}\right\}^{\frac{1}{2}}=a\qquad\qquad \left\{\frac{n^2}{\mathrm{A}(1-\mathrm{A})(m+n)}\right\}^{\frac{1}{2}}=l$$

$$\left\{\frac{m+n}{\mathrm{A}(1-\mathrm{A})}\right\}^{\frac{1}{2}}t=b$$

$$\left\{\frac{m+n}{\mathrm{A}(1-\mathrm{A})}\right\}^{\frac{1}{2}}t^2=c,\ \text{\&c.},$$

and if we adopt the summing symbol introduced by Gauss, our equations take the form

$$[aa]\alpha+[ab]\beta+[ac]\gamma+\ldots\ldots\qquad-[al]=0$$

$$[ab]\alpha+[bb]\beta+[bc]\gamma+\ldots\ldots\qquad-[bl]=0$$

$$[ac]\alpha+[bc]\beta+[cc]\gamma+\ldots\ldots\qquad-[cl]=0,\ \text{\&c.}$$

This is the form well known in the method of least squares, and it is therefore superfluous to add more here about the solution. But we must bear in mind that these equations do not give the exact values of α, β, γ . . . . but only approximate values, which perhaps need further corrections, to be found in the same way. This results from the introduction of the approximate value A instead of y.

After having found α, β, γ, ... by the preceding equations, we compute y_1, y_2, ... y_μ, and compare them with the approximate values A_1, A_2, ... If the neglected quantities E_1, E_2, ... are too considerable, the method of solution is to be repeated with the adoption of the approximate values of y_1, y_2, ..., just found, and this must be continued until E_1, E_2, ... are really small enough to be neglected.

XI.

Generally the quantities $\frac{n_1}{m_1+n_1}$, $\frac{n_2}{m_2+n_2}$, ... will be a first approximation to y_1, y_2, ... if there have not been conditions introduced which forbid us to adopt them simultaneously. The difference between these values and y_1, y_2, ... must not be too

large, or else the probability of the occurrence of the event observed would become too small.

If we put

$y=\frac{n}{m+n}+\mathrm{E}$, consequently $1-y=\frac{m}{m+n}-\mathrm{E}$, it follows that

$$\frac{m+n}{y(1-y)}=\frac{m+n}{\left(\frac{n}{m+n}+\mathrm{E}\right)\left(\frac{m}{m+n}-\mathrm{E}\right)}=\frac{(m+n)^3}{mn+\mathrm{E}(m-n)(m+n)-\mathrm{E}^2(m+n)^2}$$

$$=\frac{(m+n)^3}{mn}\left\{1-\frac{\mathrm{E}(m^2-n^2)}{mn}+\frac{\mathrm{E}^2(m+n)^2}{mn}\ldots\ldots\right\},$$

or approximately $\frac{m+n}{y(1-y)}=\frac{(m+n)^3}{mn}$. Adopting the same development as shown in No. (x), we have

$$a=\frac{(m+n)^{\frac{3}{2}}}{\sqrt{mn}}$$

$$b=\frac{(m+n)^{\frac{3}{2}}t}{\sqrt{mn}} \qquad l=\sqrt{\frac{(m+n)n}{m}}$$

$$c=\frac{(m+n)^{\frac{3}{2}}t^2}{\sqrt{mn}}, \text{ \&c.}$$

Here too the solution must be repeated, if the differences between $y_1, y_2, \ldots\ldots$ and $\frac{n_1}{m_1+n_1}, \frac{n_2}{m_2+n_2}, \ldots\ldots$ respectively, prove too large to allow of their being neglected.

XII.

If the function f is such that the constants $a, \beta, \gamma, \ldots\ldots$ appear therein in a non-linear form, the solution is to be made similarly to the analogous case in the method of least squares. Approximate values $a_1, \beta_1, \gamma_1 \ldots\ldots$ must be introduced instead of $a, \beta, \gamma, \ldots\ldots$ and their corrections computed. If the approximate values $a_1, \beta_1, \gamma_1 \ldots\ldots$ are sufficiently near to $a, \beta, \gamma, \ldots\ldots$ the higher powers of the corrections may be neglected, and thus the problem is reduced to the preceding case.

XIII.

It is interesting to notice that our investigation has led to equations which have the greatest resemblance to those dealt with by the method of least squares, and that we have found these

equations, *without introducing any hypothesis*, a result unattainable in the method of least squares. The only difference between the two sets of equations lies in the quantity, which is termed in the method of least squares, "the weight of observation", and appears there as a quantity independent of the most probable value of the measured quantity sought, but here depends on the probability to be determined. It is obvious to ask whether there are not other relations between the adjustment of measured quantities and the conclusions from observed events, but it is beyond the scope of the present paper to enter into this question.

XIV.

The fact is, that the relation $y=f(\alpha, \beta, \gamma \ldots . t)$, which has been assumed to regulate the probabilities $y_1, y_2, , \ldots y_\mu$, is very often a mere supposition, the admissibility of which is to be examined with the aid of the observation made. If the probability of an event is y_1, and n_1+m_1 trials are to be made, the most probable case will be the happening of the event $y_1(n_1+m_1)$ times, its failure $(1-y_1)(n_1+m_1)$ times; but we must reckon, as we have seen (in II.), that there will be a deviation of $\pm\sqrt{(m_1+n_1)y_1(1-y_1)}$.

If observation has shown that the event has happened n_1 times, and failed m_1 times, then the deviation from the most probable occurrence is $n_1-y_1(n_1+m_1)=n_1(1-y_1)-m_1y_1$, and we have to examine whether this is within the limits of the mean deviation or not. If it is, we can make no objection to the hypothetical value y_1, as we could not have been surprised at the occurrence of such a deviation from the most probable case, even if we had known *for certain* that the probability of the event was y_1.

If we have to do with the concurrent probabilities, $y_1, y_2, \ldots y_\mu$, and $n_1+m_1, n_2+m_2, \ldots . n_\mu+m_\mu$, trials, we must examine our hypothesis with reference to the total result. The mean deviation of the whole compound event is $\pm\sqrt{\Sigma(m+n)y(1-y)}$. If the event has occurred respectively $n_1, n_2, \ldots n_\mu$ times, and failed $m_1, m_2, \ldots m_\mu$ times, the total deviation from the most probable case is $\Sigma\{n(1-y)\}-\Sigma(my)$; and our opinion about the admissibility of the hypothesis will be materially determined by the proportion this quantity bears to the mean deviation. The reason is easily explained. In the total deviation, the single deviations which lie on different sides neutralize each other, while those lying in the same direction are added together. Now it might very well

happen, that each single deviation is smaller than the mean one, but nevertheless their sum is very considerable; because by introducing our hypothesis all the deviations, or many of them, lie in the same direction, a circumstance which would dispose us to judge unfavourably of the hypothesis. On the other hand, it is not sufficient to examine only the total deviation. On account of the terms with opposite signs, this may show the favorable result 0, while nevertheless the single deviations transgress the mean one on different sides to such a degree that we should declare the hypothesis quite unallowable.

XV.

If we examine the factor $\frac{m+n}{y(1-y)}$ more closely, we easily perceive that it would have no influence on the result of our equations, if it remained constant for all the values, $y_1, y_2, \ldots y_\mu$.

If $$\frac{m_1+n_1}{y_1(1-y_1)}=\frac{m_2+n_2}{y_2(1-y_2)}=\ldots\ldots=\frac{m_\mu+n_\mu}{y_\mu(1-y_\mu)},$$

the equation $$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)dy\right\}=0,$$

leads to $$\Sigma\left\{\left(y-\frac{n}{m+n}\right)dy\right\}=0.$$

This simpler equation may be used in determining a first approximation, if the factor $\frac{m+n}{y(1-y)}$ may be really considered *approximately* constant. But we must not forget that we are only making a preparatory step, and that the factor $\frac{m+n}{y(1-y)}$ must not be neglected, if it assumes different values.

The quantity $\frac{1}{y(1-y)}$ has its minimum value 4, when $y=\frac{1}{2}$. If m and n are constant quantities, $\frac{m+n}{y(1-y)}$ also has its minimum value, $4(m+n)$, when $y=\frac{1}{2}$. For every other value of y, $\frac{m+n}{y(1-y)}$ is greater than $4(m+n)$, and the more so, the nearer y approaches to the limits 0 and 1. An increase or a decrease in the number of the observed events has consequently a greater influence on the result, the more the probability y is different from $\frac{1}{2}$, and it is obvious that the corrections must become smaller, if the probability to be corrected is itself a small one. That this is

the case both when y approaches the limit 0 as well as 1, arises from the circumstance that the probability $(1-y)$ is always opposed to y, so that one probability always approaches 0 as the other approaches 1, and that the same correction, only in an opposite direction, is applied to y and to $1-y$.

XVI.

A geometrical interpretation of the terms of our formula will facilitate its comprehension. Starting again from the observed facts, that the event happened $n_1, n_2, \ldots . n_\mu$ times, and failed $m_1, m_2, \ldots m_\mu$ times, in $m_1+n_1, m_2+n_2, \ldots m_\mu+n_\mu$ trials, we will denote the unknown probabilities of the event in question by $y_1, y_2, \ldots . y_\mu$, and connect them with each other by the equation $y=f(\alpha, \beta, \gamma \ldots . t)$. As we have already seen, the most probable values for $y_1, y_2, \ldots y_\mu$, would be

$$\frac{n_1}{m_1+n_1}, \frac{n_2}{m_2+n_2}, \ldots \frac{n_\mu}{m_\mu+n_\mu},$$

if they were independent of each other.

Taking a pair of rectangular axes, take $t_1\ t_2 \ldots t_\mu$ as the abscissas, and $\frac{n_1}{m_1+n_1} \frac{n_2}{m_2+n_2} \ldots \frac{n_\mu}{m_\mu+n_\mu}$ as ordinates, and join the extremities of the ordinates by straight lines; then the ordinates of the broken line will represent the probabilities y, assuming there is no relation between them. But the construction is founded on the assumption that these probabilities are a function of t varying if t varies; and our construction takes no account of the variable trustworthiness which is to be attributed to the different values of y, and which is expressed by $\frac{m+n}{y(1-y)}$. The representation which the broken line gives us is only approximately correct. We know that we cannot assume that always the most probable case, which may be derived from a probability, will occur. Instead of the broken line, we shall do better to imagine a zone, extending on both sides of the line, and the true representation of y as a curve within this zone. The breadth of the zone will depend on the number of trials made, and on the value of the probability y; it will be a variable one in different parts of the line.

But as the single probabilities are to satisfy the condition $y=f(\alpha, \beta, \gamma \ldots t)$, we must determine the constants $\alpha, \beta, \gamma \ldots$, so that the values of $y_1, y_2, \ldots$ derived from the equation

$y=f(\alpha, \beta, \gamma \ldots t)$, are more probable than those from an equation with any other values of these constants.

If we construct now within our system of ordinates the curve determined by the equation $y=f(\alpha, \beta, \gamma \ldots t)$, it will agree the more closely with the broken line, the more the events observed correspond with the assumed hypothesis. The deviation of these probabilities y from what would be their most probable values, if they were not connected with each other, $y-\frac{n}{m+n}$, appears in the construction, while it excludes again the factor $\frac{m+n}{y(1-y)}$ with which these deviations have been multiplied in our equation.

XVII.

We will now illustrate the preceding considerations by some applications. Let us suppose two series of observations, in the first of which n_1 white and m_1 black balls have been drawn, and in the second one n_2 white and m_2 black balls. The relation between the two series consists in the supposition that the proportion of white and black balls in the urn has been the same in both series.

Consequently the probability y_1 becomes equal y_2.

If we put $y=a$, $\frac{dy}{da}=1$, in the equation

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)dy\right\}=0,$$

we get
$$\Sigma\frac{m+n}{a(1-a)}\left\{a-\frac{n}{m+n}\right\}=0;$$

or, suppressing the constant factor, $a\frac{1}{(1-a)}$,

$$\Sigma(m+n)\left(a-\frac{n}{m+n}\right)=0;$$

or, as there are only two terms,

$$(m_1+n_1)\left(a-\frac{n_1}{m_1+n_1}\right)+(m_2+n_2)\left(a-\frac{n_2}{m_2+n_2}\right)=0.$$

If we unite what can be united, this gives

$$(m_1+n_1+m_2+n_2)a-n_1-n_2=0;$$

consequently
$$a=\frac{n_1+n_2}{m_1+n_1+m_2+n_2}.$$

We can in this case obtain the same result by a direct solution of the problem. Since the probability of drawing a white ball is the same in both series of observations, they may be united, so that they form only one single series, wherein $(m_1+n_1+m_2+n_2)$ trials, (n_1+n_2) white, (m_1+m_2) black balls have been drawn.

The probability of the hypothesis that this event has been caused by a probability y of drawing a white ball is

$$\frac{y^{n_1+n_2}(1-y)^{m_1+m_2}dy}{\int_0^1 y^{n_1+n_2}(1-y)^{m_1+m_2}dy},$$

and this probability has its maximum value, when

$$y=\frac{n_1+n_2}{m_1+n_1+m_2+n_2}.$$

It would be quite a mistake to look on the mean value between $\frac{n_1}{m_1+n_1}$ and $\frac{n_2}{m_2+n_2}$ as the one which corresponds with the most probable hypothesis. That has, however, been occasionally done by some authors, who have put $y=\frac{1}{2}\left\{\frac{n_1}{m_1+n_1}+\frac{n_2}{m_2+n_2}\right\}$. This value coincides with the one found above only in one single case, which we will investigate for the purpose of explaining how it came that the mistake mentioned has been made. Assume, then, that

$$\frac{1}{2}\left\{\frac{n_1}{m_1+n_1}+\frac{n_2}{m_2+n_2}\right\}=\frac{n_1+n_2}{m_1+n_1+m_2+n_2},$$

and for brevity put $\frac{n_1}{m_1+n_1}=a$, $\frac{m_2}{m_2+n_2}=b$;

so that $$m_1+n_1=\frac{n_1}{a},\ m_2+n_2=\frac{n_2}{b}.$$

We get by substitution

$$\frac{1}{2}a+\frac{1}{2}b=\frac{n_1+n_2}{\frac{n_1}{a}+\frac{n_2}{b}}=\frac{(n_1+n_2)ab}{n_1b+n_2a}$$

$$\frac{1}{2}(n_1+n_2)ab+\frac{1}{2}n_1b_2+\frac{1}{2}n_2a^2=(n_1+n_2)ab$$

$$n_1b^2+n_2a^2=n_1ab+n_2ab$$

$$n_1b(b-a)=n_2a(b-a)$$

$$n_1b=n_2a \text{ or } n_1\frac{n_2}{m_2+n_2}=n_2\frac{n_1}{m_1+n_1},$$

consequently $$m_1+n_1=m_2+n_2.$$

It thus appears that the mean value of $\frac{n_1}{m_1+n_1}$ and $\frac{n_2}{m_2+n_2}$ can be considered the most probable value, only when the number of trials has been the same in both series of observations. In that case y becomes $=\frac{n_1+n_2}{2(m_1+n_1)}$, and the same result follows by our method of adjustment.

If the number of trials is different in the two cases, the method of means is not applicable. The number of trials is decisive with reference to the trustworthiness of the probabilities found, and the method of means leaves out this important element, considering both probabilities as equally trustworthy, which is allowable only if they have been derived from an equal number of trials.

XVIII.

If instead of two series of observations there exist μ series, all connected with each other by the supposition that in all series the proportion of white and black balls has been the same, or, in other words, that the probability of drawing a white ball has always been the same, then the equation

$$\Sigma\left\{\frac{m+n}{y(1-y)}\left(y-\frac{n}{m+n}\right)dy\right\}=0,$$

y being $=a$, becomes

$$\Sigma\left\{(m+n)\left(a-\frac{n}{m+n}\right)\right\}=0$$

$$a=\frac{\Sigma(n)}{\Sigma(m+n)}.$$

The correctness of this proposition will be acknowledged immediately, if we recall to mind that we may divide each series of observations into different parts, and that our conclusions must be the same, whether we derive them from the undivided series of observation or from the whole number of parts of which it is composed.

XIX.

It cannot be denied that the application of the above principles to find the most probable hypothesis, leads to troublesome and laborious calculations, especially if the probabilities to be found are of a complicated nature. But this circumstance is no sufficient justification for putting in the place of logical reasoning a more or less

arbitrary and mechanical process, which, although it may have the advantage of great simplicity and shortness, and may give a very regular series of numbers, does not offer the least certainty that the values found are the best, that is to say, the most probable ones to be derived from the observation in connection with the stipulated conditions.

For the solution of our problem, it is above all necessary that we should be clear about the conditions which are to be satisfied by the probabilities sought; for the whole problem of adjustment, as already pointed out, depends on the conditions. The introduction of these conditions is the only justification for our altering the values of $y_1, y_2, \ldots y_\mu$, which would be the most probable ones if the quantities were independent of each other. If no such conditions had been introduced, there would have been no reason to forbid any irregularity, however great. If the regularity which is desired in the series $y_1, y_2, \ldots y_\mu$, is the consequence of conditions apart from observation, it will not be denied that these conditions must be *precisely defined,* and expressed in such a form that they can be introduced into the process of calculation. Furthermore, if, as we have seen, a correct method of finding the best hypothesis, cannot dispense with taking into consideration the frequency of the trials made, we must reject every method which does not take account of that frequency. The methods of adjustment best known, besides being more or less arbitrary and mechanical, take no account of the frequency of the observed events in connection with the probability in question. This remark refers also to the calculations made for the graduation of mortality tables. Most of them have been applied to a mortality table fictitiously made up, while they ought to have been applied to the probabilities and the numbers observed.

Generally this mortality table has been calculated by means of the unadjusted probabilities of surviving a year, as they would follow from observation, if we suppose each probability to be independent of all the other ones. A table of the number living at each age having been formed by multiplying the probabilities, and transforming the product into whole numbers by multiplying with a common denominator, the further task consisted in removing the irregularities of this table on the basis of any suppositions whatever, or in a more or less mechanical way by correcting the numbers of persons living. The series thus to be corrected takes no notice at all of the frequency of the observations, all the probabilities used in framing it having been considered as of the same trust-

worthiness. But even those methods which apply the adjustment immediately to the probabilities, ignore the important influence of the frequency of the observation.

XX.

Finally, we intend to glance at some of the methods of adjustment recommended for the graduation of mortality tables.

1. Method of Filipowski (*The Insurance Record*, of 9th Dec. 1870, p. 393). This is applied to a table of the numbers living, constructed in the manner above described. The influence of the frequency of the observations in connection with the probability of surviving a year is thus entirely neglected.

Let $l_0\ l_1\ l_2 \ldots$ be the unadjusted numbers living according to the table, $l'_0\ l'_1\ l'_2 \ldots$ the adjusted numbers. Now Filipowski assumes

$$l_{\frac{1}{2}}=\frac{l_0+l_1}{2},\quad l_{1\frac{1}{2}}=\frac{l_1+l_2}{2},\quad l_{2\frac{1}{2}}=\frac{l_2+l_3}{2},\quad \text{and so on;}$$

and from this he deduces

$$l'_1=\tfrac{1}{2}\left\{\frac{l_0+l_1}{2}+\frac{l_1+l_2}{2}\right\}=\frac{l_0+2l_1+l_2}{4}$$

$$l'_2=\tfrac{1}{2}\left\{\frac{l_1+l_2}{2}+\frac{l_2+l_3}{2}\right\}=\frac{l_1+2l_2+l_3}{4},\quad \text{and so on.}$$

If the numbers $l'_1\ l'_2 \ldots$ are not sufficiently regular, this process is repeated as often as is necessary.

It cannot be denied that this method is a mere mechanical one. It is not founded on principles clearly defined, and it removes all irregularities which the observation brings out, without caring whether these irregularities are founded in the nature of the event, or whether they appear as a consequence of the insufficiency of the facts observed.

2. Method of Wittstein (*Mathematische Statistik*, p. 30, Hannover, 1867*). Wittstein starts with the most probable values, $\frac{n_1}{m_1+n_1}, \frac{n_2}{m_2+n_2}, \ldots$, and with the object of removing the irregularities in them, he considers each y to be the arithmetical mean of 5 of these values, where the index of y corresponds to that of the middle term of the 5 numbers. If the regularity acquired by this process is not sufficient, the process is repeated.

* A translation of this treatise will be found in vol. xvii of this *Journal*, pages 178, 355, 417.—ED. *J.I.A.*

Thus Wittstein finds the first time, putting for brevity $\frac{n}{m+n}=p$,

$$y_r=\frac{p_{r-2}+p_{r-1}+p_r+p_{r+1}+p_{r+2}}{5},$$

and after repeating the process,

$$y_r=\frac{p_{r-4}+2p_{r-3}+3p_{r-2}+4p_{r-1}+5p_r+4p_{r+1}+3p_{r+2}+2p_{r+3}+1p_{r+4}}{25}.$$

The same objections which we had to make to the method of Filipowski apply also to this. Wittstein himself declares (p. 31) that any other number of terms might be chosen instead of 5, the number he has adopted to form the arithmetical mean. He thus tacitly admits that his process is an arbitrary one. Two computers, of whom one should apply the number 5, the other a different number of terms, would obtain different results; but each of them claims his solution to be the correct one.

The argument upon which this method is founded, is darkened by the consideration that the condition established does not refer to the probabilities, $y_1, y_2, \ldots\ldots$ finally to be found, but to one value of y and to a series of values of $n:(m+n)$. Thus in reality no connection between the different values of y is established. It would be quite different, if the condition were, that 5 values of y are to form an arithmetical series; and we are tempted to presume that such a demand has been the real foundation of Wittstein's method, and has then been modified, to avoid the breaks which would appear in advancing from one arithmetical series to the next one. If we establish this condition, and if we apply our method of adjustment explained before, putting $y=a+6t$, but excluding the factor $\frac{m+n}{y(1-y)}$, we should find, in conformity with Wittstein,

$$y_r=\frac{p_{r-2}+p_{r-1}+p_r+p_{r+1}+p_{r+2}}{5},$$

but our values for p_{r-} , p_{r-1}, p_{r+1}, p_{r+2}, would not correspond with what he finds, because with us these values are to be regulated by the same condition, which is not the case with him.

When Wittstein's process is repeated, y_r comes to depend on 9 values of p. If we had established the condition that 9 values of $n:(m+n)$ were to be an arithmetical series, our method of adjustment (the factor $\frac{m+n}{y(1-y)}$ being always excluded) would bring out

$$y_r = \frac{p_{r-4}+p_{r-3}+p_{r-2}+p_{r-1}+p_r+p_{r+1}+p_{r+2}+p_{r+3}+p_{r+4}}{9};$$

but if we had proceeded in our solution in an erronéous way, so as to seek

a mean value from $p_{r-4}p_{r-3}p_{r-2}p_{r-1}p_r$,
another one from $p_{r-3}p_{r-2}p_{r-1}p_rp_{r+1}$,
another one from $p_{r-2}p_{r-1}p_rp_{r+1}p_{r+2}$,
another one from $p_{r-1}p_rp_{r+1}p_{r+2}p_{r+3}$,
another one from $p_rp_{r+1}p_{r+2}p_{r+3}p_{r+4}$,

and had finally determined y_r as the mean value of the 5 means before, then we would come to the same result as Wittstein, because now we have taken into consideration p_{r-4} and p_{r+4} simply, p_{r-3} and p_{r+3} twice, p_{r-2} and p_{r+2} three, p_{r-1} and p_{r+1} four, and p_r five times, which proceeding would not be justified at all. This consideration will throw some light upon the method of Wittstein. The method of Finlayson has so much resemblance with that of Wittstein, that it needs no separate discussion.

3. Method of Woolhouse (*Journal of the Institute of Actuaries*, vol. xv, p. 390). This method, like the two former, takes no account of the degree of trustworthiness of the single probabilities as derived from the frequency of the observations. It is applied to the numbers of persons living, in a mortality table constructed as before described. Taking 15 consecutive values of l, and grouping them $l_1l_6l_{11}$, $l_2l_7l_{12}$, $l_3l_8l_{13}$, $l_4l_9l_{15}$ and $l_5l_{10}l_{15}$, it considers the three l's belonging to each group as ordinates of a curve of the second degree, and calculates from each group the ordinate corresponding to the absciss for l_8, determining then l'_8 as the mean value of these five ordinates. This is open to the same objections as the other methods. It is arbitrary, and the condition does not connect the final values of y.

Possibly the original idea was that 15 consecutive values of y were to be an arithmetical series of the second order; or, in geometrical language, the ordinates of a curve of the second degree. If this were the case, the 5 curves which correspond to the 5 groups, would form one single curve of the second degree, and l'_8 would coincide with l_8. But the 5 curves could also all go through the same point, the ordinate of which is l_8, without being parts of one single curve of the second degree, and then too l'_8 would be $=l_8$, that is to say, the condition would be fulfilled without a correction.

All these different methods seem to have sprung from the same source; they resemble each other very closely, not only in their defects, but also in their processes, and, from a theoretical point of view, they are all inferior to the method of adjustment explained in this paper.

Hamburg, September 1877.

On the Analogy between an Annuity-Certain and a Life Annuity.

By GEORGE KING, F.I.A.

THE strict analogy which exists between these two classes of annuities has doubtless received the consideration of the majority of students, but I am not aware that it has ever in print had that attention which it deserves; and at any rate our usual text books hardly even refer to it. Although it is shown, in the short but suggestive article of the late Prof. De Morgan (vol. iv, p. 277), in the most general way, that years, or periods of time, may be interchanged with any other kind of status whatsoever, whether certain or contingent; yet, as successive statuses are almost exclusively spoken of, the less complex cases are liable to escape notice. Under these circumstances, the following brief notes may not be thought superfluous.

In comparing an annuity-certain with a life annuity, we can look at it from two points of view. We can consider that the status of n years certain terminates just after the expiry of n years, or, symbolically, that it consists of a period of $n+0$ years; and that the annuity of 1 is payable at the end of each year as long as the status exists, nothing being due at the end of the year in which it fails. Or we may imagine the status to terminate just before the n years run out, to consist, that is, of $n-0$ years; and the annuity to be composed of $n-1$ payments made at the end of the several years while the status lasts, together with a payment at the end of the year in which the status fails. Either of these alternative modes of regarding the annuity-certain may be selected, but perhaps the second will be found the more convenient, and it will be adopted in what follows.

We have the expression for an annuity-certain,

$$a_{\overline{n}|}=\frac{1-v^n}{i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where, on the supposition made as to the time of failure of the

status, v^n is the present value of 1 to be received at the end of the year in which the status terminates. To place the life annuity on the same footing as the annuity-certain, we must provide for 1 to be received at the end of the year of death—that is, we must treat of the annuity $v(1+a_x)$, an annuity payable at the end of each year, if the life was in being at the commencement of the year. Suitably modifying the usual equation between a_x and A_x, we have the expression for the annuity now under consideration,

$$v(1+a_x)=\frac{1-A_x}{i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which is seen to be precisely similar in principle to expression (1), A_x being the present value of 1 to be received at the end of the year in which the status terminates, thus corresponding exactly to v^n, and the other involved quantities being identical. Since then $a_{\overline{n-1}|}$ (which is $a_{\overline{n}|}$ less its last payment) and v^n bear the same relation to each other as do a_x (which is $v(1+a_x)$ less its last payment) and A_x, it follows that Orchard's tables apply to annuities-certain. Having given $a_{\overline{n-1}|}$, we can by inspection find v^n, and *vice versâ*.

Again, we have another expression for the value of an annuity-certain,

$$a_{\overline{n}|}=\frac{1}{\frac{1}{s_{\overline{n}|}}+i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $s_{\overline{n}|}$ represents the annuity accumulated, and consequently $1:s_{\overline{n}|}$ the sinking fund per unit which will replace the capital at the end of the period. It should be noticed that the payments of the sinking fund take place at the *end* of each year, and that the last instalment falls due at the end of the year in which the status fails. The corresponding expression for a life annuity is

$$v(1+a_x)=\frac{1}{P_x(1+i)+i} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Once more we see the analogy. Expressions (3) and (4) have the same import. $P_x(1+i)$ is the premium per unit, payable at the *end* of each year, including the year in which the status fails, to replace the capital at the end of the period, and it therefore has the same operation as the sinking fund; and, as before, the other quantities involved are identical.

We have seen that $a_{\overline{n-1}|}$ corresponds to a_x, and since $P_x(1+i)$ corresponds to $1:s_{\overline{n}|}$, or P_x to $v:s_{\overline{n}|}$, it follows that if Orchard's

tables of annual premiums be entered with $a_{\overline{n-1}|}$, we shall obtain $v : s_{\overline{n}|}$, that is, the sinking fund payable at the *beginning* of each year to provide for the return of capital at the end of n years. In interest tables, where the function appears at all, it is usual to display only the sinking fund payable at the *end* of the year, though in practice it is sometimes requisite to find the sinking fund payable at the beginning of the year. A reference to Orchard's tables may occasionally involve less trouble than going through the calculation which would otherwise be necessary.

The late Mr. Peter Hardy showed (vol. i, p. 4 of part 2) that to find the value of an annuity-certain, to pay the purchaser one rate i, on his investment, and to replace his capital at another rate, i', we must in expression (3) calculate $1 : s_{\overline{n}|}$ at i', the accumulative rate, and for i use the remunerative rate. Similarly, Mr. Jellicoe has shown that to find the value of a life annuity to pay certainly a given rate of interest, we must in the expression $\dfrac{1}{\mathrm{P}_x + d}$ take P_x as a practical office premium, and d at the rate of interest to be realized. In equation (4) the corresponding alteration is to substitute for $\mathrm{P}_x(1+i)$ such a premium payable at the end of each year as would be charged by an office willing to agree to the condition, and for i the rate of interest to be realized. Therefore, as regards practical transactions also, expressions (3) and (4) are proved to be exactly parallel.

Mr. Makeham, in his masterly paper on the "Theory of Annuities-Certain" (vol. xiv, p. 189), gives some formulas of perfectly general application. He shows that if there be an annuity for n years the successive payments of which are $u_1, u_2, u_3, \ldots u_n$, its present value is

$$\overset{1}{\mathrm{V}}_n u_1 + \overset{2}{\mathrm{V}}_n \Delta u_1 + \overset{3}{\mathrm{V}}_n \Delta^2 u_1 + \text{ \&c.},$$

where the symbols $\overset{1}{\mathrm{V}}_n$, $\overset{2}{\mathrm{V}}_n$, &c., represent the values of annuities of the 1st order, 2nd order, &c. By the law which connects the different orders of annuities, every payment of an annuity of one order is the difference of the corresponding payment of the next higher order: and from this it follows that in any order, say the tth, all the payments up to and including the $(t-1)$th are equal to 0; the tth payment is unity; and the succeeding payments are formed by adding continuously the successive payments of the $(t-1)$th order. When the same principles are applied to life contingency tables, we find that the annuity of the 1st order is represented by N_x or M_x, according as annuities or assurances

are in question: that an annuity of the 2nd order can be denoted by S_{x+1} or R_{x+1} as the case may be; an annuity of the 3rd order by ΣS_{x+2} or ΣR_{x+2}, where Σ means the sum of the values of S or R from that at age $x+2$ to the greatest age; and so on. For the value of a life annuity the successive payments of which are u_1, u_2, u_3, &c., we have therefore the simple expression,

$$\text{Value} = \frac{u_1 N_x + \Delta u_1 S_{x+1} + \Delta^2 u_1 \Sigma S_{x+2} + \text{ \&c.}}{D_x}.$$

The above formula furnishes a clue to guide in the solution of complex problems. Take, for instance, the following:—

An assurance is granted on the life of (x) for 100 the first year, 101 the second year, 104 the third year, &c., and generally for $\{100+(n-1)^2\}$ should death occur in the nth year. The annual premium is always to bear to the sum assured the ratio κ. Find κ.

In the above question $u_1=100$, $\Delta u_1=1$, $\Delta^2 u_1=2$, and $\Delta^3 u_1=0$. Applying the formula, we have

$$\text{Benefit side} = 100M_x + R_{x+1} + 2\Sigma R_{x+2}.$$

$$\text{Payment side} = \kappa(100N_{x-1} + S_x + 2\Sigma S_{x+1}).$$

Whence
$$\kappa = \frac{100M_x + R_{x+1} + 2\Sigma R_{x+2}}{100N_{x-1} + S_x + 2\Sigma S_{x+1}}.$$

By viewing Mr. Makeham's successive orders of annuities as the equivalents of the commutation columns, the extended scope of his paper receives illustration. The simplicity secured for life contingencies by means of the columnar method is at once reflected back on questions relating to fixed terms of years, as in the following example:—

A sum receivable at the end of n years is to be bought by an annual premium, the first payment, π, to be made now, and afterwards at the beginning of each year a regularly diminishing amount; the last premium being paid at the beginning of the nth year, after which the premium becomes extinct. Find π.

If a life contingency were involved, we should have

$$\pi = \frac{D_{x+n}}{{}_{|n}N_{x-1} - \frac{1}{n}\,{}_{|n}S_x},$$

where ${}_{|n}N_{x-1} = N_{x-1} - N_{x+n-1}$ and ${}_{|n}S_x = S_x - S_{x+n} - nN_{x+n-1}$.

Remembering now that $\overset{1}{V}$ corresponds to N_x, and consequently $(1+i)\overset{1}{V}_n$ to N_{x-1}, we have, for the value of π,

$$\pi = \frac{v^n}{(1+i)\left(\overset{1}{\mathrm{V}}_n - \frac{1}{n}\overset{2}{\mathrm{V}}_n\right)}.$$

In cases where life is involved, the columns N, S, ΣS, &c., vary somewhat in meaning according as the transaction is to extend over the whole of life or to be confined to a limited number of years; but with Mr. Makeham's annuities this difficulty is not encountered. In $\overset{1}{\mathrm{V}}_n$, $\overset{2}{\mathrm{V}}_n$, &c., all payments cease after the nth. In N_x, S_x, &c., the payments are naturally extinguished at the end of life, but if only a limited term of years be in question, the same result must be attained by assigning to the symbols suitable values.

What to do with Insolvent Life Companies.

The following Article from the *Spectator* of New York may be compared with Mr. Sprague's paper *On the Liquidation and Reconstruction of an Insolvent Life Insurance Company,* vol. xvi, p. 229, and with Mr. D. Pitcairn's letter *On the Liquidation of an Insolvent Life Office,* vol. xv, p. 385. Our readers will no doubt be much interested to learn that the legislature of the State of New York has recognized the principle of paying the policyholders of an insolvent insurance company their dividend in the shape of insurance instead of cash.

The legislature of the State of New York did a good many foolish things last winter, as the readers of the *Spectator* have been duly informed; but it did one exceedingly good thing. It passed a law the provisions of which should be made available in the interests of policyholders in insolvent life insurance companies.

The winding up of a company by distributing its assets among policyholders on the basis of the reserve on their policies, is manifestly and indisputably the most disadvantageous thing that can be done for policyholders. All the assets must be converted into cash, and this process at any time, and more especially at this time, when prices are depressed and real values are uncertain—rather because real values are not present market values—is necessarily a slow process, and one which must result in very great sacrifice. This alone would be a sufficient reason for adopting some other plan of settling the affairs of an insolvent insurance company. But there is another, and from some points of view a much stronger reason, why such a distribution of assets is undesirable and inequitable to policyholders. The policyholders have paid for insurance and should receive insurance for their payments. Many of the lives have become impaired, and could not obtain insurance at any price; but in the mass of risks carried by the

company in which they were insured before they were impaired, they will not increase the average mortality to the injury of the company. Provision has been made for just the impairment that has taken place in their lives, and the company can afford to carry them if it have the lives taken at the same time and previously, which have not become impaired.

But whatever are the facts in this regard, there can be no question of the hardship of giving policyholders their share of the assets of the company, as ascertained by the reserve on their respective policies. It is to give them a mere pittance, and at the end of a long period. What then shall be done? The law alluded to provides a method of meting out exact justice to all the policyholders, as far as the assets of the company admit of justice to them, and of giving them just what they have paid their money for, and that is insurance payable at a specified time or on the death of the insured. And this is what should be done. The following is the law in full:—

"*Section* 1.—No life insurance company organized under the laws of this State, shall insure any of its outstanding risks or policy obligations in any other life insurance company except as hereinafter provided.

"*Section* 2.—Any life insurance company, organized under the laws of this State, is hereby authorized and empowered to reinsure the whole or any part of any policy obligation in any other company or companies, provided the written consent of the owner of such policy so reinsured shall first be obtained to such reinsurance.

"*Section* 3.—It shall be lawful for any receiver of any life insurance company organized under the laws of this State, to reinsure upon the written consent of the superintendent of the insurance department and the attorney-general, the whole of the policy obligations of such company, in any solvent company or companies, organized under the laws of this State, whenever the assets of the company of which he is receiver are sufficient to effect such reinsurance; and whenever such assets are not sufficient to effect such reinsurance, such receiver, upon the like consent as above provided, may reinsure a percentage of each and every policy obligation outstanding in such company to the extent that the assets of such company may be sufficient to effect such reinsurance, provided, however, that no contract effecting such reinsurance shall be entered into except in pursuance of an order of the court in which such receiver was appointed, directing reinsurances authorized by this section, and establishing the general form of the contract to effect the same."

We repeat this is what should be done with all the insolvent companies now in the hands of receivers, or which may hereafter go into the hands of receivers. Here is plain smooth sailing. No litigation, no unnecessary expense, no hardship towards policyholders, except that they will have a less amount of insurance hereafter, but they will at the same time pay proportionately less premium. Thus their loss will be confined to the sum they have hitherto paid for insurance beyond this time in excess of the amount for which their policies are continued. And this is a very trifling loss compared with the loss inflicted upon them by any other method of closing up a company.

We can conceive of no objection to this plan of reinsurance—in whole, if practicable, *pro rata*, if necessary,—unless such objection come from the receivers, who would thus be deprived of long-continued occupation, more or less lucrative. And if we can judge by the vast number of applicants for every such position, the employment must be lucrative, or very desirable for some other reason; but we are not solicitous for the interests of receivers. Certainly the policyholders would be benefited by such reinsurance, and we hope that that plan will be adopted with all the companies now in the hands of receivers.

Life Insurance Acts of the Colonies of Tasmania, New Zealand, and Canada.

We are indebted to Mr. Jas. Valentine, of the Northern Insurance Company, for the following account of the Life Insurance Acts passed in recent years by the Colonies of Tasmania, New Zealand, and Canada.—Ed. *J.I.A.*

Tasmania.

THE Tasmania Act was passed on 18 September 1874, and is substantially the same as that of the neighbouring colony of Victoria, which is reprinted in the *Journal* for October 1876. The following are the only points of difference of any importance.

§ 19. British companies are allowed, in lieu of the abstract of the Actuary's report made in the form prescribed by the Act, which is identical with that laid down in the 8th schedule to the Victoria Act, to deposit a copy of the last preceding abstract of the same nature furnished to the Board of Trade in accordance with the 5th schedule to the British Act, whereby the necessity of making a separate valuation of the business done in the colony is avoided. It may not be uninteresting to mention that a like provision was inserted in a Bill to amend the Victoria Act, which was brought under the consideration of the Legislature of that colony in December 1874, but failed to become law, in consequence of a change of Government taking place soon after, and more pressing questions coming to the front.

§ 36. The rules for the valuation of life policies and annuities (see the 10th schedule to the Victoria Act) are made applicable in Tasmania only to companies which are being wound up. Their extension in Victoria by section 35 of the Act to cases "where application is being made to the Court to wind up a company" establishes a legal standard of solvency in that colony; but it seems

as if this was not originally intended; for section 32, which deals specially with the conditions under which companies may be ordered to be wound up, merely provides that in determining whether the company is insolvent the Court shall take into account the contingent liability under policies, just as the British Act provides. In the amending Bill of which we have spoken, it was proposed to make the same restriction of these rules as in the Tasmania Act.

§ 38. The section protecting the interest of the assured in life policies in certain cases to the extent of £1000 against creditors, contains an explanation, not in the Victoria Act, to the effect that where any policy so protected exceeds £1000, the trustee of the estate of the bankrupt may require the company to cancel the same, and to issue substituted policies of equal standing and aggregate amount; one of these to be for £1000 in favour of the bankrupt or whom he may appoint, and the other or others in favour of the trustee; and in case the company refuses or neglects to issue such new policies, the trustee may, after the expiration of one month, dispose of the original policy, but must pay to the bankrupt a rateable proportion of the proceeds, or, in other words, the sum which represents the surrender value of a policy for £1000 of equal standing.

Sched. 7. The schedule corresponding to the 7th schedule in the Victoria Act requires only the number and amounts of Life, Endowment, and Annuity policies to be stated, without calling for similar particulars in respect of Fire, Marine, and other policies. Here, again, the amending Bill proposed the same limitation in Victoria.

New Zealand.

The New Zealand Act, which was passed on 15 September 1873, differs considerably from those of Victoria and Tasmania, and is in several respects very stringent. It may indeed be said that, if the framers of the measure aimed at making the carrying on of life business almost prohibitory to foreign companies, they have produced an Act well fitted to secure their object.

§§ 21 to 24. The provisions relating to the furnishing of Accounts and Statements seem to be based upon the British Act, the schedules being the same in number and character. The only differences worth pointing out are:—that new and renewal premiums are kept distinct; that the corresponding commissions are also distinct; that certain details are required of the charges

of management, viz., agents' travelling expenses, medical fees, salaries of officials, and advertising; that outstanding claims are divided into—claims admitted but not paid, claims in process of adjustment or adjusted and not due, claims resisted by the company; and that the amount of annuities due and unpaid is required. A separate valuation of the New Zealand policies is not asked for.

Every company carrying on business is required (§ 4) to deposit at once £5,000, and to increase the deposit up to £20,000, by paying annually the amount of premiums received on New Zealand policies, whether issued before or after the commencement of the Act, less a deduction of 25 per-cent therefrom and of the net amount of claims actually paid in respect of such policies. A company may also make voluntary deposits (§ 5) of any further amounts it thinks fit. It is not provided that the deposits are to be returned when the accumulated funds out of premiums amount to a specified sum, as in the British Act and in those of the neighbouring colonies. On the contrary, the monies are to remain while there are any liabilities under the policies. As may be inferred, the conditions under which deposits may be withdrawn are very severe. In the case of a *voluntary* deposit, before the same can be withdrawn, in whole or in part, it is necessary, (§ 17):

(1) to give 6 months' notice in writing.
(2) to satisfy an official called the Public Trustee that, after the proposed withdrawal has been made, the remainder of the securities left in deposit will be equal in value to the amount of the monies assured by current policies.

In the case of a *compulsory* deposit (§ 18), the above conditions must be complied with, and, in addition, the Public Trustee must be satisfied that, from the time of giving the 6 months' notice, the company has ceased to issue new policies. It may be thought that the Act would have gone far enough had it required the Public Trustee to keep in hand so much of the deposit as would cover the *value* of the liabilities under current policies, but the words seem plainly to forbid any withdrawal unless what is left covers the sums assured themselves. The Public Trustee is required to publish every notice of withdrawal, and also his decision, at the cost of the company, in such manner as he thinks fit. § 19. A company, after notice given of withdrawal of *compulsory* deposit, cannot recommence to carry on the business until it has again made the deposit required in the first instance; and such new deposit is in that case to be treated apart from any securities of the original deposit remaining in the hands of the Public Trustee.

§ 9, &c., Deposits are to be held in trust for policyholders, in respect of contracts entered into either before or after the commencement of the Act, but only of such of them as are registered. This requirement of registration is a novel feature, which we do not remember to have come across before. Policies issued before the commencement of the Act must be registered within 12 months after such commencement; and policies issued after the commencement, within 6 months from their date. The registration is to take place with the Public Trustee, and the policyholder is required to pay a fee of five shillings to this functionary, who is empowered to appoint persons in such places as he thinks fit to receive policies from the assured for transmission to him for registration.

§ 15. The Public Trustee must prepare a report annually, or at shorter intervals, if required, of each company which has made a deposit, showing the nature and value of the securities deposited, and such particulars of each policy for which the deposit is held, as will enable the present value of the policy to be computed.

§ 28. Printed copies of the various returns must be forwarded by the company, on application, to every shareholder and policyholder, the Act in this respect following the British one, and differing from those of Victoria and Tasmania, which enjoin the company to forward copies whether applied for or not.

The Act does not contain any clauses for the regulation of amalgamations or transfers of business from one office to another. It provides, however (§ 35), for companies being ordered to be wound up, upon its being proved to the satisfaction of the Court that the funds, with such further capital as can be realized, are not equal to the liabilities, the latter including the net value of its policies according to the 17 Offices' table ("Combined Experience or Actuaries rate of mortality"), with 4 per-cent interest. This estimate of the contingent liabilities is, of course, excessive. It is provided in the British Act of 1872 that values of policies by the same data are to be taken as the measure according to which policyholders are to rank upon the estate of a company actually being wound up, and in the Tasmania Act there is a provision to the same effect, but in neither of these Acts is there any direct provision as to the legal standard of solvency. The Victoria Act, on the other hand, as already mentioned, does prescribe such a standard. However actuaries may differ if asked to lay down a standard of solvency, they will probably all agree that, in the case of any ordinary company which has charged average rates of premium, solvency is quite compatible with a much smaller

reserve than would be required by a net valuation according to the 17 Offices' Experience 4 per-cent table.

There are special provisions relating to foreign companies—that is, to all other than New Zealand companies. Every such company must (§ 40), in writing, appoint a person resident in New Zealand as General Agent, upon whom all lawful processes against the company may be served, and it must (§ 41) be stated in all policies that the company will abide by decisions of the Supreme Court of New Zealand. Policies issued by a foreign company without complying with the provisions of this part of the Act are to be valid (§ 43); but the Agent through whom they are negotiated is to be liable to a penalty not exceeding £250 for each offence; and any company which neglects to appoint and to keep appointed a General Agent is not to be able to recover premiums on policies,—which inability, by itself, as it seems to us, would not be a heavy punishment, since the policies, in the event of the premiums not being paid, would lapse; and we are not aware that any companies claim the right of enforcing the payment of premiums against unwilling policyholders.

There are also special clauses for the protection of life policies (§ 46). The interest of every person who effects a policy *bonâ fide* upon the life of himself, or for any future endowment of his wife or any child, or for an annuity for his wife or any child, and the interest of the personal representative of such wife or child, are made exempt from the claims of creditors, subject to certain restrictions. In the case of a policy for an ordinary assurance or for an endowment, the exemption commences after two years, when the assurance is free to the extent of £200. After 5 years the exemption extends to £500, after 7 years to £1000, and after 10 years to £2000. The protection, in respect of a policy for an endowment, is to be for the benefit only of the nominee; and in respect of an ordinary life assurance, for the benefit only of the personal representative of the assured; not in any case for an assignee of the assured. In the case of a life assurance, if the assured can by the terms of the policy surrender the same, and avails himself accordingly, the surrender value is not to be protected. A policy for an annuity is only protected (1) after 6 years, (2) if the annuity does not exceed £100, (3) for the benefit of the actual annuitant, and (4) to such part as is payable after the annuitant attains the age of 50.

§ 47. Married women are allowed to effect policies, and the same, if expressed to be made for their separate use, are to be free

from the debts or control of their husbands, subject, however, so far as such debts are concerned, to restrictions as to durations and amounts similar to those which have been specified as limiting the protection of policies effected by the husbands themselves.

§ 48. A policy for a life assurance not exceeding £2000 effected by a man for the whole term of his own life for the benefit of his wife, or of his wife and children, or of his children only, is to be deemed a trust for the benefit of wife or children, according to the interest so expressed, and free from the claims of creditors so long as any object of the trust remains. But it is provided that the payment of the premium shall be spread over at least seven years of the husband's lifetime by equal annual, half-yearly, or quarterly payments. In the case of a life assurance, which by the terms of the policy can be surrendered, and is surrendered, the surrender value is not to be paid to the assured, but retained by the company as payment for a paid-up policy of an equivalent amount, to be issued in terms of the original policy, and such paid-up policy is to be free from creditors.

The only approach in England to the protection of life policies from creditors afforded by this Act lies in the provisions of the Married Women's Property Act. This Act, however, being primarily for the benefit of married women, does not protect any policy unless a wife is benefited by it—at all events, to a nominal extent. Policies intended for the exclusive benefit of children, whether of married men or widowers, do not fall within its scope, nor, of course, assurances for the benefit of the assured themselves. The New Zealand Act, however, as we have shown, goes much further, and grants a very wide protection to policies.

Canada.

The Canada Act was passed on 28th April last, "to amend and consolidate certain Acts respecting Insurance" previously in force.

§§ 2 and 3. To be able to transact Life Insurance business in Canada, a company must obtain a licence, which expires on 31st March in each year, but is renewable from year to year.

§ 5. Before a licence can be got, it is necessary to deposit $50,000 (say, £10,000) with the Receiver-General, which may be in Canadian Government securities; or, in the case of a British company, in securities of the United Kingdom; or, in the case of any United States company, in securities of the United States. Other securities may also be accepted, on such terms as the Treasury Board thinks fit. § 9. It is likewise necessary to file a

copy of the company's Charter, Act of Incorporation, or Articles of Association, a power of attorney to its Agent in Canada, in which provision must be made for service of process in legal proceedings, and a statement of the condition of the company at the usual balancing day next preceding. § 13. Any person acting as agent of a company which has no licence is subject to a penalty of $1,000 (say, £200) for each offence, and, in default of payment, is liable to imprisonment for a period not exceeding six months.

§ 6. A company has the option of making further deposits, to be held subject to the same conditions as the original deposit.

§ 7. If it appear at any time, from an examination of the affairs of a company, that its liabilities in Canada (including values of policies) exceed its assets there, the company must make good the deficiency within 60 days, or the licence will be withdrawn. This being the case, it is necessary for foreign companies, as soon as the growth of their business requires it, besides making the deposits which have been mentioned, to keep other monies in Canada, of sufficient amount, with them, to meet the total liabilities in that country. Those other monies must be vested in two or more trustees resident in Canada, who are to be appointed by the company, but must be approved by the Minister of Finance, by whom also the trust-deed must be approved; and the trustees may deal with the monies in any way provided by the deed of trust, so long as the value held by them does not, along with the deposit in the hands of the Receiver-General, fall below the total amount of the liabilities. There is a proviso, however, whereby, in the case of any company which gives notice before 31st March 1878 of its intention to avail itself of the same, policies issued before that date will be exempt from the above requirement; and in that case the deposit in the hands of the Receiver-General before the passing of this Act is to be dealt with in regard to these policies as if the Act had not been passed. The object of the proviso—and it seems a reasonable one—is evidently to save companies which have already liabilities under Canadian policies in excess of their deposit from the necessity of making an additional deposit at once in respect of these policies. Under previous Acts, it may be explained, there was no provision for vesting funds in trustees, nor was a company bound to keep monies in Canada equivalent to the total liabilities there; but, in addition to the deposit of £10,000 required before commencing business, it was compulsory upon companies to hand over annually what was practically the excess of income over expenditure, until the deposit in the

hands of the Receiver-General reached the sum of £20,000. The temporary position of a company availing itself of the proviso will be, that its deposit will be held to be answerable, as far as it goes, for the liabilities under policies issued before 31st March 1878, while funds will have to be placed in the hands of the trustees to meet the values of the policies effected after that date. From time to time, as the liability under the older policies falls below the amount of the deposit, the Minister of Finance is empowered to hand over the excess to the company, until the deposit is reduced to the sum of £10,000 required by this Act, at which figure it must remain with the Receiver-General, and the company will then be in the normal position contemplated by the Act, and required to keep sufficient monies in all in Canada to meet the liabilities under the whole of its policies there.

§ 17. A company licensed under any former Act, which ceases business before the time fixed for the first renewal of its licence under this Act (which we take to be 31st March next), may collect renewal premiums, and otherwise wind up its affairs, as if this Act had not been passed.

§ 20. Every company must furnish annually statements of its condition, showing its assets and liabilities, and its income and expenditure during the previous year, and such other information as may be deemed necessary by the Minister of Finance. As the forms in which these returns are to be made are quite different from those to which we are accustomed in England, or from those in the Australasian Acts, we reprint them here in full. § 21. So far as foreign companies are concerned, particulars only of their Canadian business have to be given in these forms, the particulars of the remainder of the business being accepted in the forms prescribed by the laws of the country in which the head office of the company is situate.

SCHEDULE (A).

DETAILS OF YEARLY STATEMENTS REQUIRED BY § 20.

" A list of the stockholders, with the amount subscribed for, the amount paid thereon, and the residence of each stockholder.

" *Property or Assets held by the company, specifying Assets as per Ledger Accounts.*

" The value (as nearly as may be) of the real estate held by the company.

" The amount secured by way of loan on real estate, whether by mortgages, bonds, or any other security, distinguishing between those having first or second lien on such real estate.

"The amount of loans secured by bonds or stocks, or other collaterals.

"The amount of loans as above on which interest has not been paid within one year previous to such statement, with a schedule thereof.

"The amount of loans made in cash to policyholders on the company's policies assigned as collaterals.

"Premium-notes, loans, or liens, on policies in force, the reserve on each policy being in excess of all indebtedness thereon.

"Par and market values of Canadian and other stocks and securities owned by the company, specifying in detail the amount, number of shares, and the par and market value of each kind.

"Amount of cash at head office.

"Amount of cash in banks, with details.

"Bills receivable.

"Agents' ledger balances.

"*Other Assets.*

"Interest due and accrued.

"Rents due and accrued.

"Due from other companies for losses or claims on policies of the company, reinsured.

"Net amount of uncollected and deferred premiums.

"Commuted commissions.

"All other property owned by the company, with details.

"*Liabilities.*

"Net present value of all outstanding policies in force, with mode of computation or estimation, deducting those reinsured.

"Premium obligations in excess of net values of their policies.

"Claims for death losses and matured endowments, and annuity claims, due and unpaid, or in process of adjustment, or adjusted but not due, or resisted.

"Dividends to stockholders, and dividends of surplus or other profits to policyholders, due and unpaid.

"Amount due on account of office expenses.

"Amount of loans.

"Amount of all other claims against the company.

"*Income.*

"Amount of cash premiums received, less reinsurance. Premium-notes, loans, or liens, taken in part-payment for premiums; and premiums paid by dividends, including reconverted additions, and by surrendered policies.

"Cash received for annuities.

"Amount of interest received.

"Amount received for rents.

"Net amount received for profits on bonds, stocks, and other property actually sold.

"All other income in detail.

"*Premium-Note Account.*

"Premium-notes, loans, or liens, on hand at date of last previous statement.

"Additions and deductions in detail during the year.

"Balance, note-assets at date.

"*Expenditure.*

"Total amount actually paid for losses and matured endowments.

"Cash paid to annuitants and for surrendered policies.

"Premium-notes, loans, or liens, used in purchase of surrendered policies.

"The same voided by lapse.

"Cash surrender values, including reconverted additions applied in payment of premiums.

"Dividends paid to policyholders, or applied in payment of premiums.

"Premium-notes, loans, or liens, used in payment of dividends to policyholders.

"Cash paid stockholders for interest or dividends.

"Cash paid for commissions, salaries, and other expenses of officials.

"Cash paid for taxes, licences, fees, or fines.

"All other expenditures in detail.

"*Exhibit of Policies.*

"Number and amount of policies and additions in full at the end of the previous year.

"New policies and changes.

"Policies terminated, and the manner of termination.

"Number and amount of policies in force at date of statement.

"Reinsurances."

It will be observed that an annual valuation is necessary, as the value of outstanding policies must always be included among the liabilities. This will involve a great amount of labour; greater, we think, than is necessary—a valuation once in five years being in most, if not all cases, enough for practical purposes.

§ 23. A company may, in computing the reserve necessary to be held in respect of its Canadian policies, employ any of the standard tables of mortality used in the construction of its premiums, and any rate of interest not exceeding 4½ per-cent; but if it appears to the Superintendent of Insurance that such reserve falls below that brought out by "the mortality table of the Institute of Actuaries" (by which we presume the H^M table is meant), at 4½ per-cent interest, with certain exceptions which we shall presently mention, the Minister of Finance may direct that the reserve

on that basis, if it differs materially from the return made by the company, may be substituted in the annual statement; so that, practically, the reserve may be greater than the legal standard, but must not be materially less. The exceptions referred to above are (1) That bonus additions declared before the passing of this Act, and then valued at a rate of interest other than 4½ per-cent, may continue to be valued at such other rate; and (2) That policies (apart from bonuses), hitherto valued at 5 per-cent, may continue to be valued at the same rate for the next 10 years.

We have no information as to the circumstances which led to the insertion of these provisions; but we presume that some of the Canadian companies value their policies at 5 per-cent, and their bonuses possibly at a still higher rate. At the present time, it seems scarcely possible for a company to realize 5 per-cent on the whole of its Canadian assets, if the whole or a considerable proportion of the monies in the hands of the Receiver-General be invested in Government Stocks.

It will thus be seen that a net H^M 4½ per-cent reserve is established as the ultimate legal standard of solvency (with certain modifications in the meantime), which, if it be the case that fully that rate of interest can be realized in Canada, is, in common with the standard set up in the Victoria and New Zealand Acts, higher than is necessary.

§ 24(2). An important provision is, that once at least in every five years the Superintendent is required to value the Canadian policies of all companies at H^M 4½ per-cent, with the foregoing exception as to bonuses. This will be a check upon the valuations made by the companies themselves.

§ 24(3). The Minister of Finance is empowered to instruct the Superintendent, from time to time, to visit the head office of any foreign company, and to examine into its condition; and if the company declines to allow this, its licence is to be withdrawn.

§ 18. A company, after giving written notice of its intention to cease business, may procure the transfer, with the consent of the policyholders, of its outstanding policies to some other company licensed under this Act, or may obtain the surrender of as many of the policies as possible; and the trustees may employ any portion of the assets vested in them for these purposes. The company must file lists of the policies which have been thus transferred or surrendered, and of those which have not been so dealt with. It must also publish in the Canada *Gazette* a notice stating that it will apply to Government for the release of its assets on a certain

day, not less than three months after the date of the notice, and calling upon the policyholders who may oppose such release to file their opposition before the day so named. After that day, the Minister of Finance may direct that a portion of the assets be retained "sufficient in amount to cover the full equitable net surrender value" of the policies not transferred or surrendered, or in respect to which opposition has been filed, and order the remainder to be transferred or paid to the company. The values are to be taken by the H^M table, at 4½ per-cent, with the single exception, which seems equitable, that bonuses declared before the passing of the Act, *must* be valued at the rate at which they were originally valued. The portion of the assets retained is to be tendered *pro ratâ*, according to the values of their policies, to the remaining policyholders; and on the acceptance of the amount tendered, the policies shall be deemed to be cancelled. If the tender be refused, the amount may be paid over to the company, and the policy remain in force. The property of a company retiring from business in Canada will thus be returned to it, on certain formalities being complied with, and the dissenting policyholders be left without any special security in Canada. The Act expressly states that such policyholders are not to be barred from any recourse which they may have, in either law or equity, against the company, to compel the fulfilment of the contract; and any policyholder may make special arrangements with the company, whereby his policy may be kept in force; and then the policy may be omitted from the lists of policies, and this Act no longer apply to it. These provisions seem skilfully planned to facilitate the withdrawal of solvent foreign offices from Canada, so as to leave the field open to the operations of the native offices without outside competition.

§ 16. Upon the insolvency of any company, the Court is empowered to appoint an assignee, who is to call upon the company for a statement of its outstanding policies in Canada, and upon the policyholders to file their claims. All policyholders in Canada are to be entitled to claim for the values of their policies, and the assignee may require the Superintendent to calculate the same, taking as his basis the H^M table, and interest at 4½ per-cent (with the exception as to bonuses, which here again is very properly compulsory). When the schedule of claims in Canada is completed, the Court is empowered to cause the assets of the company in Canada, or any part of them, to be realized, and to distribute the proceeds (except what may be applied in effecting a reinsurance in some other

company of the outstanding policies, in whole or in part) *pro ratâ* amongst the claimants, the balance, if any (and there may be a balance in Canada, although the office as a whole is insolvent), to be given up to the company. If the proceeds of the assets are not sufficient to cover all claims in full, the policyholders are not to be barred from any other recourse they may have against the company.

In cases of distribution of the Canadian assets, there is, however, a very important proviso, whereby any Canadian policyholder who "has been insured on the 'mutual' principle", is not to be entitled to the benefit of the special security afforded by the Canadian funds, but must share at the same rate as the other policyholders, if it appear that the Canadian policyholders, under the laws of the country where the head office of the company is situate, are entitled to the full privileges and rights of all the other policyholders. In the absence of any definition of what is meant by the mutual principle, it seems probable that members (or participating policyholders) of mutual societies are exclusively referred to in this provision.

Each company [§ 24(4)] is required to make an annual contribution, in proportion to the gross premiums on its Canadian policies, towards paying the expenses of the Superintendent's Office; and when (§§ 16, 18, and 23) the Superintendent makes a valuation of the company's liabilities under its policies, whether with the view of testing the company's solvency, distributing its assets, or at the request of the company (to save it the trouble of making the periodical valuations itself), a further payment has to be made of three cents (about 1½*d.*) for each policy or bonus addition valued—which is certainly moderate enough.

§ 29. Any company which has commenced to make its deposit under any of the former Acts, repealed by the present one, may continue the same in the manner originally prescribed until the amount reaches £10,000 required by the present Act.

There are no clauses in this Act for the protection of policies against creditors, but under separate Acts such provision is afforded to a similar extent as is done in England by the Married Women's Property Act; and on this point the reader is referred to the article under "Canada" in the first volume of Mr. Walford's *Cyclopædia*.

This Act is for the regulation of Life companies only, but it may be mentioned that there is another Act in force, passed in 1875, for the regulation of Fire and Inland Marine companies.

In conclusion, it may interest our readers to learn that the

Act throughout speaks of life *insurance*—not assurance—the practice in this respect being the same as in the United States of America.

Since this article was written, we have been favored with a copy of a report by the Superintendent to the Minister of Finance, containing, amongst other matter, a synopsis of this Act, in which it is stated that an exact actuarial computation of the reserve is not required every year, an "estimate" of the amount for the years between the dates of the periodical investigations of the company being sufficient. This, of course, will greatly lessen the labor.

CORRESPONDENCE.

ON THE PURCHASE OF A COMPLETE ANNUITY—INVESTIGATION OF FORMULAS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I have had occasion to investigate some formulas for calculations connected with the purchase (as an investment) of a complete annuity, and as they differ, I believe, from the other formulas that have been suggested for use in similar cases, I venture to send you a note of them, that you may insert it in the *Journal* if you consider the matter of sufficient interest.

In purchasing a complete annuity as an investment, the annual income is held to consist of two parts, (1) the interest on the capital advanced, and (2) the premium for assuring its return. It is also assumed that the apportionment receivable at the death of the annuitant will amount on the average to six months' premium and six months' interest. Now as the premium ceases with the payment at the beginning of the year of death, while interest continues to accrue till payment of the sum assured at, say, the $\frac{1}{m}$th part of a year after death, the assurance in connection with an outlay of 1 should be $1-\frac{\pi}{2}+\frac{i}{m}$,—where π is the premium actually payable at the rate P per 1. The value of π is formed from the equation

$$\pi=\mathrm{P}\left(1-\frac{\pi}{2}+\frac{i}{m}\right), \text{ which gives } \pi=\frac{\mathrm{P}\left(1+\frac{i}{m}\right)}{1+\frac{\mathrm{P}}{2}}.$$

Again, out of an outlay of 1 the vendor receives only $1-\pi$, since the first year's premium has to be deducted from the amount; and for this he requires to grant an annuity of $\pi+i$. It follows, therefore, since

$$\pi+i:1-\pi::1:\frac{1-\pi}{\pi+i},$$

that the present value of an annuity of 1 is $\frac{1-\pi}{\pi+i}$, where

$$\pi=\frac{P\left(1+\frac{i}{m}\right)}{1+\frac{P}{2}} \text{ as above.}$$

We have still to find the annual premium and the policy required in connection with the transaction. Calling the former π' and putting $\mathring{a}$ for the present value of the annuity $\left(\frac{1-\pi}{\pi+i}\right)$, we get the equation $\pi'=\pi(\mathring{a}+\pi')$; whence the annual premium $\pi'=\frac{\pi\mathring{a}}{1-\pi}$. Lastly, the sum to be assured $=\frac{\pi'}{P}$, that is $=\frac{\pi}{P}\cdot\frac{\mathring{a}}{1-\pi}$.

It will be obvious that the expressions now given for the price, the premium and the policy may be stated in terms of P. But the resulting formulas are not nearly so practically convenient as those under the present arrangement, where the numerical value of π is ascertained at first, and used throughout all the succeeding calculations.

I may explain, in closing, that my reason for undertaking the investigation was, that I wished to make allowance for the sum assured being payable four months after death; on the assumption that the time between proof of death and payment of the claim was three months, and that one month would be required for proof. In laying the results before your readers, however, I have exhibited the expressions in the more general form.

I am, Sir,

Your most obedient servant,

Edinburgh, December 1877. JAMES SORLEY.

HOME AND FOREIGN INTELLIGENCE.

THE SCOTTISH PROVINCIAL ASSURANCE COMPANY.

Established 1825.

EXTRACT FROM THE REPORT OF THE DIRECTORS.

LIFE INVESTIGATION.

Another Quinquennial period for investigating the business of the Life Department terminated at 31st January last (1877).

The Directors felt bound to employ, in this investigation, the latest and most reliable table of the expectancy of life, and they had no hesitation, after full enquiry into the practice of other offices and the opinion of the best actuaries, in adopting the actuary's view, that the Carlisle Table, formerly employed, cannot be safely relied on. The most recent and best exponent of the value of assured lives, namely, the Institute of Actuaries H^M Table, has been accordingly employed as the basis in valuing the life policies; and the latest Government Annuity Tables (1860) for the valuation of the annuities.

PARTICIPATION BRANCH.

It will be seen by the actuary's report, that on the participation life branch there is a surplus of £48,067. 14*s*. 5*d*.; and had the valuation been made on the basis of the Carlisle Table, this would have been larger by about £38,697. As the immediate effect of the change has been so severe, the Directors propose to apply the whole surplus of £48,067. 14*s*. 5*d*. to the participation policies. They are also satisfied, and propose that a reversionary bonus ought to be declared at the same rate as that in 1872, on the original amounts assured by all policies current at 31st January 1877, not being on an ascending scale of premiums.

The following are the rates of Bonus thus proposed to be declared:—

Retrospective bonuses of

£1 per-cent per annum on original amounts by all participating policies under 5 years' standing.

£1	2	6	do.	of 5 and under	10	years' standing.
1	5	0	do.	of 10 ,,	15	do.
1	7	6	do.	of 15 ,,	20	do.
1	10	0	do.	of 20 years' standing and over.		

It is necessary, according to the actuary's calculations, to provide £45,416. 10*s*. 7*d*. to meet the rate of bonus, in addition to the surplus of £48,067. 14*s*. 5*d*. on the participation branch; but this can be provided for without any undue strain upon the company's resources, by carrying £40,000 from the reserve fund to the credit of the participation life branch, and £6,000 from the general surplus revenue of the year, as after stated.

The Directors beg to report that, following the system which has recommended itself to other large offices, they have resolved to adjust their rates of premiums on new life policies to those found practically safe and necessary; and accordingly new tables are being prepared, based on the approved tables adopted in the investigation. They will also give effect to the actuary's suggestion that no bonus should vest on new policies until they have been five years in existence.

EXTRACTS FROM THE REPORT BY THE ACTUARY (MR. C. GORDON).

The valuation has been made in the form best suited for the purpose of the returns required by the Board of Trade, namely, by showing on the one hand the present value of the sums payable by the company, and on the other, the present value of the net premiums receivable, the difference between these sums being the present liability of the company.

The policies for the whole term of life have been classified according to the age of the assured at the date of the valuation, and the values taken in the middle of the year. Similar classifications have been made in certain of the special classes where the grouping system would not interfere with the accuracy of the result. The rest of the policies have been valued individually.

All the calculations have been made in duplicate, and for the most important classes in triplicate, the results being carefully compared,

and at every stage of the classification of policies, and tabulating of results, every check that could be devised to test the accuracy of entries was brought to bear.

The premium valued as receivable in the future is only the net or risk premium according to the H^M Table, the loading or difference between this and the full premium actually payable being left intact, to meet future expenses and to form a source of profit for succeeding investigations. This margin averages 30·06 per-cent on participation, and 13·02 on non-participation policies. In addition to the values thus brought out, I have reserved one-half of the year's income from extra premiums to meet unexpired extra risks, and have also set aside, in respect of paid-up policies and those whose premiums cease after a limited number of payments, a sum sufficient to put these on a similar footing, as regards future profits and expenses, to those policies where the premiums are payable and the margins consequently receivable by the company during the whole continuance of the risk.

Throughout the whole valuation every principle whereby negative values could be produced has been studiously avoided.

In order that the Directors may see the effect of the change from the Carlisle Table employed at former investigations to the H^M, I have made a valuation by the former table, equally strict as regards the largest and most important classes, and approximating to the values of the more special policies, and find the net difference between the two valuations to be as follows, the H^M Table showing the higher reserve:—

Participation Branch	£38,697	16	0
Non-Participation Branch	6,023	4	0
	£44,721	0	0

The immediate annuities for whole term of life were classified in the same manner as the policies—the sexes of annuitants being distinguished, and the annuities for each sex valued by the corresponding table. The annuities for special terms were valued individually. The results of the valuation of annuities by the Government Tables and $3\frac{1}{2}$ per-cent interest, are in total almost identical with what would have been reserved by the Carlisle Table and 3 per-cent, the basis of former valuations, the present valuation being higher by £37.

Memorandum of Results of Investigation.

PARTICIPATION BRANCH.

This Branch comprises 9,784 Policies, assuring £3,699,223, to which £224,003 of Reversionary Bonuses have been added, and yielding an annual revenue of £112,871. 17*s.* 2*d.* of ordinary premiums and £963. 9*s.* of extras. The reassurances are for £167,585, with £10,686 of bonus additions. The sum of £5,695. 2*s.* 6*d.* is payable annually as premiums for the said reassurances.

The state of affairs is as follows:—

				£	s.	d.
Participation Life Fund				621,599	15	6
Bonus Fund (after payment of £5,115. 11*s*. of prospective bonuses during the Quinquennial Period) . .				116,136	0	11
				737,735	16	5
Net Liability under Policies . .	£568,532	1	0			
Liability for Bonuses . . .	120,258	3	0			
Half-year's Income from extras .	473	12	0			
Additional Reserve in respect of Limited Payment Policies, see report	404	6	0			
				689,668	2	0
Surplus				£48,067	14	5

NOTE.—To provide a bonus at the same rates as declared at the valuation in 1872 would require a sum of £93,484. 5*s*.

NON-PARTICIPATION BRANCH.

This Branch comprises 1,269 Policies, assuring £673,330, and Contingent Annuities of £131. 4*s*. 8*d*. The annual revenue is £21,717. 10*s*. 5*d*. of ordinary premiums and £233. 17*s*. 10*d*. of extras. The reassurances are for £96,821, the annual premiums thereon being £3,291. 12*s*. 1*d*.

The state of affairs is as follows:—

				£	s.	d.
Non-Participation Life Fund				118,141	5	11
Net Liability under Policies .	£123,099	4	2			
Extra Premiums, as above . .	116	19	0			
Additional Reserve in respect of Limited Payment Policies .	57	14	6			
				123,278	17	8
Deficiency				£5,132	11	9

NOTE.—The total annual margin reserved for future expenses and profit, in terms of the report, amounts to £27,901. 2*s*., the present value of which is £379,190.

ANNUITY FUND.

This Branch comprises 175 policies, assuring annuities of £4,220. 11*s*. 10*d*.

The state of affairs is as follows:—

				£	s.	d.
Annuity Fund				40,615	11	0
Net Liability under Policies .	£38,889	6	2			
Payments outstanding on 31st January 1877 . . .	68	6	8			
				38,957	12	10
Surplus				£1,657	18	2

DECLARATION OF BONUS.

The vested additions to policies for £1000 are as under:—

Policies opened before Annual Balance in	Bonus Vested at 10th Feb. 1847.			10th Feb. 1852.			31st Jan. 1857.			31st Jan. 1862.			31st Jan. 1867.			31st Jan. 1872.			31st Jan. 1877.			Total Vested Bonuses in 1877.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
1841	87	10	0	67	19	5	79	8	9	84	18	0	68	15	0	75	0	0	75	0	0	538	11	2
1848	...			62	10	0	73	0	11	78	1	4	68	15	0	75	0	0	75	0	0	432	7	3
1853	...			...			68	15	0	73	9	6	68	15	0	68	15	0	75	0	0	354	14	6
1858	...			...			...			68	15	0	68	15	0	62	10	0	68	15	0	268	15	0
1863	...			...			...			...			68	15	0	56	5	0	62	10	0	187	10	0
1868	...			...			...			...			...			50	0	0	56	5	0	106	5	0
1873	...			...			...			...			...			...			50	0	0	50	0	0

The following additional particulars are extracted from the Returns under the 5th and 6th schedules of the Life Assurance Companies Act, 1870.

Consolidated Revenue Account for the five years ending 31*st Jan.* 1877.

	£	s.	d.
Amount of Funds on 31st Jan. 1872, the beginning of the quinquennial period	662,993	0	11
Premiums (after deduction of Reassurance Premiums)	582,877	15	4
Consideration for Annuities granted	16,127	17	11
Interest and Dividends	154,294	11	9
	£1,416,293	5	11
Claims under Policies (after deduction of Sums re-assured)	351,205	18	5
Surrenders	41,940	12	7
Annuities	19,907	11	1
Commission	28,702	15	5
Expenses of Management	57,187	2	2
Medical Fees and Policy Stamps	5,213	7	3
Dividends and Bonuses to Shareholders	...	...	
Expenses applicable to the Investigations of 1872 and 1877	609	9	0
Profit belonging to Shareholders, as shown by Investigation of 1872, and transferred to General Profit and Loss Account, April 1872	15,033	16	8
Amount of Funds on 31st January 1877, the end of the period	896,492	13	4
	£1,416,293	5	11

SUMMARY *and* VALUATION (see p. 460).

There are no investments specially set apart for the Life Assurance and Annuity Funds, but these accounts have been accredited annually throughout the quinquennium with interest at 4 per-cent.

The minimum values allowed for the surrender of policies for the whole term of life, with uniform annual premiums, on which three full years' premiums have been paid, are 40 per-cent of the ordinary

Summary and Valuation of the Policies of the Scottish Provincial Assurance Company, as at 31st January 1877.

DESCRIPTION OF TRANSACTIONS.	PARTICULARS OF THE POLICIES FOR VALUATION.				VALUATION. Value by Hm and Government Annuitants' Tables, 1860—Interest 3½ per-Cent.			
	Numbers of Policies.	Sums Assured and Bonuses.	Office Yearly Premiums.	Net Yearly Premiums.	Sums Assured and Bonuses.	Office Yearly Premiums.	Net Yearly Premiums.	Net Liability.
ASSURANCES.								
I.—With Participation in Profits.		£	£	£	£	£	£	£
Whole Term—Uniform Premiums . .	8,057	3,379,339	94,005	72,786	1,630,859	1,330,484	1,018,277	612,582
Do. Limited Payments . .	231	102,283	2,893	2,189	45,580	25,289	19,086	26,494
* * * *	*	*	*	*	*	*	*	*
Total Assurances, with Profits . .	9,784	3,923,226	113,835	86,859	1,908,011	1,560,883	1,189,251	719,234
II.—Without Participation in Profits.								
Whole Term—Uniform Premiums . .	907	479,275	16,064	14,451	269,618	182,390	162,052	107,566
Do. Limited Payments . .	21	5,491	58	48	3,121	313	260	2,861
* * * *	*	*	*	*	*	*	*	*
Total Assurances, without Profits .	1,269	673,330	21,951	19,194	351,992	240,615	211,125	140,984
Total Assurances	11,053	4,596,556	135,786	106,053	2,260,003	1,801,498	1,400,376	860,218
Deduct Reassurances . . .	...	275,092	8,986	7,326	140,582	114,776	92,814	47,738
Net amount of Assurances	...	4,321,464	126,800	98,727	2,119,421	1,686,722	1,307,532	812,480
Additional Reserve for Expenses on Paid-up and Limited Payment Policies .	...	...	...	...	...	...	...	462
	11,053	4,321,464	126,800	98,727	2,119,421	1,686,722	1,307,532	812,942

premiums received by the office, increased by the cash value—according to the published bonus table issued by the company—of whatever bonuses may have been added thereto.

The minimum surrender value allowed for Endowment Assurances is also 40 per-cent of the premiums paid, it being however the practice of the Office to allow on their surrender, *two-thirds* or upwards of the Office Reserve, according to their standing on the books; and in the case of Participation Policies, in addition, the cash value of any bonuses that may have been declared thereon.

We take this opportunity of printing the following extracts from the Reports of the Directors at previous investigations of the Company.

1. As at 31 January 1862:—

From the profits realized on the Life Business, the Directors recommend that a retrospective bonus of £1. 7*s*. 6*d*. per-cent per annum, for the last five years, be given on the sums originally assured, increased by the previous bonuses added thereto, on all Participation Policies in force at 31st January last, for the number of full years standing of the respective assurances since 31st January 1857.

Also that a further prospective bonus of £1 per-cent per annum on the amount of the policies with said bonuses, be declared on all such of said policies as may become claims by death before the next investigation at 31st January 1867, for every complete year for which premiums may have been paid. The present value of the bonuses just referred to is £31,690. 15*s*.

The claims by death, including bonus additions, paid during the quinquennial period, have amounted to £93,975. 6*s*. 6*d*.

The total number of policies in force at 31st January last was 4,759, assuring the sum of £1,955,950.

In valuing the policies it will be noticed that each has been separately computed, and that nett premiums only have been used, thus leaving a surplus of annual revenue of £11,447. 16*s*. 7*d*., to meet future expenses and to form a source of profit for next investigation.

From the report of the Consulting Actuary (Mr. J. J. Downes) it appears that the respective values of the sums assured, and the *nett* values *only* of the future premiums, were computed to the nearest month; and that in all cases the age was taken as that for which the premium was charged at the date of entry.

That the law of mortality and the rate of interest employed were the same as were adopted in the construction of the company's tables of assurance premiums,—namely, Carlisle 3½ per-cent—except for immediate annuities.

"The effect of taking credit for the *nett* values of the future premiums only, is to limit any surplus that may arise to the amount of profits *actually realized*.

"This principle places old and new assurers upon an equal footing

in regard to future bonuses, as the present assured will contribute towards the future expenses and contingencies in a like ratio to new members; while new assurers have a guarantee of a fair and full participation in future profits, in a just proportion to their contribution to the common fund.

"If, in reporting the results of the valuation, it should be decided to state the present value of the sums assured on one side, and the present value of the future premiums on the other, I would suggest that it should be stated that those results were not obtained by the application of any lumping process founded upon an assumption of an average age, but that they are the aggregates of the separate results obtained for each individual policy."

Statement of the Company's Business during the Quinquennial Period ending 31*st January* 1862.

Year ending 31st January.	Amount of New Policies Issued.	No. of Policies Issued.	New Annual Premiums.
	£		£ s. d.
1858	136,180	356	3,989 18 0
1859	169,171	477	5,492 1 6
1860	283,920	659	8,776 17 9
1861	270,746	736	7,971 17 6
1862	359,680	981	11,170 1 1
Totals .	1,219,697	3,209	37,400 15 10

Memorandum of Result of Investigation at 31*st January* 1862.

	£ s. d.	£ s. d.
I.—PARTICIPATION LIFE FUND	150,939 8 10	
Value of Policies, as per Abstract of Valuation States	111,813 11 10	
Surplus . .		39,125 17 0
II.—BONUS FUND	19,866 16 3	
Value of Bonuses, as per said Abstract .	17,661 11 3	
Surplus . .		2,205 5 0
III.—NON-PARTICIPATION LIFE FUND . .	74,175 17 7	
Value of Policies, as per said Abstract .	69,870 9 3	
Surplus . .		4,305 8 4
DEDUCT—		
VII.—ANNUITANTS' FUND	30,344 3 3	
Value of Policies, as per Valuation States	30,624 2 9	
		£279 19 6

The rate of bonus now declared is equivalent in the case of the older policies to £1. 14*s*. per-cent per annum on the original sum assured.

2. As at 31 January 1867:—

The valuations and relative calculations have been conducted on the same principles as were adopted on last occasion, as then recommended by Mr. James John Downes.

The Directors recommend that £55,527. 7*s.* 10*d.* be set apart to declare a retrospective bonus at the rate of £1. 7*s.* 6*d.* per-cent per annum, for the last five years, on the original sums assured. on all policies on the Participation Branch, not on an ascending scale of premiums, in force at 31st January last, for their number of full years standing on the books since 31st January 1862.

During the last five years, the Office has issued 4,845 policies, assuring £2,092,973, and yielding an annual revenue of £62,452. 2*s.* 2*d.*

The claims by death during the Quinquennial period amount to £172,963. 3*s.*

The total number of life policies in force at 31st January last was 7,662, assuring the sum of £3,250,000—yielding in annual premiums £95,263. 11*s.* 11*d.*

Memorandum of Result of Investigation.

	£	*s.*	*d.*	£	*s.*	*d.*
I.—Participation Life Fund	260,789	16	4			
Value of Policies, as per Abstract of Valuation States	229,413	12	0			
Surplus . .				31,376	4	4
II.—Bonus Fund	42,228	17	2			
Value of Bonuses, as per said Abstract .	39,829	5	6			
Surplus . .				2,399	11	8
III.—Non-Participation Life Fund . .	91,896	7	11			
Value of Policies, as per said Abstract .	86,537	9	0			
Surplus . .				5,358	18	11
IV.—Annuitants' Fund	37,048	14	1			
Value of Policies, as per Valuation States	36,470	3	10			
Surplus . .				£578	10	3

Note.—In the valuation of policies, the *pure* premiums only having been used, a surplus of annual revenue, amounting to £18,433. 12*s.* 3*d.*, remains unvalued, to meet future charges and to form a source of profit for next investigation.

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3. As at 31 January 1872:—

Quinquennial Investigation.

As on previous occasions, the valuations and relative calculations have been conducted on the same principles as were adopted in 1862 (and again in 1867), on the recommendation of Mr. James John Downes, late Actuary of the Economic Life Assurance Society of London, each policy having been separately valued in duplicate, and the calculations carefully compared. The valuations were based on the Carlisle Table of Mortality, and the rate of interest assumed was $3\frac{1}{2}$ per-cent. The net or pure premiums have alone been dealt with, and the loading, amounting to £22,052. 1*s.* 10*d.* *per annum*, has been reserved, in accordance with the practice of the company, to meet future charges, and to form a source of profit for next investigation.

In the Participation Branch, the balance is £75,459. 17*s.* 5*d.*, as per memorandum attached. From this sum the Directors propose to apply *one-tenth* part, namely, £7,545. 19*s.* 9*d.* to the credit of Profit and Loss, £67,584. 10*s.* 3*d.* to provide for bonuses, as follows, and to carry forward the balance of £329. 7*s.* 5*d.*

Retrospective bonuses of—

£1 per-cent per annum on all participating policies under 5 years' standing.

£	s.	d.			
£1	2	6	do.	do.	of 5 and under 10 years' standing.
1	5	0	do.	do.	of 10 and under 15 years' standing.
1	7	6	do.	do.	of 15 and under 20 years' standing.
1	10	0	do.	do.	of 20 years' standing and over.

Such bonus to be for the last five years on the original sums assured, on all policies on the Participation Branch, *not being on an ascending scale of premiums* (these are separately dealt with in regard to bonus), in force at 31st January last. The £1 per-cent applicable to the policies on the first of the above periods to be according to the number of full years premiums paid thereon as at that date.

The surpluses on the Non-Participation Branch and on the Annuity Department amounted respectively to £3,114. 3*s.* 5*d.* and £4,373. 13*s.* 6*d.*

Memorandum of Results of Investigation.

	£	s.	d.	£	s.	d.
I.—PARTICIPATION LIFE FUND (after payment of £3,740 for Prospective Bonuses during the quinquennial period)				420,419	10	7
Additional Interest for the 5 years, to bring the rate of accumulation up to 4½ per-cent				8,055	12	1
Increase on the Value of Investments				8,900	0	0
				437,375	2	8
Value of Policies, as per Abstract of Valuation States				364,730	6	10
Surplus				72,644	15	10
Bonus Fund	£73,783	3	6			
Additional Interest, as above	4,324	1	5			
	78,107	4	11			
Value of Bonuses, as per said Abstract	75,292	3	4			
Surplus				2,815	1	7
Total Surplus on Participation Branch				£75,459	17	5
II.—NON-PARTICIPATION FUND				103,700	18	2
Additional Interest, as above				2,422	1	3
				106,122	19	5
Value of Policies, as per said Abstract				103,008	16	0
Surplus				£3,114	3	5
III.—ANNUITY FUND				41,387	13	11
Value of Policies, as per said Abstract				37,014	0	5
Surplus				£4,373	13	6

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END OF VOL. XX.

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THE

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TO THE

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PREFACE.

In drawing up the following index to the first 20 volumes of the *Journal of the Institute of Actuaries and Assurance Magazine*, I have adopted the plan which my experience has led me to believe would be most useful to persons who wish to refer to any particular paper, or to ascertain what has been written upon any subject which they are at the time studying. With this view, I have laid down the general rule that every paper is to be indexed at least twice,—(1) under the author's name, and (2) under the subject.

Under the author's name are arranged in order of date all the communications from him, whether long or short, whether they appeared in the form of a paper, a short note, or a letter to the Editor; and in each case the exact title adopted by the author is given, some explanation being occasionally added when this seemed desirable, in order to indicate more clearly the nature of the subject dealt with in the communication. At the end of these communications are arranged any other entries that it appeared desirable to make under the author's name, such, for instance, as translations by him of foreign articles, reviews of works written by him, and, in the case of the two Editors, editorial remarks.

It has been a much more troublesome matter to arrange the articles according to the nature of the subject; and in doing this I have had to be guided principally by my own judgment, and to a comparatively small extent by the wording of the title of the paper. For instance, the numerous papers upon the subject of the graduation of life tables are arranged under the heading of "Graduation", although in the titles of some of them that word does not appear, the process being

called "Adjustment". Under the heading "Adjustment of Tables", I have given a cross reference to "Graduation". In cases of this kind, where references to many papers are collected under one heading, I have contented myself with giving the author's name and the volume and page of his article, leaving the reader to refer to the author's name in the index if he wishes for further particulars.

In many cases, references to a particular paper have been given under two or three different headings descriptive of the subject.

In addition to references of the kind just named, I have endeavoured to give under each paper references to subsequent papers by different authors in which the paper is mentioned or discussed. Although a great deal of time and trouble has been bestowed upon this part of the work, I can scarcely hope that it has been done with any approach to completeness; but I hope that what has been done will be found to be of material assistance to students and others consulting the index.

It may now be useful to mention some of the headings in the index under which the entries are most numerous.

Bonus (or Investigation) Reports. Under this heading are placed the names of all the companies whose bonus or investigation reports have appeared in the *Journal* since 1867, when the plan was adopted of systematically printing these reports under the title of "Home and Foreign Intelligence". As I stated in the *Journal* (October 1867), I believe that such reports have for various reasons a permanent interest attaching to them; but since the Life Assurance Companies Act, 1870, has made it compulsory on all the life offices in the country to publish full particulars regarding their valuations and distributions of profits, which particulars are annually published by the Board of Trade in the form of a Blue-book, it has seemed undesirable to continue the practice of reprinting all such reports of British companies in the *Journal*; and, during recent years, only a few of more than usual interest have been reprinted, more particularly such as contained information not given in the Blue-books. I have, however, printed a considerable number of bonus reports of Australian offices, which are not so readily accessible as those of

British companies. The names of the companies in the index are arranged alphabetically under the above-mentioned heading, and I have considered it unnecessary to insert the name of the company a second time in its alphabetical place in the index.

CORRESPONDENCE *(anonymous)*. Only those letters appear here to which the author's name was not appended. All letters which were signed by the author, have been entered, like other communica-cations, under his name.

DISCUSSIONS. Under this heading are given references to the abridged reports of the discussions at the Meetings of the *Institute*, which have appeared in the *Journal* since 1869, and for which, as is well known, we are indebted to the public spirit of the proprietors of the *Insurance Record*. The full reports of the discussions given in that paper were, as they appeared, carefully revised and condensed, the persons who took part in the discussion being sometimes referred to for assistance when this was thought desirable; and although this involved a considerable expenditure of time and labour on the part of myself and the gentlemen who kindly assisted me, I believe that the time and labour were well bestowed, as the reports of the discussions have been found very useful for reference, and are generally appreciated by students of the *Journal*. The various discussions are arranged in the alphabetical order of the names of the authors whose papers were discussed.

FIRE INSURANCE. Here are inserted references to all the communications on the subject that have appeared in the *Journal*. A similar course has been followed as regards MARINE INSURANCE; but not with the corresponding heading LIFE INSURANCE, because the greatest part of the *Journal* is occupied with this subject. Instead of this, I have given the headings, ANNUITIES, ANNUITY- (*certain*, *continuous*, *increasing*, &c., &c.), ASSURANCE, PREMIUMS, VALUES OF POLICIES, and others of a like kind.

FOREIGN INTELLIGENCE. Under this general heading are entered all the numerous articles as to Fire and Life Insurance business in foreign countries, with the exception of the reports of companies, which are put under the general heading REPORTS OF COMPANIES—FOREIGN. The entries are divided under the sub-headings, *Life*

Insurance, Fire Insurance, Marine Insurance, Miscellaneous Branches of Insurance, General Articles; and again, under each of these, according to the names of the country to which they relate, *Austria, Belgium, France,* &c.

MORTALITY. The various articles relating to this subject have been arranged under the headings (1) *Mortality Experience of various Offices* (the names of Offices being placed alphabetically); (2) *Mortality, General Remarks;* (3) *Mortality Observations,* (being observations as to the rate of mortality in different countries and among different classes of persons); (4) *Mortality Tables, Construction of.*

ORIGINAL TABLES. A great number of original tables have been at various times contributed to the *Journal;* and as the use of these will be much facilitated by having a complete list of them in a convenient form, I have bestowed much pains in the description of them under the above general heading. I first give an alphabetical list of the names of the persons whose tables are printed in the *Journal,* with a reference to the volume and page. I then, in a second division, give an alphabetical list of the subjects to which the tables relate, adding the author's name and the volume and page. In a third division, arranged in chronological order, I have given, besides the author's name, a full description of each table, containing such particulars as I thought likely to be of service to persons using the index.

REPORTS OF COMPANIES. In the early volumes of the *Journal* it was usual to give summaries of the annual reports of Insurance Offices, both British and Foreign. References to all these summaries are entered under the above general heading with the three sub-headings—*British—Colonial and Indian—Foreign,* the last being subdivided under the names of the various countries. In each section the names of the companies are arranged alphabetically. As in the case of Bonus Reports, I have considered it unnecessary to enter the names of the companies again in their alphabetical place in the index.

REVIEWS. Under this heading are arranged in alphabetical order the names of all the authors whose works have been reviewed in the *Journal,* each being followed by the title of the book.

The task of preparing the index has been extremely laborious; but I undertook it and persevered with it in spite of many hindrances which have greatly delayed the completion of the work, because I believed that it was very important it should be done by a person familiar with the subject, and that the value of the work to the profession and the student, would be thereby greatly increased. I trust that this opinion will be shared and confirmed by those who shall hereafter have occasion to consult the index; and if so, I shall feel well rewarded for the labour and time the preparation of it has cost me.

T. B. SPRAGUE.

EDINBURGH,
28 *March* 1883.

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